Turbulent Transport in Tokamak Plasmas with Rotational Shear

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Nonlinear gyrokinetic simulations are conducted to investigate turbulent transport in tokamak plasmas with rotational shear. At sufficiently large flow shears, linear instabilities are suppressed, but transiently growing modes drive subcritical turbulence whose amplitude increases with flow shear. This leads to a local minimum in the heat flux, indicating an optimal $\mathbf{E} \times \mathbf{B}$ shear value for plasma confinement. Local maxima in the momentum fluxes are observed, implying the possibility of bifurcations in the $\mathbf{E} \times \mathbf{B}$ shear. The critical temperature gradient for the onset of turbulence increases with flow shear at low flow shears; at higher flow shears, the dependence of heat flux on temperature gradient becomes less stiff. The turbulent Prandtl number is found to be largely independent of temperature and flow gradients, with a value close to unity.

Introduction.—Experimental measurements in magnetic confinement fusion devices indicate that sheared mean $\mathbf{E} \times \mathbf{B}$ flows can significantly reduce and sometimes fully suppress turbulent particle, momentum, and heat fluxes [1,2]. Since these fluxes determine mean plasma density and temperature profiles, their reduction leads to a local increase in the profile gradients. This increase can be dramatic: transport barriers in both the plasma core and edge have been measured with radial extents of only tens of ion Larmor radii [3]. The associated increase in core density and temperature results in increased fusion power. Thus, understanding how shear flow layers develop and what effect they have on turbulent fluxes is both physically interesting and practically useful.

This Letter reports a numerical study of the influence of sheared toroidal rotation on turbulent heat and momentum transport in tokamak plasmas. Two main effects of sheared rotation were identified in previous numerical work [4–9]: suppression of turbulent transport by shear in the perpendicular (to the mean magnetic field) velocity and linear destabilization due to the parallel velocity gradient (PVG). While the former observation indicates that a finite flow shear improves plasma confinement, the latter raises the question of whether more shear is always beneficial. Prior simulation results with low to moderate flow shear indicated that the PVG linear growth rate increases monotonically with flow shear [6,9]. Below we report that the PVG-driven linear instability [10] is stabilized at larger flow shear values, leading to a dip in the heat flux. However, transiently growing modes driven by the PVG eventually give rise to subcritical turbulence for sufficiently large flow shear. The fluxes associated with this subcritical turbulence increase with flow shear, leading to an optimal value of flow shear for each temperature gradient comparable to that found in many experimental transport barriers [11].

In the absence of flow shear, a small increase in temperature gradient leads to a large increase in heat flux ("stiff transport"). Recent experimental evidence [12] suggests that flow shear may reduce this sensitivity in configurations with low magnetic shear. Our results indicate that at low flow shears, both the critical temperature gradient for the onset of turbulence and the stiffness increase. At high flow shears, the opposite behavior is observed (stiffness and critical temperature gradient both decrease).

Model.—A closed set of fluid conservation equations determines the evolution of mean plasma density, flow, and pressure in tokamak plasmas [13,14]. The fluxes appearing in these equations are typically dominated by turbulent contributions, which we calculate here. We restrict our attention to electrostatic fluctuations and assume a modified Boltzmann response [15] for the electron distribution. The particle flux then vanishes, and the radial components of the turbulent heat flux of species s, Q_s, and toroidal angular momentum flux, Π, are

$$Q_s = \overline{\int d^3\mathbf{v}\left(\frac{m_s v^2}{2}\right)\left(\mathbf{v}_E \cdot \frac{\nabla\psi}{|\nabla\psi|}\right)\delta f_s}, \qquad (1)$$

$$\Pi = \overline{\sum_s m_s R^2 \int d^3\mathbf{v}(\mathbf{v} \cdot \nabla\phi)\left(\mathbf{v}_E \cdot \frac{\nabla\psi}{|\nabla\psi|}\right)\delta f_s}, \quad (2)$$

with R the major radius of the torus, ψ a flux-surface label, ϕ the toroidal angle, m_s the particle mass, $\mathbf{v}$ its velocity in the mean flow frame, $\mathbf{v}_E$ the fluctuating $\mathbf{E} \times \mathbf{B}$ drift velocity, δf_s the deviation of the distribution function from a local Maxwellian, and the overline a spatial average over a thin annulus encompassing a flux surface.

