Unidirectional Mechanical Amplification as a Design Principle for an Active Microphone

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Amplification underlies the operation of many biological and engineering systems. Simple electrical, optical, and mechanical amplifiers are reciprocal: the backward coupling of the output to the input equals the forward coupling of the input to the output. Unidirectional amplifiers that occur often in electrical and optical systems are special nonreciprocal devices in which the output does not couple back to the input even though the forward coupling persists. Here we propose a scheme for unidirectional mechanical amplification that we utilize to construct an active microphone. We show that amplification improves the microphone's threshold for detecting weak signals and that unidirectionality prevents distortion.

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Maxwell's reciprocity theorem in mechanics states that a force applied at a point A, such as an amplifier's input, produces a displacement at a point B, say the amplifier's output, that equals the displacement at A caused by an equal force at B [1,2] [Fig. 1(a)]. Similar reciprocal relations govern electrical circuits [3] and optics [1]. They represent a potential problem in the design of amplifiers, for an ideal amplifier should operate unidirectionally: although the signal at the input must control the output, to avoid feedback and distortion the output should not couple back to the input [Fig. 1(b)]. Reciprocity can be violated in electrical circuits through the use of semiconductors and in optical systems through Faraday rotation [1,4]. Amplifiers in electrical engineering and microwave devices therefore employ these effects to achieve unidirectionality [4,5].

Mechanical amplification can enhance the detection of a weak signal by raising its amplitude above the noise level. Biology employs this strategy in hearing, for mechanosensitive hair cells in the vertebrate inner ear actively amplify weak sounds and thereby greatly lower the threshold of hearing [6,7]. In contrast, microphones—the ear's technological analogues—are passive devices that do not employ mechanical amplification but rely on subsequent electronic signal processing. One difficulty in implementing mechanical amplification in microphones is the reciprocity described above, which leads to undesired feedback and hence highlights the need for a mechanism of unidirectional mechanical amplification. We have recently described how such a mechanism may operate in the mammalian inner ear, where hair cells provide piezoelectric coupling [8] between the input and output that can foster unidirectionality [9,10]. Here we show that a similar strategy can be used to implement unidirectional mechanical amplification in a microphone.

A dynamic microphone functions by the same principle as a dynamic speaker [11]: a diaphragm is attached to a coil that moves in a magnetic field [Fig. 1(c)]. Sound vibrates the diaphragm and thereby causes oscillations of the coil that electromagnetically induce a voltage. Such a system serves as a speaker when an oscillatory electrical signal is applied to the coil; the consequent Lorentz force vibrates the coil and hence the diaphragm, emitting sound. The system's dual function as microphone and speaker therefore results from two potential forces. An external sound force $F_{\rm ext}$ acts on the diaphragm, representing the microphone's input, and the internal Lorentz force $F_{\rm int} = IlB$

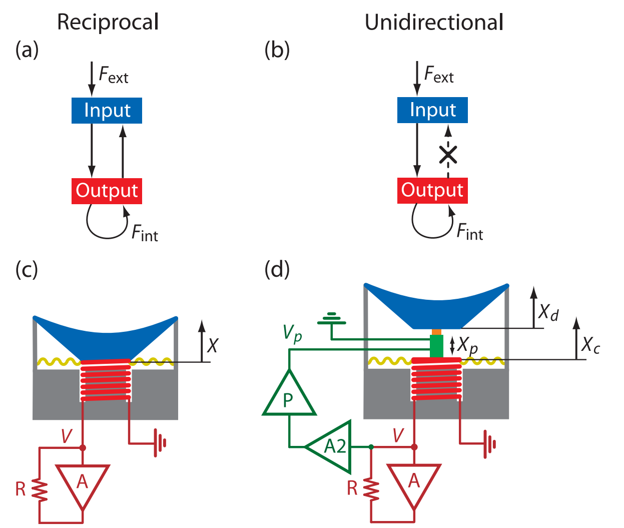

FIG. 1 (color). Amplifier and microphone designs. (a), (b) Schematic diagrams of reciprocal and unidirectional amplifiers. (c) In a dynamic microphone the spider (yellow) and diaphragm (blue) hold the voice coil (red) in a magnetic field. The coil's displacement can be amplified through amplifying the coil voltage V (amplifier A) and feeding it back into the coil through a resistance R. (d) Unidirectionality results from a piezoelectric element (green) in series with an elastic element (orange) placed between the coil and diaphragm. The piezoelectric element's length change X_p is controlled by a voltage V_p that results from the coil voltage V and that is adjusted through an amplifier $A2$ and a phase shifter P.

