Proton Size Anomaly

Vernon Barger,¹ Cheng-Wei Chiang,^{2,3,1} Wai-Yee Keung,⁴ and Danny Marfatia^{5,1}

¹Department of Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706, USA
²Department of Physics and Canter for Mathematics and Theoratical Physics, National Cantral Uni

 2 Department of Physics and Center for Mathematics and Theoretical Physics, National Central University,

Chungli, Taiwan 32001, Republic of China
³Institute of Physics, Academia Sinica, Nankana, Tainei 11025

Institute of Physics, Academia Sinica, Nankang, Taipei 11925, Republic of China ⁴

⁴Department of Physics, University of Illinois, Chicago, Illinois 60607, USA

 5 Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045, USA

(Received 18 November 2010; published 11 April 2011)

A measurement of the Lamb shift in muonic hydrogen yields a charge radius of the proton that is smaller than the CODATA value by about 5 standard deviations. We explore the possibility that new scalar, pseudoscalar, vector, and tensor flavor-conserving nonuniversal interactions may be responsible for the discrepancy. We consider exotic particles that, among leptons, couple preferentially to muons and mediate an attractive nucleon-muon interaction. We find that the many constraints from low energy data disfavor new spin-0, spin-1, and spin-2 particles as an explanation.

DOI: [10.1103/PhysRevLett.106.153001](http://dx.doi.org/10.1103/PhysRevLett.106.153001) PACS numbers: 31.30.jr, 14.80.j

Lamb shift.—The success of quantum electrodynamics (QED) is apparent in the explanation of the Lamb shift [\[1\]](#page-3-0) which is the observation that the $2S_{1/2}$ state of hydrogen is higher than the $2P_{1/2}$ state by about 1 GHz [[2\]](#page-3-1). Precision measurements in atomic spectra have tested bound-state QED to the extent that the charge distribution of the proton needs to be taken into account [[3](#page-3-2)]. The root-mean-square charge radius of the proton compiled by CODATA from the spectroscopy of atomic hydrogen and electron-proton scattering is [\[4](#page-3-3)]

$$
\langle r_p^2 \rangle^{1/2} = 0.8768 \pm 0.0069 \text{ fm}, \tag{1}
$$

provided there are no new long-range e - p interactions [[5\]](#page-3-4). It has been a long-held goal to measure the corresponding Lamb shift in muonic hydrogen which is even more sensitive to the structure of the proton due to its smaller Bohr radius $(\alpha m_\mu)^{-1}$ (where $\alpha \sim 1/137$ is the electromagnetic fine structure constant and $m_\mu \approx 105$ MeV). Becently, the fine structure constant and $m_{\mu} \approx 105$ MeV). Recently, the
 $2P^{F=2} \rightarrow 2S^{F=1}$ Lamb shift in muonic hydrogen was mea- $2P_{\frac{3}{2}}^{F=2} \rightarrow 2S_{\frac{1}{2}}^{F=1}$ Lamb shift in muonic hydrogen was mea-sured to be [\[6](#page-3-5)]

$$
\Delta \tilde{E} = E(2P_{3/2}^{F=2}) - E(2S_{1/2}^{F=1})
$$

= 206.2949 ± 0.0032 meV, (2)

while the predicted value is [\[7,](#page-3-6)[8](#page-3-7)]

$$
\Delta \tilde{E} = 209.9779(49) - 5.2262 \langle r_p^2 \rangle + 0.0347 \langle r_p^2 \rangle^{3/2}, \quad (3)
$$

where radii are in femtometers and energy in meV and the number in parentheses indicates the 1σ uncertainty of the last two decimal places of the given number. (Note that ΔE is defined to be positive.) Equations [\(2](#page-0-0)) and [\(3\)](#page-0-1) yield the order of magnitude more precise result [\[6\]](#page-3-5)

$$
\langle r_p^2 \rangle^{1/2} = 0.84184 \pm 0.00067 \text{ fm}, \tag{4}
$$

which is smaller than the CODATA value by about 5σ . A partial resolution of the discrepancy may be found in a correlation between $\langle r_p^2 \rangle$ and the r_p^3 -dependent third Zemach moment (since they contribute to the Lamb shift with opposite signs) and perhaps unreliable extractions of these from electron-proton scattering data [[9](#page-3-8)]. Nevertheless, a 4σ difference remains. The possibility that the 4% difference is a hint of a new gauge interaction with a natural scale αm_{μ} has been entertained in Ref. [[10](#page-3-9)].

