Compressible Turbulence: The Cascade and its Locality

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We prove that interscale transfer of kinetic energy in compressible turbulence is dominated by local interactions. In particular, our results preclude direct transfer of kinetic energy from large-scales to dissipation scales, such as into shocks, in high Reynolds number turbulence as is commonly believed. Our assumptions on the scaling of structure functions are weak and enjoy compelling empirical support. Under a stronger assumption on pressure dilatation cospectrum, we show that mean kinetic and internal energy budgets statistically decouple beyond a transitional conversion range. Our analysis establishes the existence of an ensuing inertial range over which mean subgrid scale kinetic energy flux becomes constant, independent of scale. Over this inertial range, mean kinetic energy cascades locally and in a conservative fashion despite not being an invariant.

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Turbulence is a phenomenon that pervades most liquid, gas, and plasma flows in engineering and nature, ranging from high-speed engines, nuclear fusion power reactors, and spacecraft reentry, to star formation in molecular clouds, and supernovae. While the traditional Richardson-Kolmogorov-Onsager picture is a successful theory of incompressible turbulence, all aforementioned systems are characterized by significant compressibility effects. We will lay a rigorous framework in [1] to study scale coupling in compressible flows and to analyze transfer of kinetic energy between different scales. The purpose of this Letter is to explore if such transfer of energy takes place through a cascade process and whether the cascade is scale local.

Kolmogorov's 1941 theory of incompressible turbulence makes the fundamental assumption of a scale-local cascade process in which modes all of a comparable scale $\sim\ell$ participate predominantly in the transfer of energy across scale ℓ. If, furthermore, the cascade steps are chaotic processes then it is expected that any "memory" of large-scale features of the system, such as geometry and large-scale statistics, or the specifics of microscopic dissipation, will be "forgotten." This gives rise to an inertial scale-range over which turbulent fluctuations have universal statistics and the flow evolves under its own internal dynamics without *direct* communication with the largest or smallest scales in the system.

Therefore, scale locality of the cascade is crucial to justify the existence of universal statistics and to warrant the concept of an inertial range. It is, furthermore, necessary for the physical foundation of large-eddy simulation (LES) modeling of turbulence. It motivates the belief that models of subscale terms in the equations for large scales can be of general utility, independent of the particulars of turbulent flows under study. While scale locality in incompressible turbulence stands on firm theoretical [2,3] and numerical [4,5] grounds, no similar results exist for compressible turbulence. In fact, there is a widespread belief especially common in the astrophysical literature which maintains that a "finite portion" of energy at a given scale must be dissipated directly into shocks through nonlocal transfer in scale (see, for example, [6]). Moreover, the idea of a cascade itself is without physical basis since kinetic energy is not a global invariant of the inviscid dynamics. Hence, the notion of an inertial cascade-range in compressible turbulence remains tenuous and unsubstantiated.

In this Letter, we prove under modest assumptions that transfer of kinetic energy is indeed local in scale. Under a stronger assumption, we will further show that kinetic energy cascades conservatively despite not being an invariant. We reach these results by a direct analysis of the compressible Navier Stokes equations, without use of any closure approximation. The equations are those of continuity and momentum:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0,$$

$$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla P + \mu \nabla \cdot \left(\nabla \mathbf{u} + \frac{1}{3} \nabla \cdot \mathbf{u}\, \mathbf{I} \right) + \rho \mathbf{f},$$

and either internal or total energy, supplemented with an equation of state for the fluid. Here, $\mathbf{f}$ is an external acceleration field stirring the fluid, and we have assumed a constant dynamic viscosity, μ.

Our analysis is based on a coarse-graining (or filtering) approach expounded in [1], in which we observe that any scale decomposition aimed at studying inertial-range dynamics must satisfy an *inviscid criterion*; i.e., it must guarantee that viscous momentum diffusion and kinetic energy dissipation are negligible at large scales. We prove in [1] that a Favre decomposition meets such a requirement. Using classically filtered fields, $\bar{\mathbf{a}}_\ell(\mathbf{x}) \equiv \int d^3\mathbf{r}\, G_\ell(\mathbf{r})\mathbf{a}(\mathbf{x}+\mathbf{r})$, with kernel $G_\ell(\mathbf{r}) = \ell^{-3}G(\mathbf{r}/\ell)$ that is smooth and decays sufficiently rapidly for large r, a Favre filtered field is weighted by density as $\tilde{\mathbf{a}}_\ell(\mathbf{x}) \equiv \overline{\rho \mathbf{a}}_\ell(\mathbf{x})/\bar{\rho}_\ell(\mathbf{x})$. In what follows, we take the liberty of dropping subscript ℓ. From coarse-grained continuity and momentum equations, we can write down a large-scale kinetic energy (KE) budget,