The distribution function δf_s is calculated by solving the standard δf gyrokinetic equation in the limit where the plasma flow speed, u, is ordered comparable to the ion

thermal speed, $v_{th} \equiv \sqrt{T/m}$, with T the species temperature [14,16]. In this "high-flow" regime, the flow velocity is constrained to be $\mathbf{u} = R^2 \omega(\psi)\nabla\phi$, where ω is the rotational frequency [17]. To focus on the effect of flow shear, we consider the limit in which the flow is small, but the flow shear is large (comparable to the fluctuation frequency). We thus ignore effects arising from the centrifugal and Coriolis forces. Using $(\mathbf{R}, E, \mu, \vartheta)$ coordinates, with $\mathbf{R}$ the guiding center position, $E = mv^2/2$, $\mu = mv_\perp^2/2B$, and ϑ the gyro angle, the gyrokinetic equation is

$$\frac{d\langle\delta f\rangle}{dt} + (v_\parallel \hat{\mathbf{b}} + \mathbf{v}_B + \langle\mathbf{v}_E\rangle)\cdot\nabla\left(\langle\delta f\rangle + \frac{e\langle\varphi\rangle}{T}F_0\right)$$
$$= \langle C[\delta f]\rangle - \langle\mathbf{v}_E\rangle\cdot\nabla\psi\left(\frac{\partial F_0}{\partial\psi} + \frac{mv_\parallel}{T}\frac{RB_\phi}{B}\frac{d\omega}{d\psi}F_0\right),$$
$$(3)$$

with $d/dt = \partial/\partial t + \mathbf{u}\cdot\nabla$, e the particle charge, φ the electrostatic potential, $\mathbf{v}_E = (c/B^2)\mathbf{B}\times\nabla\varphi$, F_0 a Maxwellian distribution in the frame rotating with frequency ω, C the collision operator, B the magnetic field strength, B_ϕ its toroidal component, $\mathbf{v}_B = \hat{\mathbf{b}}/\Omega \times (v_\parallel^2\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}} + v_\perp^2\nabla B/2B)$, Ω the Larmor frequency, $\hat{\mathbf{b}} = \mathbf{B}/B$, and $\langle.\rangle$ the average over ϑ at fixed $(\mathbf{R}, E, \mu)$.

Equation (3) is solved in the rotating reference frame using the local, nonlinear gyrokinetic code GS2 [18]. Flow shear enters the calculation through the convective time derivative, which depends on the shear in the perpendicular flow, and the $d\omega/d\psi$ drive term in Eq. (3), which accounts for the shear in the parallel flow. Because the flow is purely toroidal, these two components are related by the geometric factor (qR_0/r) [9]. The flow shear is thus controlled by a single parameter, $\gamma_E = (\psi/q)(d\omega/d\psi)R_0/\sqrt{2}v_{th} = (M/q)(d\ln\omega/d\ln r)$, with q the safety factor, R_0 the major radius at the center of the flux surface, and r the half-diameter of the flux surface (both measured at the height of the magnetic axis).

We study a system whose magnetic geometry corresponds to the widely used Cyclone base case [19] (unshifted, circular flux surface with $q = 1.4$, magnetic shear $\hat{s} = d\ln q/d\ln r = 0.8$, $r/R_0 = 0.18$, and $R_0/L_n = 2.2$, where $L_n^{-1} = -d\ln n/dr$). The shearing rate, γ_E, and the normalized inverse temperature gradient scale length, $\kappa \equiv R_0/L_T$, were varied over a wide range of values in a series of linear and nonlinear simulations.

Linear stability.—In tokamaks, mean $\mathbf{E}\times\mathbf{B}$ shear advects fluctuations through regions where the magnetic field line curvature alternates between stabilizing and destabilizing. The resulting modes, called Floquet modes, exhibit periodic oscillations superposed on mean growth or decay (the time domain analog of Bloch states in periodic media). The mean linear growth rates, γ, obtained from our simulations are given in Fig. 1. We find that γ decreases discontinuously when γ_E increases from zero [20] before increasing to a local maximum and subsequently decreasing to zero. For $\gamma_E \gtrsim 0.25$, the system is linearly stable
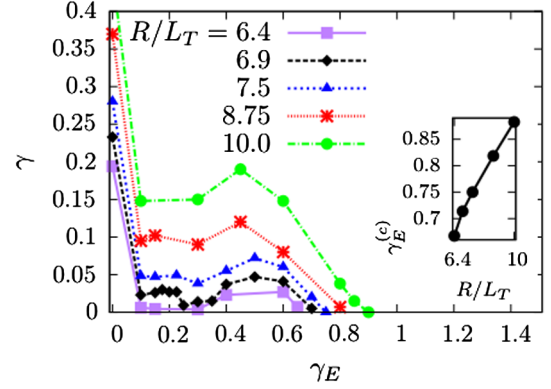