acts on the coil, corresponding to the output. Here I denotes the current in the coil, l the coil's length, and B the magnetic field. To linear order the diaphragm's displacement X then obeys

$$m\partial_t^2 X + \lambda\partial_t X + KX = F_{\text{int}} + F_{\text{ext}}, \tag{1}$$

in which m, λ, and K denote, respectively, the mass, damping coefficient, and stiffness of the coil together with the diaphragm and the entrained air. The voltage V induced by coil motion is

$$V = Bl\partial_t X. \tag{2}$$

Because of the system's dual function as both speaker and microphone it can readily be converted into a mechanically active microphone [Fig. 1(c)]. Indeed, positive feedback of the electrical signal in the coil can employ the internal Lorentz force to increase the coil's displacement resulting from an external sound force. To this end we have constructed a circuit in which the electrical signal in the coil is amplified and fed back into the coil through a resistance R [Fig. 1(c)]. The electrical amplification is unidirectional with an adjustable gain G achieved through an operational amplifier in the noninverting feedback configuration (supplemental material [12]). The amplifier's output voltage V_A in response to the input V is then $V_A = GV$ and fullfills $V_A - V = IR$; these relations may be employed to express the voltage V in terms of the current I:

$$V = \left(\frac{R}{G-1}\right)I. \tag{3}$$

To analyze Eqs. (1)–(3), we consider a pure tone of frequency f that produces an oscillatory external force $F_{\text{ext}} = \tilde{F}_{\text{ext}}e^{i\omega t} + \text{c.c.}$ in which the tilde connotes the Fourier component, $\omega = 2\pi f$ and "c.c." denotes the complex conjugate. Because the equations are linear, the resulting displacement X, voltage V, and current I oscillate at the same frequency f: $X = \tilde{X}e^{i\omega t} + \text{c.c.}$, $V = \tilde{V}e^{i\omega t} + \text{c.c.}$, and $I = \tilde{I}e^{i\omega t} + \text{c.c.}$ The amplitudes follow as

$$\tilde{X} = \frac{1}{i\omega Z}\tilde{F}_{\text{ext}}, \qquad \tilde{V} = \frac{Bl}{Z}\tilde{F}_{\text{ext}}, \qquad \tilde{I} = \frac{(G-1)Bl}{RZ}\tilde{F}_{\text{ext}} \tag{4}$$

with the impedance

$$Z = i(m\omega - K/\omega) + \lambda - l^2B^2(G-1)/R. \tag{5}$$

Equation (5) defines a resonant frequency $\omega_0 = 2\pi f_0 = \sqrt{K/m}$ at which the imaginary part of the impedance vanishes. The real part results from the damping λ counteracted by the positive feedback. At a critical value of the gain $G_c = 1 + \lambda R/(l^2B^2)$, the real part of the impedance vanishes; this value therefore defines a transition from damped to undamped oscillation [13,14]. Nonlinearities control the system's behavior at this bifurcation and yield a transition from damped to stable limit-cycle oscillations consistent with a supercritical Hopf bifurcation (supplemental material [12]).

Operation at the Hopf bifurcation enhances the detection of sounds at the resonant frequency [5,15–17]. The active microphone's response is sharply tuned to the resonant frequency f_0 (supplemental material [12]) at which the linear part of the response vanishes [Eq. (5)] and the response becomes nonlinear. In our active microphone the response follows a power-law relation to stimulus amplitude with an exponent of about 0.5 [Fig. 2(b), supplemental material [12]]. Faint signals cause a significantly larger response than in the passive microphone. Amplification acts only on the noise component at the resonant frequency and not on the remaining noise spectrum and thus improves the signal-to-noise ratio and lowers the threshold for signal detection at the resonance [Figs. 2(a) and 2(b)]. Because an active microphone constructed in this way detects only a small band of frequencies near its resonance, an ensemble of such systems tuned to different resonant frequencies is required to cover a broader frequency range. Such an array of active oscillators indeed underlies hearing in the vertebrate inner ear [6,7].