$$\partial_t \bar{\rho} \frac{|\tilde{\mathbf{u}}|^2}{2} + \nabla \cdot \mathbf{J}_\ell = -\Pi_\ell - \Lambda_\ell + \bar{P}_\ell \nabla \cdot \bar{\mathbf{u}}_\ell - D_\ell + \epsilon^{\mathrm{inj}}.$$

The KE budget describes instantaneous kinetic energy evolution at every point $\mathbf{x}$ in the flow and at scales $>\ell$, for arbitrary ℓ. Our approach, therefore, allows the simultaneous resolution of dynamics both in scale and in space, and admits intuitive physical interpretation of all terms. Here $\mathbf{J}_\ell(\mathbf{x})$ is spatial transport of large-scale kinetic energy, $-\bar{P}\nabla \cdot \bar{\mathbf{u}}$ is large-scale pressure dilatation, $D_\ell(\mathbf{x})$ is viscous dissipation acting on scales $>\ell$, and $\epsilon^{\mathrm{inj}}(\mathbf{x})$ is the energy injected due to external stirring (see [1] for details). We prove in [1] that $D_\ell(\mathbf{x})$ is negligible at scales $\ell \gg \ell_\mu$, where ℓ_μ denotes the dissipation scale. We have also shown that mean kinetic energy injection can be localized to the largest scales $L \gg \ell$ by proper stirring. Over an intermediate scale range $L \gg \ell \gg \ell_\mu$, the only relevant terms in the large-scale KE budget are inertial processes. The subgrid scale (SGS) flux terms are defined as

$$\Pi_\ell(\mathbf{x}) = -\bar{\rho}\partial_j \tilde{u}_i \tilde{\tau}(u_i, u_j), \qquad \Lambda_\ell(\mathbf{x}) = \frac{1}{\bar{\rho}} \partial_j \bar{P}\, \bar{\tau}(\rho, u_j),$$

and act as sinks in the large-scale KE budget, transferring large-scale kinetic energy to scales $<\ell$. Deformation work, Π_ℓ, is due to large-scale strain, $\nabla \tilde{\mathbf{u}}_\ell$, acting against turbulent stress, $\bar{\rho}_\ell \tilde{\tau}_\ell(\mathbf{u}, \mathbf{u}) = \bar{\rho}(\widetilde{(\mathbf{u}\mathbf{u})}_\ell - \tilde{\mathbf{u}}_\ell \tilde{\mathbf{u}}_\ell)$, while baropycnal work, Λ_ℓ, is due to large-scale pressure-gradient force, $\nabla \bar{P}_\ell / \bar{\rho}_\ell$, acting against turbulent mass flux, $\bar{\tau}_\ell(\rho, \mathbf{u})$ (see [1] for a more detailed discussion of the physics). We employ the notation $\bar{\tau}_\ell(f, g) \equiv \overline{(fg)}_\ell - \bar{f}_\ell \bar{g}_\ell$ for 2nd order central moments of fields $f(\mathbf{x})$, $g(\mathbf{x})$ [7].

There are three facts crucial for proving scale locality of interscale transfer. First is the observation that deformation work, Π_ℓ, and baropycnal work, Λ_ℓ, represent the only two processes capable of direct transfer of kinetic energy *across* scales. Pressure dilatation, $-\bar{P}_\ell \nabla \cdot \bar{\mathbf{u}}_\ell$, does not contain any modes at scales $<\ell$ and does not vanish in the absence of subscale fluctuations. It, therefore, cannot participate in transferring kinetic energy directly across scales and only contributes to conversion of large-scale kinetic energy into internal energy. This observation allows us to circumvent analyzing the internal energy budget which does not couple to large-scale kinetic energy via viscous dynamics, as we prove in [1].

