## New Measurement of the $\pi^0$ Radiative Decay Width

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High precision measurements of the differential cross sections for  $\pi^0$  photoproduction at forward angles for two nuclei,  $^{12}\mathrm{C}$  and  $^{208}\mathrm{Pb}$ , have been performed for incident photon energies of 4.9–5.5 GeV to extract the  $\pi^0 \to \gamma\gamma$  decay width. The experiment was done at Jefferson Lab using the Hall B photon tagger and a high-resolution multichannel calorimeter. The  $\pi^0 \to \gamma\gamma$  decay width was extracted by fitting the measured cross sections using recently updated theoretical models for the process. The resulting value for the decay width is  $\Gamma(\pi^0 \to \gamma\gamma) = 7.82 \pm 0.14 (\mathrm{stat}) \pm 0.17 (\mathrm{syst})$  eV. With the 2.8% total uncertainty, this result is a factor of 2.5 more precise than the current Particle Data Group average of this fundamental quantity, and it is consistent with current theoretical predictions.

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The  $\pi^0 \to \gamma \gamma$  decay represents one of the key processes in the anomaly sector of QCD. This process provides the main test of the chiral anomaly [1,2] and at the same time a test of the Nambu-Goldstone nature of the  $\pi^0$  meson. The  $\pi^0 \to \gamma \gamma$  decay amplitude is determined by the chiral anomaly resulting from the coupling of quarks to the electromagnetic field. In the limit of vanishing quark masses

(chiral limit) the amplitude is exactly predicted and is expressed in terms of the fine structure constant, the  $\pi^0$  decay constant, and the number of colors of QCD [1,2]. In the real world there are corrections due to the nonvanishing quark masses. These corrections are primarily a result of state mixing effects in the  $\pi^0$  meson that result from the isospin symmetry breaking by  $m_u < m_d$  [3,4]. The corrections have

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been analyzed in the framework of chiral perturbation theory (ChPT) [3–5] up to order  $p^6$  [next-to-leading order (NLO) in Fig. 1] and are shown to lead to an enhancement of about 4.5% in the  $\pi^0$  decay width with respect to the case where state mixing is not included (LO in Fig. 1). A recent calculation done in the framework of SU(2) ChPT considering next-to-next-to-leading order (NNLO) corrections [6] agrees with the earlier NLO results. The estimated uncertainty in the ChPT prediction is on the level of 1% [4,6]. Corrections to the chiral anomaly have also been performed in the framework of QCD using dispersion relations and sum rules [7] (Ioffe07 in Fig. 1). The fact that the corrections to the chiral anomaly are small and are known at the 1% level makes the  $\pi^0 \rightarrow \gamma \gamma$  decay channel a benchmark process to test one of the fundamental predictions of QCD.

Three different experimental methods have been used in the past to measure the neutral pion decay width, the Primakoff, the direct, and the collider methods. The  $\pi^0 \to \gamma \gamma$  decay can be considered as a time-reversal process to  $\gamma \gamma \to \pi^0$ , which can be experimentally realized in the coherent photoproduction of pions in the Coulomb field of a target nucleus at forward angles—the Primakoff effect [8]. Using the fact that the decay width is inversely proportional to the mean lifetime, several experiments measured the decay length distribution (proportional to the lifetime) of the pions produced in thin targets by high energy beams—the direct method. In the collider experiments one is using the fusion of two quasireal photons from

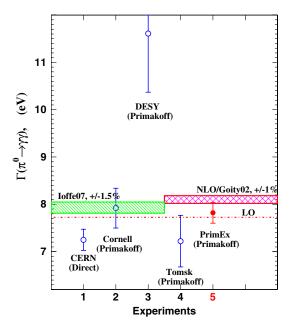


FIG. 1 (color online).  $\pi^0 \rightarrow \gamma \gamma$  decay width in eV. The dashed horizontal line is the LO chiral anomaly prediction. NLO ChPT prediction [4] is shown as the shaded band on the right-hand side. The left-hand side shaded band is the prediction from Ref. [7]. The experimental results, included in the PDG average, are for (1) the direct method [13], (2–4) the Primakoff method [10–12], and (5) the current PrimEx result.

electron and positron beams to produce the pion that is subsequently detected by its two real decay photons.