In this Letter, we postulate the existence of a new interaction between muons and nucleons and study its nature, bearing in mind the many experimental constraints. The interaction must be attractive since ΔE measured in muonic hydrogen is larger than expected, signaling that the $2S_{1/2}$ state is subject to a stronger attraction than electromagnetic.

Scalar and spin-2 boson exchanges produce an attractive potential, giving positive contributions to $\Delta \tilde{E}$. Pseudoscalar boson exchange is a derivative interaction involving the spins and velocities of the lepton and the nucleus, which becomes insignificant in the nonrelativistic limit and irrelevant to the Lamb shift. A boson with both scalar and pseudoscalar couplings violates CP conservation. Such a scenario faces strong constraints from electric dipole moment measurements of leptons and nucleons and is disfavored [[11](#page-3-10)]. Vector boson exchange (like photon exchange) can produce an attractive potential if the quantum numbers associated with the lepton and the nucleus are opposite in sign. Then Lamb shift phenomenology is like that of scalar exchange. Axial-vector exchange couples the spins of the lepton and the nucleus in the nonrelativistic limit [with an effective potential $-\alpha_{\chi}(\boldsymbol{\sigma}_{\mu} \cdot \boldsymbol{\sigma}_{p})e^{-m_{\chi}r}/r$ [\[12\]](#page-3-11)] and affects the hyperfine structure (but not the Lamb shift) so that the correction to the hyperfine splitting between the $2P_{3/2}^{F=2}$ and $2S_{1/2}^{F=1}$ levels for $m_{\chi} \gg \alpha m_{\mu}$ is $\alpha_{\chi}\frac{\alpha m_r}{20}$ $\frac{1+10[m_\chi/(am_r)]^2}{[1+(m_\chi/am_r)]^4}$, in the notation defined below.

However, since the axial-vector current is not conserved, the propagator gives a very singular contribution for $m_{\chi} \le$ αm_{μ} , which is unphysical. Absent a well-defined model, we do not consider the axial-vector case any further.

Suppose the interaction between fermions f and χ is given by $C_f^{S,V,T} \bar{f} f \chi$, where S, V, and T denote scalar, vector, and tensor χ , respectively, and f can be a muon μ or a nucleon *n*; we assume isospin is conserved. Throughout, we take the couplings C to be real and positive. In the nonrelativistic limit, the muon-nucleon interaction is given by the Yukawa-type potential

$$
\Delta V(r) = -\alpha_{\chi} \frac{e^{-m_{\chi}r}}{r},\tag{5}
$$

where $\alpha_{\chi} = C_{\mu}^{S,V,T} C_{n}^{S,V,T} / (4\pi)$ and m_{χ} is the mass of the particle χ . Physical systems in which tensor interactions particle χ . Physical systems in which tensor interactions (by which we mean spin-2 exchange) are governed by a Yukawa potential allow the identification $C_f^T \equiv C_f^S$. This will be valid for all our constraints except those that involve the anomalous magnetic moment of the muon $a_{\mu} \equiv (g_{\mu} - g_{\mu})$ 2)/2. The correction to the muonic Lamb shift is $[7,13]$ $[7,13]$ $[7,13]$

$$
\delta(\Delta \tilde{E}) = \alpha_{\chi} m_{\chi} \frac{\frac{m_{\chi}}{\alpha m_{r}}}{2(1 + \frac{m_{\chi}}{\alpha m_{r}})^{4}},
$$
(6)

where m_r is the reduced mass of the muon-proton system, and its use is numerically important as m_r is smaller than m_{μ} by more than [1](#page-1-0)0%. The green shaded region in Fig. 1 shows the 95% C.L. region that accommodates the differ-ence between Eqs. ([1](#page-0-2)) and ([4\)](#page-0-3). We do not consider m_{χ} > 10 GeV since the required α_{χ} becomes larger than 20 α , in the nonperturbative regime.