The remaining two parts of our proof build upon previous studies in incompressible hydrodynamic [3] and magnetohydrodynamic [8] turbulence, with some technical modifications. The second ingredient we use is the fact that SGS kinetic energy flux across ℓ, $\Pi_\ell + \Lambda_\ell$, depends on the fields only through their increments, $\delta f(\mathbf{x}; \mathbf{r}) = f(\mathbf{x} + \mathbf{r}) - f(\mathbf{x})$, for separation distances $|\mathbf{r}| < \ell$ (or some moderate multiple of ℓ) and does not depend on the absolute field $f(\mathbf{x})$. Baropycnal work, Λ_ℓ, can be expressed in terms of increments by noting that gradient fields and central moments are related to increments as

$$\nabla \bar{f}_\ell = \mathcal{O}[\delta f(\ell)/\ell], \qquad f'_\ell = \mathcal{O}[\delta f(\ell)],$$
$$\bar{\tau}_\ell(f, g) = \mathcal{O}[\delta f(\ell) \delta g(\ell)], \tag{1}$$

where symbol $\mathcal{O}$ stands for "same order-of-magnitude as," and $f'_\ell = f - \bar{f}_\ell$ is the fine-scale field. For rigorous details, see [3,9]. In order to express deformation work, Π_ℓ, in terms of increments, we need the following identities which are straightforward to verify:

$$\nabla \tilde{\mathbf{u}} = \nabla \bar{\mathbf{u}} + \bar{\rho}^{-1} \nabla \bar{\tau}(\rho, \mathbf{u}) - \bar{\rho}^{-2} \bar{\tau}(\rho, \mathbf{u}) \nabla \bar{\rho},$$

$$\tilde{\tau}(\mathbf{u}, \mathbf{u}) = \bar{\tau}(\mathbf{u}, \mathbf{u}) + \bar{\tau}(\rho, \mathbf{u}, \mathbf{u})/\bar{\rho} - \bar{\tau}(\rho, \mathbf{u}) \bar{\tau}(\rho, \mathbf{u})/\bar{\rho}^2.$$

We are finally able to express Π_ℓ in terms of increments using (1) and two additional relations,

$$\nabla \bar{\tau}_\ell(f, g) = \mathcal{O}[\delta f(\ell) \delta g(\ell)/\ell],$$
$$\bar{\tau}_\ell(f, g, h) = \mathcal{O}[\delta f(\ell) \delta g(\ell) \delta h(\ell)], \tag{2}$$

whose rigorous details are in our longer work [9]. The relation of 3rd order central moments, $\bar{\tau}(f, g, h) \equiv \overline{(fgh)}_\ell - \bar{f}_\ell \bar{\tau}_\ell(g, h) - \bar{g}_\ell \bar{\tau}_\ell(f, h) - \bar{h}_\ell \bar{\tau}_\ell(f, g) - \bar{f}_\ell \bar{g}_\ell \bar{h}_\ell$, to increments is unpublished and due to Eyink [10].

Since Π_ℓ and Λ_ℓ can be expressed in terms of velocity, pressure, and density increments, it thus becomes sufficient to show that these increments themselves are scale-local. To establish this, we need the third requirement crucial for locality—that scaling properties of structure functions are constrained by

$$\|\delta \mathbf{u}(\mathbf{r})\|_p \sim u_{\mathrm{rms}} A_p (r/L)^{\sigma_p^u}, \qquad 0 < \sigma_p^u < 1, \tag{3}$$

$$\|\delta P(\mathbf{r})\|_p \sim P_{\mathrm{rms}} B_p (r/L)^{\sigma_p^P}, \qquad \sigma_p^P < 1, \tag{4}$$

$$\|\delta \rho(\mathbf{r})\|_p \le \rho_{\mathrm{rms}} C_p (r/L)^{\sigma_p^\rho}, \qquad 0 < \sigma_p^\rho \tag{5}$$

for some dimensionless constants A_p, B_p, and C_p. The root-mean-square of a field $f(\mathbf{x})$ is denoted by $f_{\mathrm{rms}} \equiv \langle f^2 \rangle^{1/2}$, where $\langle \ldots \rangle$ is a space average. Here, the pth power of an L_p norm $\| \cdot \|_p^p = \langle | \cdot |^p \rangle$ is just the traditional structure function. For example, $\sigma_p^u = 1/3$ within Kolmogorov's 1941 theory. We remark that condition (5) on the scaling of density increments is only an upper bound. It only stipulates that the intensity of density fluctuations decays at smaller scales, which is a very mild requirement and is readily satisfied in incompressible or nearly incompressible flows. Heuristically, assumptions (3)–(5) characterize the roughness of fields: $\sigma^f < 1$

specifies that the field $f(\mathbf{x})$ is "rough enough", while $\sigma^f > 0$ states that $f(\mathbf{x})$ is "smooth enough."