FIG. 1 (color online). Average linear growth rates (normalized by R_0/v_{th}) vs flow shear. Inset: Critical flow shear vs R_0/L_T.

unless both the ion temperature gradient and the PVG are nonzero (cf. [6–9]). The system becomes linearly stable for larger values of γ_E, with the critical γ_E for stability, γ_{Ec}, increasing approximately linearly with κ. Beyond γ_{Ec}, there are no linearly unstable modes, which is similar to the result from fluid theory in a slab [21]. While γ_{Ec} is larger than the γ_E values found for standard operating conditions in many tokamaks, it is comparable to or smaller than the γ_E values observed in many transport barriers [11].

Heat flux.—The turbulent heat flux calculated from nonlinear simulations is given in Fig. 2. For all κ values, the heat flux follows the same trend as growth rates when $\gamma_E < \gamma_{Ec}$. For $\gamma_E > \gamma_{Ec}$, nonlinear simulations initialized with low-amplitude noise develop no turbulent transport. However, for finite initial fluctuation amplitudes, the turbulence does not necessarily decay away: for sufficiently large values of κ and initial amplitude, the flux reaches steady state values in excess of those found for γ_E just below γ_{Ec} [22]. The flux then increases monotonically with γ_E. For the range of γ_E considered here, the heat flux associated with a given temperature gradient is thus minimized at a finite value of flow shear.

Subcritical turbulence.—Because the turbulence is present in the absence of linear instability, we refer to it as subcritical. When $\gamma_E > \gamma_{Ec}$, our linear simulations exhibit transient growth, with order unity increases in the initial fluctuation amplitudes over times of several R_0/v_{th} before subsequent decay (Fig. 3). The duration of growth decreases with increasing γ_E, but the transient growth rate increases so that the amplification factor of the initial perturbation amplitude grows with flow shear, consistent with analytic theory [21]. This provides an energy source for the turbulence, which can be maintained by the nonlinearity through redistribution of energy among other modes. As seen in Fig. 2, the γ_E beyond which subcritical turbulence is maintained depends on κ (and likely on magnetic geometry as well).

The phenomenon reported here is physically distinct from the subcritical turbulence observed previously (cf. [6]), which is sustained by the transient growth of
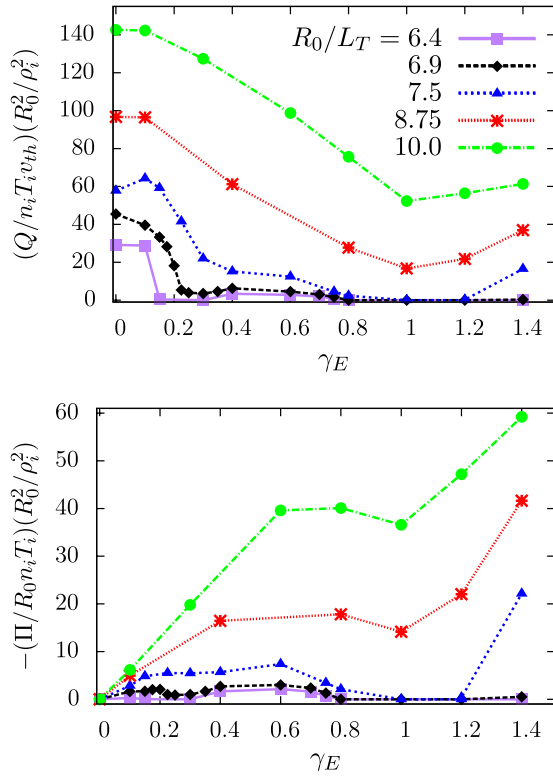
FIG. 2 (color online).   Turbulent heat (top) and toroidal angular momentum (bottom) fluxes vs flow shear for a range of R_0/L_T values. For $\gamma_E > \gamma_{Ec}$ (see Fig. 1), the turbulence is subcritical, sustained by transiently growing modes.