Upsilon decay.—For scalar χ in the mass range $2m_{\mu}$ – 9.3 GeV, the nonobservance of radiative decays

FIG. 1 (color online). The 95% C.L. range of α_{χ}/α required to reproduce the muonic Lamb shift is indicated by the green shaded region. The black solid, red dashed, and blue dot-dashed lines are the upper limits for vector, scalar, and spin-2 particles, respectively, from a combination of $n - {}^{208}Pb$ scattering data and the anomalous magnetic moment of the muon. The black dotted curve is the upper bound obtained from atomic x-ray transitions. All bounds are at the 95% C.L.

of the $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances $\Upsilon \rightarrow \gamma \chi, \chi \rightarrow$ $\mu^+\mu$ $\mu^+ \mu^-$ strongly constrains the $\Upsilon - \chi$ coupling [[14](#page-3-13)], which we expect to be no smaller than C_n^S ; for a Higgs-like χ , the coupling is naturally $\mathcal{O}(m_b/m_n) \times C_n^S$, while for a universal interaction, it should be $\mathcal{O}(C_n^S)$. We conservatively take the $Y - \chi$ coupling to be C_n^S . In obvious notation [\[15\]](#page-3-14),

$$
\frac{\text{BF}(\Upsilon \to \gamma \chi)}{\text{BF}(\Upsilon \to \mu^+ \mu^-)} = \frac{(C_n^S)^2}{4\pi \alpha} \left(1 - \frac{m_\chi^2}{m_\Upsilon^2}\right). \tag{7}
$$

Under our assumption that the branching fraction of $\chi \rightarrow \mu^+ \mu^-$ is unity, the 90% C.L. upper limit on C_n^S range is $(0.94-9.4) \times 10^{-3}$ [\[14\]](#page-3-13), where the lower end of the range corresponds to smaller m_{χ} . The values of α_{χ}/α needed to explain the muonic Lamb shift constrain C_{μ}^{S} to lie above $\mathcal{O}(1)$, $\mathcal{O}(10)$, and $\mathcal{O}(100)$ for $m_{\chi} \sim 2m_{\mu}$, 1 GeV, and Ω GeV respectively couplings which are too large 9 GeV, respectively, couplings which are too large.

A vector χ can mediate leptonic decays of spin-1 quarkonia. Since the only lepton that χ couples to is the muon, one expects nonuniversality in leptonic decays. For $\Upsilon(1S)$ decays, $R_{\tau\mu} \equiv \Gamma_{\tau\tau}/\Gamma_{\mu\mu} = 1.005 \pm 0.013 \pm 0.022$ [[16\]](#page-3-15),
whereas the standard model (SM) expectation is 0.002 whereas the standard model (SM) expectation is 0.992 [\[17\]](#page-3-16). The inclusion of χ modifies the SM value of $R_{\tau\mu}$ by a factor

$$
\left[\left(1 \pm \frac{\alpha_{\chi}}{\alpha Q_b}\right) - (m_{\chi}/m_{\Upsilon})^2\right]^2 \left[1 - (m_{\chi}/m_{\Upsilon})^2\right]^{-2}, \quad (8)
$$

where the $+ (-)$ sign corresponds to destructive (constructive) interference and Q_b is the electric charge of the b quark. For $m_{\chi} \le 1$ GeV, a conservative 95% C.L. (onesided) upper bound on α_{γ}/α (assuming the SM and χ contributions destructively interfere) is 8.8 \times 10⁻³. In the range 1 GeV $\leq m_{\chi} < m_{\Upsilon}$, the upper bound becomes even more stringent, falling monotonically with m_y . The mass of a vector χ is restricted to be less than about 230 MeV in order to explain the muonic Lamb shift.