Under conditions (3)–(5), proving scale locality of the SGS flux becomes simple and follows directly from scale-locality of increments [3]. For example, the contribution to any increment $\delta f(\ell)$ from scales $\Delta \geq \ell$ is represented by $\delta \bar{f}_\Delta(\ell)$. Here, $f(\mathbf{x})$ can denote either velocity or pressure field. Since the low-pass filtered field $\bar{f}_\Delta(\mathbf{x})$ is smooth, its increment may be estimated by Taylor expansion, and (1) and (3) or (4), as

$$\|\delta \bar{f}_\Delta(\ell)\|_p \simeq \|\ell \cdot (\nabla \bar{f}_\Delta)\|_p = O\left[\left(\frac{\ell}{\Delta}\right)^{1-\sigma_p^f}\right], \qquad (6)$$

and this is negligible for $\Delta \gg \ell$ as long as $\sigma_p^f < 1$. The notation $O(\ldots)$ denotes a big-O upper bound. On the other hand, the contribution to any increment $\delta f(\ell)$ from scales $\delta \leq \ell$ is represented by $\delta f'_\delta(\ell)$. Here, $f(\mathbf{x})$ can denote either velocity or density field. Since $f'_\delta = \mathcal{O}[f(x+\delta) - f(x)]$ from (1), scaling conditions (3) or (5) imply

$$\|\delta f'_\delta(\ell)\|_p \leq 2\|f'_\delta\|_p = O\left[\left(\frac{\delta}{\ell}\right)^{\sigma_p^f}\right], \qquad (7)$$

and this is negligible for $\delta \ll \ell$ as long as $\sigma_p^f > 0$. For more details and for the careful proofs of these statements, see [3] and our longer work [9].

Notice that, unlike for the velocity and pressure fields, we do not stipulate that $\rho(\mathbf{x})$ be "rough enough." Contributions to the flux across scale ℓ from the largest density scales $L \gg \ell$ need not be negligible, yet the flux can still be scale local. The underlying physical reason is simple; an energy flux across scale ℓ at a point $\mathbf{x}$ will depend on the mass in a ball of radius ℓ around $\mathbf{x}$. Mass is proportional to average density, $\bar{\rho}_\ell(\mathbf{x})$, in the ball which is dominated by large scales: $\bar{\rho}_\ell(\mathbf{x}) = \mathcal{O}[\bar{\rho}_L(\mathbf{x})] = \mathcal{O}[\rho_{\mathrm{rms}}]$. Furthermore, we do not require that the pressure field be "smooth enough" even though we expect $\sigma_p^P > 0$. This is because pressure only appears as a large-scale pressure-gradient in Λ_ℓ, with no contributions from scales $\delta \ll \ell$.

The ultimate source of scaling properties (3)–(5) is empirical evidence from numerical simulations, experiments, and astronomical observations. Several independent numerical studies of compressible turbulence at supersonic turbulent Mach numbers such as [11–13] report power-law scaling exponents well within our required constraints, $0 < \sigma_p^\rho$ and $0 < \sigma_p^u < 1$ for $1 \leq p \leq 6$. Alongside numerical evidence, astronomical observations by [14] show that $\sigma_2^\rho \doteq 0.3$, and measurements from molecular clouds by [15,16] also yield $0 < \sigma_p^u < 1$ for $1 \leq p \leq 6$.