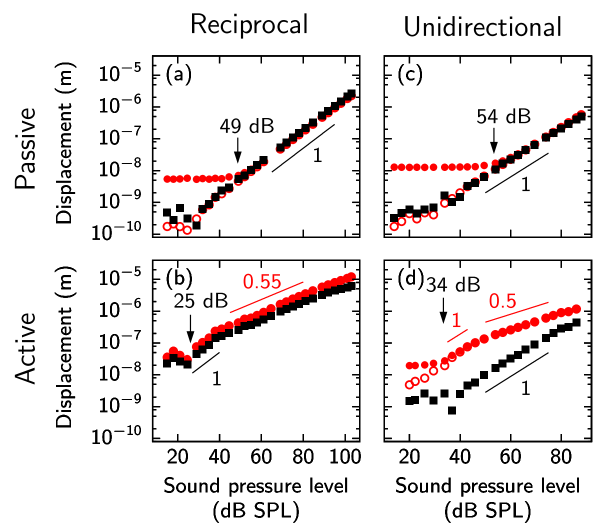

FIG. 2 (color online). Linear and nonlinear responses. The diaphragm's displacement (black squares) and the coil's displacement (red or grey circles) are presented as functions of the sound-pressure level for the reciprocal passive (a), the reciprocal active (b), the unidirectional passive (c), and the unidirectional active microphone (d) stimulated at the resonant frequencies of their active versions. The diaphragm's displacement is obtained from interferometric measurements for which we report the Fourier component. The coil's displacement is inferred from the coil's voltage; we display the Fourier component (red or grey open circles) as well as the root-mean-square value multiplied by $\sqrt{2}$ (red or grey full circles). Arrows indicate the detection thresholds at which the response enters the noise floor.

The active microphone described above operates reciprocally: the motion of the diaphragm, representing the input, is amplified by the coil's vibration, which corresponds to the output. Unidirectionality can be achieved by placing a piezoelectric element, controlled by the coil's electrical signal, between the diaphragm and coil [Fig. 1(d)]. The element's length change $\tilde{X}_p$ is then proportional to the coil displacement $\tilde{X}_c$: $\tilde{X}_p = -\alpha \tilde{X}_c$ with a complex coefficient α determined by the electrical circuit. Because the external sound force acts on the diaphragm and the internal Lorentz force on the coil, the diaphragm's displacement $\tilde{X}_d$ and the coil's displacement $\tilde{X}_c$ follow from

$$i\omega A \begin{pmatrix} \tilde{X}_d \\ \tilde{X}_c \end{pmatrix} = \begin{pmatrix} \tilde{F}_{\text{ext}} \\ \tilde{F}_{\text{int}} \end{pmatrix} \tag{6}$$

with the matrix

$$A = \begin{pmatrix} Z_d + Z & -(1+\alpha)Z \\ -Z & Z_c + (1+\alpha)Z \end{pmatrix}. \tag{7}$$

Here Z_d denotes the impedance of the diaphragm, Z_c the impedance of the coil, and Z the impedance of a coupling element positioned between the piezoelectric element and the diaphragm [Fig. 1(d)].

The piezoelectrical coupling breaks reciprocity: when $\alpha \neq 0$, the coupling of the coil to the diaphragm, given by the matrix element A_{12}, differs from the coupling of the diaphragm to the coil, represented by A_{21}. At a critical value $\alpha_* = -1$ the coupling becomes unidirectional: the matrix element A_{12} vanishes and with it the coupling from the coil to the diaphragm, whereas A_{21} and thus the coupling from the diaphragm to the coil remains nonzero. Setting the coefficient α to its critical value α_* requires adjustment of both its amplitude and phase, which can be achieved through amplifying and subsequently phase-shifting the coil voltage [Fig. 1(d) and supplemental material [12]]. At the critical value α_* the displacements are

$$\tilde{X}_d = \frac{1}{i\omega(Z_d + Z)} \tilde{F}_{\text{ext}}, \tag{8}$$

$$\tilde{X}_c = \frac{Z}{i\omega Z_c(Z_d + Z)} \tilde{F}_{\text{ext}} + \frac{1}{i\omega Z_c} \tilde{F}_{\text{int}}. \tag{9}$$

Unidirectional coupling is manifest in these equations because the coil is displaced both by the external sound force and by the internal Lorentz force [Eq. (9)], whereas only the sound force acts on the diaphragm [Eq. (8)].