The current average experimental value for the  $\pi^0$  decay width given by the Particle Data Group (PDG) [9] is  $\Gamma(\pi^0 \to \gamma \gamma) = 7.74 \pm 0.55$  eV. This value is an average of four experiments with much larger dispersion between both the decay width values and their quoted experimental uncertainties, as shown in Fig. 1. The most precise Primakoff-type measurement was done at Cornell by Browman et al. [10] with a 5.3% quoted total uncertainty:  $\Gamma(\pi^0 \to \gamma \gamma) = 7.92 \pm 0.42$  eV. This result agrees within experimental uncertainty with the theoretical predictions. Two other measurements [11,12] with relatively large experimental uncertainties ( $\simeq 11\%$  and  $\simeq 7\%$ ) differ significantly from each other and do not agree with the theoretical predictions. The most precise measurement of the  $\pi^0$  decay width, prior to the current PrimEx experiment, was made by Atherton et al. [13] using the direct method of measuring the mean decay length of  $\pi^0$ 's produced by a high energy proton beam at CERN. Their result with the quoted 3.1% total uncertainty,  $\Gamma(\pi^0 \rightarrow \gamma \gamma) = 7.25 \pm 0.18 \pm 0.14 \text{ eV}$ , is  $\sim 4\sigma$  lower than the ChPT predictions of Refs. [4,6].

Clearly, a new Primakoff-type experiment with a precision comparable to, or better than, the direct method measurement [13] was needed to address the experimental situation on this fundamental quantity.

The PrimEx experiment [14] was performed at the Thomas Jefferson National Accelerator Facility using the Hall B high precision photon tagging facility [15] together with a newly developed high-resolution electromagnetic calorimeter. The combination of these two techniques greatly improved not only the angular resolutions, which are critical for Primakoff-type measurements, but significantly reduced the systematic uncertainties that were present in previous experiments.

Tagged photons with known timing and energy were incident on two 5% radiation length targets of  $^{12}\mathrm{C}$  and  $^{208}\mathrm{Pb}$  [16]. The relative photon tagging efficiencies were continuously measured during the experiment with a  $e^+e^-$  pair spectrometer (PS) consisting of a  $\sim$ 1.7 T·m large aperture dipole magnet and two telescopes of scintillating counters located downstream of the targets. The absolute normalization of the photon flux was done periodically with a total absorption counter at low beam intensities.

The decay photons from  $\pi^0 \rightarrow \gamma \gamma$  were detected in a multichannel hybrid electromagnetic calorimeter (HyCal) located 7.5 m downstream from the targets to provide a large geometrical acceptance ( $\sim 70\%$ ). HyCal consists of 1152 PbWO<sub>4</sub> crystal shower detectors ( $2.05 \times 2.05 \times 18.0 \text{ cm}^3$ ) in the central part surrounded by 576 lead glass Cherenkov counters ( $3.82 \times 3.82 \times 45.0 \text{ cm}^3$ ). Four crystal detectors were removed from the central part of the calorimeter ( $4.1 \times 4.1 \text{ cm}^2$  hole in size) for passage of the high intensity ( $\sim 10^7 \text{ y/s}$ ) incident photon beam through the calorimeter [17]. Twelve 5-mm-thick scintillator

counters, located in front of HyCal, provided rejection of charged particles and effectively reduced the background in the experiment. To minimize the decay photon conversion in air, the space between the PS magnet to HyCal was enclosed by a helium bag at atmospheric pressure. The photon beam's position stability was monitored during the experiment by an *X-Y* scintillating-fiber detector located downstream of HyCal.