Under an additional assumption concerning the cospectrum of pressure dilatation, which is, albeit reasonable, not as weak as scaling conditions (3)–(5), our proof of a scale-local SGS flux implies a scale-local *conservative* cascade of mean kinetic energy despite the latter not being an invariant. The requirement on pressure dilatation cospectrum, $E^{\mathrm{PD}}(k) \equiv \sum_{k-0.5<|\mathbf{k}|<k+0.5} -\hat{P}(\mathbf{k})\widehat{\nabla \cdot \mathbf{u}}(-\mathbf{k})$, is that it decays fast enough at large k,

$$|E^{\mathrm{PD}}(k)| \leq C u_{\mathrm{rms}} P_{\mathrm{rms}} (kL)^{-\beta}, \qquad \beta > 1. \qquad (8)$$

Here, C is a dimensionless constant and L is an integral scale. In the limit of large Reynolds number, assumption (8) implies that *mean* pressure dilatation, $\mathrm{PD}(\ell) \equiv -\langle \bar{P}_\ell \nabla \cdot \bar{\mathbf{u}}_\ell \rangle$, converges to a finite constant, $\theta \equiv -\langle P \nabla \cdot \mathbf{u} \rangle$, and becomes independent of ℓ. In other words, we have for wave number $K \approx \ell^{-1}$,

$$\lim_{\ell \to 0} \mathrm{PD}(\ell) = \lim_{K \to \infty} \sum_{0<k<K} E^{\mathrm{PD}}(k) = \theta. \qquad (9)$$

We remark that condition (8) is sufficient but not necessary for the convergence of $\mathrm{PD}(\ell)$ in the limit of $\ell \to 0$. The series $\sum_{k<K} E^{\mathrm{PD}}(k)$ can converge with $K \to \infty$ at a rate faster than what is implied by assumption (8) due to indefiniteness in the sign of $E^{\mathrm{PD}}(k)$.

If mean pressure dilatation saturates as in (9), then this would imply that its role is to exchange *large-scale* mean kinetic and internal energy over a transitional conversion scale-range. At smaller scales beyond the conversion range, mean kinetic and internal energy budgets statistically decouple. In other words, taking $\ell_\mu \to 0$ first, then $\ell \to 0$, steady-state mean kinetic energy budget becomes,

$$\langle \Pi_\ell + \Lambda_\ell \rangle = \langle \epsilon^{\mathrm{inj}} \rangle - \theta. \qquad (10)$$

We stress that such a decoupling is statistical and does not imply that small scales evolve according to incompressible dynamics. Small scale compression and rarefaction can still take place pointwise, however, they yield a vanishing contribution to the space average.

We denote the largest scale at which such statistical decoupling occurs by ℓ_c. It may be defined, for instance, as $\ell_c \equiv \sum_k k^{-1} E^{\mathrm{PD}}(k) / \sum_k E^{\mathrm{PD}}(k)$. We expect ℓ_c to depend on the scale at which an external compressive forcing is applied. It may also be a decreasing function of Mach number and/or the ratio of compressive-to-solenoidal components of the velocity field. Over the ensuing scale range, $\ell_c > \ell \gg \ell_\mu$, net pressure dilatation does not play a role, and if, furthermore, $\langle \epsilon^{\mathrm{inj}} \rangle$ in (10) is localized to the largest scales as shown in [1], then $\langle \Pi_\ell + \Lambda_\ell \rangle$ will be a constant, independent of scale ℓ.

A constant SGS flux implies that mean kinetic energy cascades conservatively to smaller scales, despite not being an invariant of the governing dynamics. This is one of the main conclusions of this Letter. In particular, kinetic energy can only reach dissipation scales via the SGS flux, $\Pi_\ell + \Lambda_\ell$, through a scale-local cascade process. We are therefore justified in calling scale-range $\ell_c > \ell \gg \ell_\mu$ the inertial range of compressible turbulence.

Needless to say, the scaling of pressure dilatation cospectrum is easily measurable from numerical simulations. We note that condition (8) does not require a power-law scaling—only that $E^{\mathrm{PD}}(k)$ decays at a rate faster than $\sim k^{-1}$. It is not at all trivial why one should expect $\mathrm{PD}(\ell) = -\langle \bar{P}_\ell \nabla \cdot \bar{\mathbf{u}}_\ell \rangle$ to converge at small scales. How can this be reconciled with the expectation that compression, as quantified by $\nabla \cdot \mathbf{u}$, would get more intense at smaller scales? Indeed, [17] observed numerically that $(\nabla \cdot \mathbf{u})_{\mathrm{rms}}$ is an increasing function of Reynolds number. The key point here is that our assumption (8) concerns *spatially averaged* pressure dilatation. It is true that $\nabla \cdot \mathbf{u}(\mathbf{x})$, being a gradient, derives most of its contribution from the smallest scales in the flow. Since $P\nabla \cdot \mathbf{u}$ is not sign definite, however, major cancellations can occur when space averaging. The situation is very similar to helicity cospectrum in incompressible turbulence, where the pointwise vorticity, $\boldsymbol{\omega}(\mathbf{x}) = \nabla \times \mathbf{u}$, can also become unbounded in the limit of infinite Reynolds number. Yet, numerical evidence shows that $\langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle$ remains finite and the helicity cospectrum decays at a rate $\sim k^{-n}$, with $n \approx 5/3 > 1$ [18,19].