Floquet oscillations as modes are advected through the unfavorable curvature region by the $\mathbf{E} \times \mathbf{B}$ flow. The subcritical turbulence presented here is driven by the PVG term in Eq. (3) ($d\omega/d\psi$ term on the right-hand side), as the turbulence disappears when the PVG is artificially set to zero; thus, it is present even for a straight magnetic field. This is physically similar to subcritical turbulence in Couette and Poiseuille flows, where strong flows drive turbulence in the absence of linear (and nonlinear) instabilities [23].

Momentum flux.—The momentum flux also mimics the linear growth rate, except near $\gamma_E = 0$ [24]. This is understood by expressing the momentum flux as $\Pi = -m_i v_{\text{th}}(qR_0/r)\nu_i(\kappa, \gamma_E)\gamma_E$, where ν_i is the turbulent viscosity. For small γ_E, ν_i is approximately constant and $\Pi \propto \gamma_E$. For larger γ_E, turbulent amplitudes drop rapidly, so Π decreases, resulting in the local maxima seen in Fig. 2. This suppression of Π is due to linear stabilization as γ_E approaches γ_{Ec} from below. The momentum flux increases monotonically when $\gamma_E > \gamma_{Ec}$ due to subcritical turbulence. The maxima in Π may lead to a bifurcation in flow shear, discussed in the Conclusions.

Prandtl number.—The turbulent Prandtl number can be calculated from the values of Q_i and Π. It is defined as $\text{Pr} = \nu_i/\chi_i$, where the turbulent thermal diffusivity, χ_i, is given by $Q_i = -\chi_i dT_i/dr$. Figure 4 shows that Pr is approximately independent of κ and only has strong dependence on γ_E for small values of γ_E [25]. This is despite the fact that both ν_i and χ_i individually have strong dependence on κ and γ_E. For $\gamma_E \gtrsim 0.4$ the Prandtl number is close to unity, in good agreement with experimental measurements at low Mach numbers [27].

Stiff transport.—A serious impediment to confinement is the strong sensitivity of the heat flux to small changes in κ. This stiffness of the transport makes it difficult to increase κ, and therefore the core temperature, beyond the critical value, κ_c, at which turbulence is excited. Recent experimental results indicate that the stiffness, $\partial Q_i/\partial\kappa$, may be reduced at low magnetic shear and large values of γ_E [12]. For the Cyclone base case considered here ($\hat{s} = 0.8$), we find a complicated dependence of stiffness on flow shear, shown in Fig. 5. At low values of γ_E ($\lesssim 0.3$), the critical κ shifts to higher values, but the stiffness increases. For $\kappa/\gamma_E \to \infty$, the heat flux tends to the curve corresponding to $\gamma_E = 0$; since κ_c increases with γ_E, the average value of $\partial Q_i/\partial\kappa$ increases with γ_E.
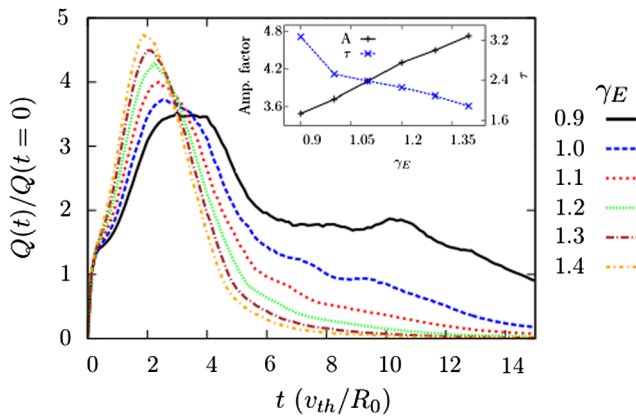