The experimental trigger was formed by requiring coincidences between the photon tagger in the upper energy interval (4.9–5.5 GeV) and HyCal with a total deposited energy greater than 2.5 GeV. The combination of the photon tagger and the calorimeter defined the following main event selection criteria in this experiment: (1) the timing between the incident photon and the decay photons in the calorimeter ( $\sigma_t = 1.1$  ns), (2) the ratio of the total energy in the calorimeter and the tagger energy, "elasticity" ( $\sigma_{\rm el} = 1.8\%$ ), and (3) the invariant mass of the two photons ( $M_{\gamma\gamma}$ ) reconstructed in the calorimeter (shown in Fig. 2).

The event yield (number of  $\pi^0$  events for each production angle bin) was obtained from the data by applying the selection criteria described above and fitting the experimental distributions of elasticity and  $M_{\gamma\gamma}$  for each angular bin. Two groups within the PrimEx Collaboration independently analyzed the experimental data. They implemented different methods for event selection and slightly different fitting procedures for extraction of the decay width from the measured cross sections. For each angular bin analysis group applied a kinematical constraint on the energies of the two decay photons to satisfy the elasticity condition for each event. The resulting  $M_{\gamma\gamma}$  distributions were fit with a Gaussian plus polynomial functions to determine the  $\pi^0$ event yields for all angular bins. In the analysis by group II, the data were sliced into both angular and elasticity bins. For each two-dimensional slice, an invariant mass

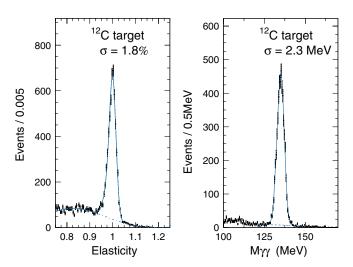


FIG. 2 (color online). Typical distribution of reconstructed elasticity (left-hand panel) and  $M_{\gamma\gamma}$  (right-hand panel) for one angular bin.

distribution was fit with a Gaussian peak and a polynomial background to determine the  $\pi^0$  yields.

The typical background in the event selection process was only a few percent of the real signal events (see Fig. 2). However, the uncertainty of 1.6% in the background extraction in this much upgraded experiment still remained one of the largest contributions to the total systematic uncertainty.

The extraction of differential cross sections from the experimental yields requires an accurate knowledge of the total photon flux for each tagger energy bin, the number of atoms in the target, the acceptance of the experimental setup, and the inefficiencies of the detectors. The uncertainty reached in the photon flux measurement, as described above, was at the level of 1% [18]. Different techniques have been used to determine the number of atoms in both targets with an uncertainty less than 0.1% [16]. The acceptance and detection efficiencies and their uncertainties were calculated by a GEANT-based Monte Carlo code that included accurate information about the detector geometry and response of each detector element. Other than accidental backgrounds, some physics processes with an energetic  $\pi^0$  in the final state can potentially contribute to the extracted yield. Monte Carlo simulation of the reaction processes showed that the  $\omega$ photoproduction with the subsequent  $\omega \to \pi^0 \gamma$  decay is the dominant contribution to the background. The fit of the experimental data, as described below, with the subtracted physics background changes the extracted  $\pi^0$  decay width by 1.4% with an uncertainty of 0.25%.

The resulting experimental cross sections for <sup>12</sup>C and <sup>208</sup>Pb are shown in Figs. 3 and 4 along with the fit results

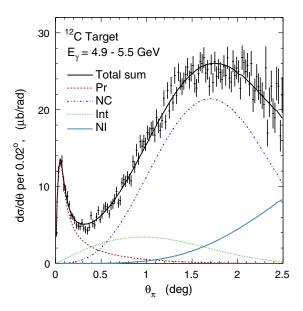


FIG. 3 (color online). Differential cross section as a function of the  $\pi^0$  production angle for  $^{12}$ C together with the fit ( $\chi^2/N_{df}=152/121$ ) results for the different physics processes (see text for explanations).