Neutron scattering.—Very precise neutron scattering experiments on heavy nuclei in the keV regime have been performed to study the electric polarizability of the neutron. The goal is to measure interference effects between the nuclear potential and the r^{-4} potential produced by electric polarizability. One can then see that a Yukawa potential $\pm A(C_n^{S,V,T})^2 e^{-m_X r}/(4\pi r)$ may also be probed by such ex-
periments: the minus and plus signs apply to scalar or tensor periments; the minus and plus signs apply to scalar or tensor and vector interactions, respectively. Stringent bounds are obtainable because the p -wave amplitude due to the shortrange strong interaction depends linearly on energy and differs markedly from that due to the new longer-range interaction. A $n - {}^{208}Pb$ scattering experiment [[18](#page-3-17)] in the neutron energy range 1–26 keV measured the differential cross section (under the assumption that the scattering amplitude can be expanded in s and p waves) to be

$$
d\sigma/d\Omega = \sigma_0 (1 + \omega E \cos \theta) / (4\pi), \tag{9}
$$

with $\sqrt{\sigma_0/4\pi} \approx 10$ fm and $\omega = (1.91 \pm 0.42) \times 10^{-3}$ keV⁻¹. The measured values are in line with 10^{-3} keV⁻¹. The measured values are in line with expectations so that the Yukawa potential contribution ought to be subdominant. Denoting the strong interaction contribution to ω by ω_s and the contribution of the new interaction by $\Delta \omega$, clearly, $\omega = \omega_s + \Delta \omega$ with [[19](#page-3-18)]

$$
\Delta \omega = \mp \frac{16}{m_{\chi}^4} \frac{(C_n^{S,V,T})^2}{4\pi} \frac{Am_n^2}{\sqrt{\sigma_0/4\pi}},
$$
(10)

in the Born approximation (not valid for $m_x \le 0.1$ MeV), m_n is the neutron mass, and A is the atomic mass number. For scalar or tensor exchange, it is possible that a cancellation between ω_s and $\Delta \omega$ produces the experimental result. However, this cannot be the case for a vector χ . A conservative 95% C.L. (one-sided) upper limit can be obtained by requiring that $\Delta \omega \le 2.6 \times 10^{-3}$ keV, i.e.,

$$
C_n^V \le (m_\chi/206)^2,\tag{11}
$$

with m_x in MeV. It is the shaded region of Fig. [2.](#page-2-0)

While reliable bounds for a scalar or tensor χ are not extractable from the differential cross section, the total cross section measured between 10 eV and 10 keV [\[20](#page-3-19)[,21\]](#page-3-20) may be employed with confidence. The energy dependence of the $n - {}^{208}Pb$ cross section for neutron energies below 10 keV can be parameterized by

$$
\sigma(k) = \sigma(0) + \sigma_2 k^2 + \mathcal{O}(k^4),\tag{12}
$$

where $k = 2.1968 \times 10^{-4} \sqrt{E}A/(A+1)$ is the wave vector
of the incoming neutron (with k in fm⁻¹ and E in eV). The of the incoming neutron (with k in fm^{-1} and E in eV). The cross section in the limit of vanishing momentum transfer $\sigma(0)$ is directly related to the scattering length, and σ_2 gives the effective range of the potential. The $\mathcal{O}(k)$ contribution to $\sigma(k)$ arises from the electric field of the nuclear charge distribution and is negligible. The measured values $\sigma(0) = 12.40 \pm 0.02$ b and $\sigma_2 = -448 \pm 3$ b fm² give a 0.5% C.I. bound on $C^{S,T}$ [211 that is almost identical to 95% C.L. bound on $C_n^{S,T}$ [[21](#page-3-20)] that is almost identical to Eq. ([11](#page-2-1)) in the mass range of interest (and is not shown separately in Fig. [2](#page-2-0)) but without the ambiguity from the cancellation mentioned above.