FIG. 3 (color online).   Heat flux versus time obtained from linear simulations for $R_0/L_T = 8.75$ and several γ_E values at which subcritical turbulence is observed. Inset: Transient amplification factor, A, and growth time, τ, versus γ_E.
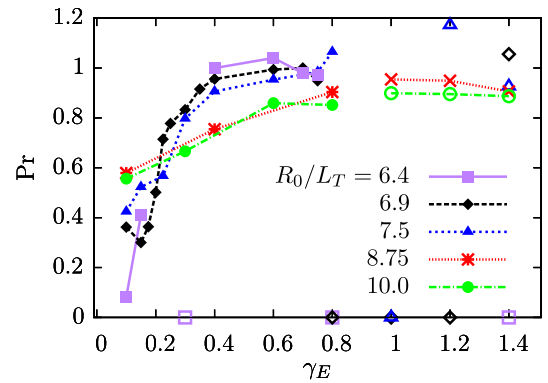

FIG. 4 (color online).   Turbulent Prandtl number as a function of flow shear for a range of R_0/L_T values. Unfilled points correspond to linearly stable flow shear values. Zero Pr points represent fully suppressed turbulence.
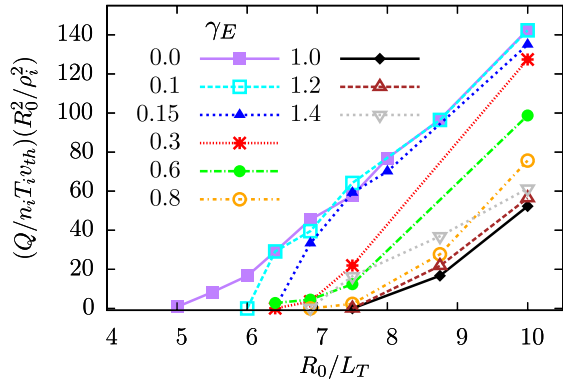
FIG. 5 (color online).   Turbulent heat flux vs R_0/L_T for various
values of the flow shear. Though there is some minimal relaxa-
tion of the profile stiffness for $0.3 \lesssim \gamma_E \lesssim 0.8$, the predominant
effect of the shear is to shift the critical temperature gradient.

For $0.3 \lesssim \gamma_E < \gamma_{Ec}$, both κ_c and the profile stiffness
decrease. This is a result of the change in nature of the
linear instability, which is now driven by the PVG. The
relaxation of stiffness is modest, and it only occurs for κ
near κ_c. When $\gamma_E > \gamma_{Ec}$, κ_c initially shifts upwards with
increasing γ_E, and stiffness increases for κ near κ_c.
However, for even larger γ_E, both the stiffness and κ_c
decrease. This is to be expected, as the subcritical turbu-
lence is driven by the velocity, not temperature, gradient
and thus has a weaker dependence on κ. Finally, Fig. 5
shows that, for the parameters considered here, $\gamma_E = 1$ is
the overall optimal value of flow shear, for which any given
input Q results in the largest possible κ.

Conclusions.—We have shown that PVG-driven turbu-
lence exists at large flow shears, but only if the initial
fluctuation amplitudes are sufficiently large. This has a
number of potentially important implications. First, it in-
dicates that linear stability analysis is insufficient to deter-
mine critical gradients in rotating plasmas. It also implies
the existence of an optimal flow shear for confinement
corresponding to each temperature gradient. Furthermore,
it shows that the system can undergo hysteresis: an equi-
librium state with large flow shear and temperature gra-
dient (e.g., $\gamma_E = 1$ and $R_0/L_T = 10$) has large turbulent
fluxes if the temperature gradient is obtained at low flow
shear, where fluctuations grow due to linear instability and
are maintained by subcritical turbulence drive when flow
shear is increased. If a large flow shear is obtained before
increasing the temperature gradient, then the initial fluc-
tuation amplitudes will not be large enough to achieve
subcritical turbulence. In experiment a certain heat flux is
maintained via external sources, so the only way to main-
tain the quiescent solution is for the temperature gradient to
get sufficiently large that the heat flux is collisional.

The existence of local maxima of the toroidal angular
momentum flux provides the potential for bifurcations in
γ_E [28] and corresponding bifurcations in temperature
gradient. However, a detailed transport analysis is neces-

sary to study such a bifurcation for experimentally relevant
conditions [29]. Finally, because the ratio of the $\mathbf{E} \times \mathbf{B}$
suppression and PVG drive term is proportional to (qR_0/r)
[9], our results, which should be qualitatively robust, may
undergo considerable quantitative variation with changes
in magnetic configuration.

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