Because amplification in the unidirectional active microphone acts through the internal Lorentz force on the coil but not on the diaphragm, the coil's displacement exhibits a Hopf bifurcation at a critical gain whereas the motion of the diaphragm does not (supplemental material [12]). The system's response at the Hopf bifurcation reflects this difference: the coil's displacement and voltage exhibit nonlinear behavior because their linear responses vanish at the bifurcation [Fig. 2(d)]. The diaphragm's displacement, however, shows a linear response for it does not encounter the amplification.

Amplification in the unidirectional active microphone lowers the threshold for detecting signals at the resonant frequency. Indeed, a passive unidirectional microphone that omits amplification through elimination of the active microphone's positive-feedback circuit exhibits a considerably higher threshold [Figs. 2(c) and 2(d)]. Because the piezoelectrical coupling introduces additional noise, the signal-detection thresholds in the passive and active unidirectional microphones are slightly increased compared to their reciprocal analogues (Fig. 2).

What is the benefit of unidirectionality in an active microphone? Amplification of the diaphragm requires energy that is spared with unidirectional coupling. The critical gain G_c of the amplifier at which the Hopf bifurcation emerges is therefore lower in the unidirectional microphone than in the reciprocal one (supplemental material [12]).

Another important advantage is prevention of distortion. The nonlinear response of an active microphone near its Hopf bifurcation causes the formation of distortion products [18]: when stimulated at two frequencies f_1 and f_2 the microphone also reports linear combinations of these frequencies such as $2f_1 - f_2$ and $2f_2 - f_1$. A strong response results if one distortion product coincides with the microphone's resonant frequency f_0 [Fig. 3(a)]. When amplification is reciprocal this distortion product is emitted from the active microphone because the coil transmits it to the diaphragm and a sound results [Figs. 3(a)–3(c)]. Unidirectional amplification, in contrast, prevents the emission of such a distortion product. Although the distortion product appears in the coil's voltage and displacement owing to the coil's operation near a Hopf bifurcation, the distortion product is not transmitted to the diaphragm and is therefore not emitted as sound [Figs. 3(d)–3(f)].

An ensemble of distortion products results when amplification is reciprocal. The number and frequency of distortion products to which an active microphone responds are determined by the form of the nonlinearities that dominate near the bifurcation. For example, the normal form of the Hopf bifurcation induces only the cubic distortion products $2f_1 - f_2$ and $2f_2 - f_1$ in response to stimulation at f_1 and f_2. However, as described above, detection of signals across a certain frequency range requires an array of active microphones with distinct resonances. Presentation of f_1 and f_2 to such an array results in a cascade of combination tones at frequencies $f_1 \pm n(f_2 - f_1)$, $n \in \mathbb{N}$, for the distortion products at $2f_1 - f_2$ and $2f_2 - f_1$ are emitted by microphones tuned to these frequencies and interact with the stimuli at frequencies f_1 and f_2 as well as themselves to create other distortion products such as $3f_1 - f_2$, $3f_2 - f_1$, and so on. Such cascades of distortion