FIG. 2 (color online). The curves are 95% C.L. upper bounds on the muonic couplings C_{μ}^{S} , C_{μ}^{V} , and C_{μ}^{T} from Δa_{μ} . The green
shading marks the values of the nucleon coupling $C_{\mu}^{S,V,T}$ as shading marks the values of the nucleon coupling $C_n^{S,V,T}$ excluded by $n - {}^{208}Pb$ scattering at the 95% C.L.

Muon anomalous magnetic moment.—We now consider the independent constraint on C_{μ} from a_{μ} . In fact, since the experimental value of a_{μ} is above the SM expectation by more than 3 standard deviations, $\Delta a_{\mu} \equiv a_{\mu}^{\text{exp}} - a_{\mu}^{\text{th}} =$
(20 + 0) × 10⁻¹⁰ [22], the new interaction may explain $(29 \pm 9) \times 10^{-10}$ [22], the new
this difference. From Ref. [\[23\]](#page-3-22), $(29 \pm 9) \times 10^{-10}$ [[22](#page-3-21)], the new interaction may explain

$$
\Delta a_{\mu} = \frac{(C_{\mu}^{S,V})^2}{8\pi^2} \int_0^1 \frac{2x^2 - \beta x^3}{x^2 + (m_X^2/m_{\mu}^2)(1-x)} dx, \qquad (13)
$$

where $\beta = 1$ for a scalar and $\beta = 2$ for a vector. For a vector of the result of tensor interaction, we trivially modify the result of Ref. [\[24\]](#page-3-23). In the limit $m_{\chi} \ll m_{\mu}$,

$$
C_{\mu}^{S} = 4\pi (\Delta a_{\mu}/3)^{1/2} \le 4.8 \times 10^{-4}, \quad (14)
$$

$$
C_{\mu}^{V} = 4\pi (\Delta a_{\mu}/2)^{1/2} \lesssim 5.9 \times 10^{-4}, \quad (15)
$$

$$
C_{\mu}^{T} = 4\pi (3\Delta a_{\mu}/20)^{1/2} \lesssim 3.2 \times 10^{-4}, \quad (16)
$$

where the one-sided upper bounds are at the 95% C.L. From Fig. [2](#page-2-0), it is evident that Eqs. [\(14](#page-2-2))–([16](#page-2-3)) apply for $m_{\chi} \leq 10$ MeV.

The bound in Eq. ([11](#page-2-1)) can be combined with those in Eqs. ([14](#page-2-2))–[\(16\)](#page-2-3) to give the following 95% C.L. constraints for $m_{\chi} \leq 10$ MeV:

$$
\alpha_{\chi}/\alpha \le (m_{\chi}/2847)^2 \qquad \text{scalar}, \tag{17}
$$

$$
\alpha_{\chi}/\alpha \lesssim (m_{\chi}/2573)^2
$$
 vector, (18)

$$
\alpha_{\chi}/\alpha \le (m_{\chi}/3477)^2 \qquad \text{tensor}, \tag{19}
$$

with m_x in MeV. A similar (numerical) procedure can be applied for the entire range of m_x to obtain the upper bounds shown in Fig. [1.](#page-1-0) We see that a vector χ with mass between 25 (with $\alpha_{\chi} \approx 10^{-4} \alpha$) and 210 MeV (with $\alpha_{\chi} \sim 10^{-2} \alpha$) is a viable candidate. While a scalar χ with mass between 70 and 210 MeV [with $\alpha_{\chi} \sim (10^{-3} - 10^{-2})\alpha$] is marginally allowed, a spin-2 χ is excluded.