We can offer a physical argument on why $\mathrm{PD}(\ell)$ is expected to converge for $\ell \to 0$ as a result of cancellations from space-averaging. The origin of such cancellations can be heuristically explained using decorrelation effects very similar to those studied in [5,20]. While pressure in $\langle P\nabla \cdot \mathbf{u} \rangle$ derives most of its contribution from the largest scales, $\nabla \cdot \mathbf{u}$ is dominated by the smallest scales. Therefore, pressure varies slowly in space, primarily at scales $\sim L$, while $\nabla \cdot \mathbf{u}$ varies much more rapidly, primarily at scales $\ell_\mu \ll L$, leading to a decorrelation between the two factors. More precisely, the pressure $\bar{P}_\ell$ in $\mathrm{PD}(\ell)$ may be approximated by $\bar{P}_\ell = \mathcal{O}[P_{\mathrm{rms}}] = \mathcal{O}[\bar{P}_L]$ such that

$$\langle \bar{P}_\ell \nabla \cdot \bar{\mathbf{u}}_\ell \rangle \approx \langle \bar{P}_L \nabla \cdot (\overline{(\bar{\mathbf{u}}_\ell)}_L + (\bar{\mathbf{u}}_\ell)'_L) \rangle$$
$$\approx \langle \bar{P}_L \nabla \cdot \bar{\mathbf{u}}_L \rangle + \langle \bar{P}_L \rangle \langle \nabla \cdot (\bar{\mathbf{u}}_\ell)'_L \rangle.$$

The first term in the last expression follows from $\overline{(\bar{\mathbf{u}}_\ell)}_L \approx \bar{\mathbf{u}}_L$, while the second term is due to an approximate statistical independence between $\bar{P}_L$ and $\nabla \cdot (\bar{\mathbf{u}}_\ell)'_L \sim \delta u(\ell)/\ell$ which varies primarily at much smaller scales $\sim \ell \ll L$. If there is no transport beyond the domain boundaries or if the flow is either statistically homogeneous or isotropic, we get $\langle \nabla \cdot (\bar{\mathbf{u}}_\ell)'_L \rangle = 0$. The heuristic argument finally yields that pressure dilatation,

$$\mathrm{PD}(\ell) = \langle \bar{P}_\ell \nabla \cdot \bar{\mathbf{u}}_\ell \rangle \approx \langle \bar{P}_L \nabla \cdot \bar{\mathbf{u}}_L \rangle, \qquad (11)$$

becomes independent of ℓ, for $\ell \ll L$. Expression (11) corroborates our claim that the primary role of pressure dilatation is conversion of *large-scale* kinetic energy into internal energy and does not take part in the cascade dynamics beyond a transitional conversion scale range.

In summary, we conclude that there exists an inertial range in high Reynolds number compressible turbulence over which kinetic energy reaches dissipation scales through a conservative and scale-local cascade process. This precludes the possibility for transfer of kinetic energy from the large-scales directly to dissipation scales, such as into shocks, at arbitrarily high Reynolds numbers as is commonly believed. We make several assumptions and predictions which are amenable to empirical scrutiny. Our locality results concerning the SGS flux can be verified in a manner very similar to what was done in [5,8]. We also invite empirical tests of assumption (8) on the scaling of pressure dilatation cospectrum. Preliminary numerical results by [21] of compressible isotropic turbulence indicate that indeed the cospectrum decays at a rate faster than k^{-1}. Verifying (8) or (9) under a variety of controlled conditions would substantiate the idea of statistical decoupling between mean kinetic and internal energy budgets. In a follow-up study, we shall show through rigorous analysis and physical reasoning how the scaling of velocity, density, and pressure structure functions can be inferred from relation (10).

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