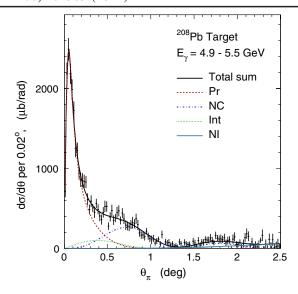


FIG. 4 (color online). Differential cross section as a function of the  $\pi^0$  production angle for  $^{208}\text{Pb}$  together with the fit ( $\chi^2/N_{df}=123/121$ ) results for the different physics processes (see text for explanations).

for individual contributions from the different  $\pi^0$  production mechanisms. Two elementary amplitudes, the Primakoff (one photon exchange)  $T_{\rm Pr}$  and the strong (hadron exchange)  $T_{\rm S}$  contribute coherently, as well as incoherently in  $\pi^0$  photoproduction from nuclei at forward angles. Therefore, the cross section of this process can be expressed by four terms: the Primakoff (Pr), the nuclear coherent (NC), the interference between strong and Primakoff amplitudes (Int), and the nuclear incoherent (NI):

$$\frac{d\sigma}{d\Omega}|T_{\rm Pr}+e^{i\varphi}T_{\rm S}|^2+\frac{d\sigma_{\rm NI}}{d\Omega}=\frac{d\sigma_{\rm Pr}}{d\Omega}+\frac{d\sigma_{\rm NC}}{d\Omega}+\frac{d\sigma_{\rm Int}}{d\Omega}+\frac{d\sigma_{\rm NI}}{d\Omega},$$

where  $\varphi$  is the relative phase between the Primakoff and the strong amplitudes. The Primakoff cross section is proportional to the  $\pi^0$  decay width, the primary focus of this experiment [10]:

$$\frac{d\sigma_{\rm Pr}}{d\Omega} = \Gamma(\pi^0 \to \gamma\gamma) \frac{8\alpha Z^2}{m^3} \frac{\beta^3 E^4}{Q^4} |F_{\rm EM}(Q)|^2 {\rm sin}^2 \theta_\pi, \label{eq:sin_Prop}$$

where Z is the atomic number, m,  $\beta$ ,  $\theta_{\pi}$  are the mass, velocity, and production angle of the pion, E is the energy

of the incident photon, Q is the four-momentum transfer to the nucleus, and  $F_{\rm EM}(Q)$  is the nuclear electromagnetic form factor corrected for final state interactions (FSI) of the outgoing pion. The FSI effects for the photoproduced pions, as well as the photon shadowing effect in nuclear matter, need to be accurately included in the cross sections before extracting the Primakoff amplitude. To achieve this, and to calculate the NC and NI cross sections, a full theoretical description based on the Glauber method was developed, providing an accurate calculation of these processes in both light and heavy nuclei [19,20]. For the NI process, an independent method based on the multicollision intranuclear cascade model [21] was also used to check the model dependence of the extracted decay width.

The sensitivity of the extracted decay width from these two different models was shown to be 0.12%. To check the dependence of the decay width on the physical parameters used inside the models, their values were changed at the few  $\sigma$  level, and the fitting procedure was repeated. For example, the variation of  $\pi^0 N$  total cross section at the  $2\sigma$ level resulted in only 0.1% change in the decay width. The incident photon shadowing in the nuclei [19] is one of the processes that contributes sizably to the model uncertainty—mostly because up to now the shadowing parameter was experimentally poorly determined. We have used the value 0.25 for the shadowing parameter taken from Meyer et al. [22]. Varying this parameter at the  $\pm 30\%$  level changed the decay width not more than 0.13%. The uncertainty from using different nuclear densities for the form factor calculations was shown to be less than 0.1%. Overall, the uncertainty in the decay width from model dependence and parameters inside the models was estimated to be 0.3%.