Muonic atom transitions.—Measurements of the muonic $3D_{5/2}$ -2 $P_{3/2}$ x-ray transition in ²⁴Mg and ²⁸Si atoms directly constrain α_{χ} for scalar, vector, and tensor particles [\[25\]](#page-3-24). For the Yukawa form of Eq. [\(5](#page-1-1)) with the coupling enhanced by a factor of A, the shift in the difference in energy levels from the QED expectation is [\[25\]](#page-3-24)

$$
\frac{\Delta \mathcal{E}}{\mathcal{E}} = \frac{2\alpha_{\chi} A}{5\alpha Z} [9f(2) - 4f(3)],\tag{20}
$$

where $f(j) = [1 + jm_{\chi}/(2\alpha Zm_{\mu})]^{-2j}$, Z is the atomic
pumber and *i* is the principle quantum number of the number, and j is the principle quantum number of the muonic state. The measured value obtained by averaging the results for ²⁴Mg and ²⁸Si, $\Delta E/E = (0.2 \pm 3.1) \times 10^{-6}$
[25] gives the 95% C.L. bound (dotted curve) in Fig. 1. No [\[25\]](#page-3-24), gives the 95% C.L. bound (dotted curve) in Fig. [1](#page-1-0). No additional area of the relevant parameter space is excluded by this constraint.

 J/ψ decay.—For $m_{\chi} < 2m_{\mu}$, the decay of scalar
 $\rightarrow u^+u^-$ is kinematically forbidden so that a constraint $\chi \rightarrow \mu^+ \mu^-$ is kinematically forbidden so that a constraint from the nonobservance of the decay $J/\psi \rightarrow \gamma \chi$, with χ invisible [[26](#page-3-25)], may be employed to exclude the marginally allowed region with 70 MeV $\lt m_x \lt 210$ MeV; preliminary data also exist for the decay $\Upsilon(3S) \rightarrow \gamma \chi$ [\[27\]](#page-3-26). A trivial modification of Eq. ([7\)](#page-1-2) applies to J/ψ decay. The 90% C.L. upper limit on BF $(J/\psi \rightarrow \gamma \chi)$ is $\sim 4.5 \times 10^{-6}$ [\[26\]](#page-3-25), which when combined with $BF(J/\psi \to \mu^+ \mu^-) =$
(5.93 + 0.06) × 10⁻² [28] gives $C^s < 0.029$ Then $(5.93 \pm 0.06) \times 10^{-2}$ [\[28\]](#page-3-27) gives $C_n^S < 0.029$. Then,
the muonic I amb shift dictates that C_2^S be larger than the muonic Lamb shift dictates that C_{μ}^{S} be larger than 3.4×10^{-3} , which is excluded at the 95% C.L. by a_{μ} ;
see Fig. 2. Thus scalars are also disfavored see Fig. [2.](#page-2-0) Thus, scalars are also disfavored.

Pion decay.—The 90% C.L. experimental upper limit on the decay $\pi^0 \to \gamma \chi$, where χ is a vector particle, is $(3.3-1.9) \times 10^{-5}$ for m_{χ} ranging from 0 to 120 MeV
[30] Equivalently $C_V^V < 4.5 \times 10^{-4}(1 - m^2/m^2)^{-3/2}$ [\[29\]](#page-3-28). Equivalently, $C_N^V < 4.5 \times 10^{-4} (1 - m_\chi^2/m_\pi^2)^{-3/2}$
[30] and the corresponding values of C_V^V required to ex-[\[30\]](#page-3-29), and the corresponding values of C^V_μ required to explain the muonic Lamb shift are excluded by a_{μ} . This leaves m_x between 120 and 230 MeV.

Eta decay.—For vector χ , the 90% C.L. experimental upper limit on invisible decays, $BF(\eta \rightarrow \chi \chi)/BF(\eta \rightarrow \gamma \gamma)$ 1.65×10^{-3} [\[31\]](#page-3-30), translates into $C_V^V \approx 0.05$ [[32\]](#page-3-31). The corresponding C_V^V for 120 MeV $\lt m \lt m \lt m/2 \approx 274$ MeV is responding C_{μ}^{V} for 120 MeV $\lt m_{\chi} \lt m_{\eta}/2 \sim 274$ MeV is excluded by a_{η} so that vector ν is ruled out excluded by a_{μ} , so that vector χ is ruled out.