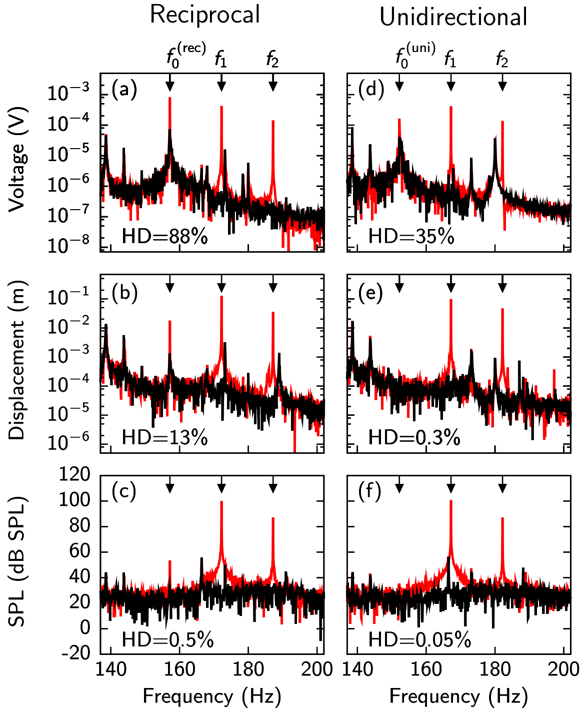

FIG. 3 (color online). Distortion products. We report the Fourier spectrum of the coil's voltage (a),(d), of the diaphragm's displacement (b),(e), and of the sound-pressure level detected by an external microphone (c),(f) for the reciprocal and the unidirectional active microphone. The system's response to stimulation at two frequencies f_1 and f_2 with $f_0 = 2f_1 - f_2$ is shown in red or grey, and the background signal without stimulation in black. The amount of distortion can be quantified through the harmonic distortion (HD, supplemental material [12]).

products have been recorded from the high-frequency region of the mammalian inner ear, where amplification is reciprocal [19].

Unidirectional coupling prevents the cascade of distortion products. Although distortion products such as $2f_1 - f_2$ and $2f_2 - f_1$ are formed in the coil, they are not emitted and hence cannot create further distortion products. The resulting reduction in the number of distortion products represents a significant advantage of the unidirectional active microphone over its reciprocal counterpart. Theoretical considerations indicate that the mammalian ear may employ the same strategy for detection of frequencies below about 2 kHz [9], which are employed predominantly in human speech [20] and music [21].

In summary, we have described a scheme for unidirectional mechanical amplification as well as its implementation in an active microphone that is ultrasensitive and nondistorting. This technology has potential applications in contexts that require the sensitive detection of mechanical signals at specific frequencies, for example, sonar, sonography, and perhaps the detection of seismic or even gravitational waves.

[1] R. J. Potton, Rep. Prog. Phys. **67**, 717 (2004).
[2] J. R. Barber, *Elasticity (Solid Mechanics and Its Applications)* (Springer, New York, 2004), 3rd ed.
[3] B. I. Bleaney and B. Bleaney, *Electricity and Magnetism* (Oxford University Press, Oxford, 1976), 3rd ed.
[4] D. M. Pozar, *Microwave Engineering* (Wiley, New York, 2005), 3rd ed.
[5] N. J. McCullen, T. Mullin, and M. Golubitsky, Phys. Rev. Lett. **98**, 254101 (2007).
[6] L. Robles and M. A. Ruggero, Physiol. Rev. **81**, 1305 (2001).
[7] A. J. Hudspeth, Neuron **59**, 530 (2008).
[8] J. Ashmore, Physiol. Rev. **88**, 173 (2008).
[9] T. Reichenbach and A. J. Hudspeth, Proc. Natl. Acad. Sci. U.S.A. **107**, 4973 (2010).
[10] T. Reichenbach and A. J. Hudspeth, Phys. Rev. Lett. **105**, 118102 (2010).
[11] J. M. Eagle, *Loudspeaker Handbook* (Chapman & Hall, New York, 1997), 1st ed.
[12] See supplemental material at http://link.aps.org/supplemental/10.1103/PhysRevLett.106.158701.
[13] S. H. Strogatz, *Nonlinear Dynamics and Chaos* (Westview, Nashville, 1994).
[14] S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos* (Springer, New York, 1990), 1st ed.
[15] V. M. Eguíluz *et al.*, Phys. Rev. Lett. **84**, 5232 (2000).
[16] S. Camalet *et al.*, Proc. Natl. Acad. Sci. U.S.A. **97**, 3183 (2000).
[17] A. Kern and R. Stoop, Phys. Rev. Lett. **91**, 128101 (2003).
[18] F. Jülicher, D. Andor, and T. Duke, Proc. Natl. Acad. Sci. U.S.A. **98**, 9080 (2001).
[19] L. Robles, M. A. Ruggero, and N. C. Rich, J. Neurophysiol. **77**, 2385 (1997).
[20] R. J. Baken and R. F. Orlikoff, *Clinical Measurement of Speech and Voice* (Singular Publishing Group, San Diego, 2000), 2nd ed.
[21] H. F. Olson, *Music, Physics, and Engineering* (Dover, New York, 1967), 1st ed.