The  $\Gamma(\pi^0 \to \gamma \gamma)$  decay width was extracted by fitting the experimental results with the theoretical cross sections of the four processes mentioned above folded with the angular resolutions ( $\sigma_{\theta_{\pi^0}} = 0.4$  mrad) and the measured energy spectrum of the incident photons (4.9–5.5 GeV). In the fitting process, four parameters,  $\Gamma(\pi^0 \to \gamma \gamma)$ ,  $C_{\rm NC}$ ,  $C_{\rm NI}$ ,  $\varphi$ , were varied to calculate the magnitude of the Primakoff, NC, NI cross sections and the phase angle, respectively. The fit results of the two analysis groups for the decay widths, as well as for the other three parameters ( $C_{\rm NC}$ ,  $\varphi$ ,  $C_{\rm NI}$ ), are presented in Table I for both targets,

TABLE I. The fit values extracted from the measured cross sections on  $^{12}$ C and  $^{208}$ Pb from two analysis groups. The uncertainties shown here are statistical only including the fitting uncertainties (see text for details).

Target	Analysis group	$\Gamma(\pi^0 \to \gamma \gamma) \text{ (eV)}$	$C_{ m NC}$	$\varphi$ (rad)	$C_{ m NI}$
<sup>12</sup> C	I	$7.67 \pm 0.18$ $7.91 \pm 0.15$	$0.83 \pm 0.02$ $0.85 \pm 0.01$	$0.78 \pm 0.07$ $1.01 \pm 0.05$	$0.72 \pm 0.06$ $0.69 \pm 0.05$
<sup>208</sup> Pb	I	$7.91 \pm 0.13$ $7.72 \pm 0.23$	$0.83 \pm 0.01$ $0.69 \pm 0.04$	$1.01 \pm 0.03$ $1.25 \pm 0.07$	$0.69 \pm 0.03$ $0.68 \pm 0.12$
	II	$7.99 \pm 0.17$	$0.57 \pm 0.05$	$1.13 \pm 0.08$	$0.44 \pm 0.44$

 $^{12}\mathrm{C}$  and  $^{208}\mathrm{Pb}$ . The uncertainties shown in this table are statistical only including the fitting uncertainties. The  $C_{\mathrm{NI}}$  term in analysis II was not constrained for  $^{208}\mathrm{Pb}$  since the fit was applied only up to  $\theta_{\pi^0}=1.75^\circ$  due to the specifics of the event selection procedure. Analysis I was able to constrain this term since the fits were carried out to 2.5°. The value of  $\Gamma(\pi^0 \to \gamma\gamma)$  decay width is not sensitive to the nuclear incoherent contribution since it is negligible in the Primakoff peak region. The weighted averages of the extracted decay widths for the two targets from the two analyses are  $\Gamma(\pi^0 \to \gamma\gamma)=7.79\pm0.18(\mathrm{stat})~\mathrm{eV}$  for  $^{12}\mathrm{C}$  and  $\Gamma(\pi^0 \to \gamma\gamma)=7.85\pm0.23(\mathrm{stat})~\mathrm{eV}$  for  $^{208}\mathrm{Pb}$ . The statistical uncertainties shown are the larger ones of the two analyses which included a more stable data set.

Our result for the extracted decay width combined for the two targets is  $\Gamma(\pi^0 \to \gamma \gamma) = 7.82 \pm 0.14 \text{(stat)} \pm 0.14 \text{(stat)}$ 0.17(syst) eV. The quoted total systematic uncertainty (2.1%) is the quadratic sum of all the estimated uncertainties in this experiment. The systematic uncertainties were verified by measuring the cross sections of the Compton scattering and the  $e^+e^-$  production processes. The extracted cross sections for these well-known processes agree with the theoretical predictions at the level of 1.5% and will be published separately. The PrimEx result, with a total experimental uncertainty of 2.8%, is the most precise Primakoff-type measurement of the  $\Gamma(\pi^0 \to \gamma \gamma)$  to date. It is a factor of two-and-a-half more precise than the current average value quoted in the Particle Data Group for this important fundamental quantity. As a single experimental result, it directly confirms the validity of the chiral anomaly in OCD at the few percent level. The goal of the PrimEx experiment has been to test the chiral anomaly and the corrections to it in the  $\pi^0$  decay width with high precision. The second phase of this experiment has recently been performed at Jefferson Lab to achieve the projected 1.4% precision.

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