Conclusions.—We have found that new spin-0, spin-1, or spin-2 particles that mediate flavor-conserving nonuniversal spin-independent interactions are excluded by several low energy constraints as an explanation of the proton radius anomaly. We assumed that, among leptons, the new particles couple only to the muon so as to avoid the large number of constraints involving the interaction of the electron with exotica. We also supposed that the coupling of the new particle to nucleons represents the minimal hadronic coupling and employed it to mesons.

There are ways to relax some of the bounds at the expense of introducing complication. For example, since the contributions of scalars and pseudoscalars to a_{μ} are opposite in sign, allowing both a scalar boson and a pseudoscalar boson with appropriately tuned couplings can lead to a cancellation that permits a rather large muonic coupling. Then, although the hadronic couplings are highly restricted, the muonic Lamb shift can be accommodated. Another possibility is that the new interaction violates isospin or CP, so that additional freedom is garnered.

This work was supported by DOE Grants No. DE-FG02- 84ER40173, No. DE-FG02-95ER40896, and No. DE-FG02-04ER41308, by NSF Grant No. PHY-0544278, by NSC Grant No. 97-2112-M-008-002-MY3, by NCTS, and by the WARF.

- [1] W. Lamb and R. Retherford, Phys. Rev. 79[, 549 \(1950\).](http://dx.doi.org/10.1103/PhysRev.79.549)
- [2] The dominant contributions to the Lamb shift arise from vacuum polarization and the vertex charge form factor of

the lepton. Vacuum polarization contributes negatively to $\Delta E \equiv E(2S_{1/2}) - E(2P_{1/2})$ since more of the lepton's bare charge is revealed for the S state (than the P state) due to its greater overlap with the nucleus. On the other hand, the vertex charge form factor is related to the zitterbewegung of the lepton which causes the effective Coulomb potential to be smeared out and less attractive. The effect is greater for the S state, so that the contribution to ΔE is positive. The latter contribution is dominant for ordinary hydrogen but plays a minor role in muonic hydrogen because of the smaller Compton wavelength of the muon. Consequently, ΔE is positive in ordinary hydrogen and negative in muonic hydrogen.

- [3] For a review, see S.G. Karshenboim, [Phys. Rep.](http://dx.doi.org/10.1016/j.physrep.2005.08.008) 422, 1 [\(2005\)](http://dx.doi.org/10.1016/j.physrep.2005.08.008).
- [4] P.J. Mohr, B.N. Taylor, and D.B. Newell, [Rev. Mod.](http://dx.doi.org/10.1103/RevModPhys.80.633) Phys. 80[, 633 \(2008\).](http://dx.doi.org/10.1103/RevModPhys.80.633)
- [5] S. G. Karshenboim, Phys. Rev. D **82**[, 073003 \(2010\).](http://dx.doi.org/10.1103/PhysRevD.82.073003)
- [6] R. Pohl et al., [Nature \(London\)](http://dx.doi.org/10.1038/nature09250) 466, 213 (2010).
- [7] E. Borie and G. A. Rinker, [Rev. Mod. Phys.](http://dx.doi.org/10.1103/RevModPhys.54.67) **54**, 67 (1982).
- [8] K. Pachucki, Phys. Rev. A 53[, 2092 \(1996\)](http://dx.doi.org/10.1103/PhysRevA.53.2092); A. P. Martynenko, Phys. Rev. A 71[, 022506 \(2005\)](http://dx.doi.org/10.1103/PhysRevA.71.022506); [Phys. At.](http://dx.doi.org/10.1134/S1063778808010146) Nucl. 71[, 125 \(2008\)](http://dx.doi.org/10.1134/S1063778808010146).
- [9] A. De Rujula, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2011.01.025) 697, 26 (2011).
- [10] W. Maricano, in Proceedings of INT, Seattle, 2010 (unpublished).
- [11] For a review, see W. Bernreuther and M. Suzuki, [Rev.](http://dx.doi.org/10.1103/RevModPhys.63.313) Mod. Phys. 63[, 313 \(1991\)](http://dx.doi.org/10.1103/RevModPhys.63.313); 64[, 633\(E\) \(1992\)](http://dx.doi.org/10.1103/RevModPhys.64.633).
- [12] S. G. Karshenboim, Phys. Rev. Lett. **104**[, 220406 \(2010\)](http://dx.doi.org/10.1103/PhysRevLett.104.220406).
- [13] J. Jaeckel and S. Roy, Phys. Rev. D **82**[, 125020 \(2010\).](http://dx.doi.org/10.1103/PhysRevD.82.125020)
- [14] B. Aubert et al. (BABAR Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.103.081803) 103[, 081803 \(2009\)](http://dx.doi.org/10.1103/PhysRevLett.103.081803).
- [15] F. Wilczek, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.39.1304) **39**, 1304 (1977).
- [16] P. del Amo Sanchez et al. (BABAR Collaboration), [Phys.](http://dx.doi.org/10.1103/PhysRevLett.104.191801) Rev. Lett. 104[, 191801 \(2010\).](http://dx.doi.org/10.1103/PhysRevLett.104.191801)
- [17] R. Van Royen and V. F. Weisskopf, [Nuovo Cimento A](http://dx.doi.org/10.1007/BF02823542) 50, [617 \(1967\)](http://dx.doi.org/10.1007/BF02823542); 51[, 583\(E\) \(1967\)](http://dx.doi.org/10.1007/BF02902203).
- [18] Y.A. Aleksandrov, G.S. Samosvat, J. Sereeter, and Tsoy Gen Sor, JETP Lett. 4, 134 (1966).
- [19] R. Barbieri and T. Ericson, [Phys. Lett. B](http://dx.doi.org/10.1016/0370-2693(75)90073-8) 57, 270 (1975).
- [20] J. Schmiedmayer, P. Riehs, J. A. Harvey, and N.W. Hill, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.66.1015) 66, 1015 (1991).
- [21] H. Leeb and J. Schmiedmayer, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.68.1472) 68, 1472 [\(1992\)](http://dx.doi.org/10.1103/PhysRevLett.68.1472).
- [22] For a review, see F. Jegerlehner and A. Nyffeler, [Phys.](http://dx.doi.org/10.1016/j.physrep.2009.04.003) Rep. 477[, 1 \(2009\)](http://dx.doi.org/10.1016/j.physrep.2009.04.003).
- [23] J. P. Leveille, [Nucl. Phys.](http://dx.doi.org/10.1016/0550-3213(78)90051-2) **B137**, 63 (1978).
- [24] M.L. Graesser, Phys. Rev. D 61[, 074019 \(2000\).](http://dx.doi.org/10.1103/PhysRevD.61.074019)
- [25] I. Beltrami et al., Nucl. Phys. A451[, 679 \(1986\)](http://dx.doi.org/10.1016/0375-9474(86)90299-X).
- [26] J. Insler et al. (CLEO Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.81.091101) 81, [091101 \(2010\).](http://dx.doi.org/10.1103/PhysRevD.81.091101)
- [27] B. Aubert et al. (BABAR Collaboration), [arXiv:0808.0017.](http://arXiv.org/abs/0808.0017)
- [28] K. Nakamura et al. (Particle Data Group), [J. Phys. G](http://dx.doi.org/10.1088/0954-3899/37/7A/075021) 37, [075021 \(2010\).](http://dx.doi.org/10.1088/0954-3899/37/7A/075021)
- [29] J. Altegoer et al. (NOMAD Collaboration), [Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(98)00402-X) 428[, 197 \(1998\).](http://dx.doi.org/10.1016/S0370-2693(98)00402-X)
- [30] S. N. Gninenko and N. V. Krasnikov, [Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(98)00358-X) 427, [307 \(1998\)](http://dx.doi.org/10.1016/S0370-2693(98)00358-X).
- [31] M. Ablikim et al. (BES Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.97.202002) 97[, 202002 \(2006\)](http://dx.doi.org/10.1103/PhysRevLett.97.202002).
- [32] P. Fayet, Phys. Rev. D **74**[, 054034 \(2006\).](http://dx.doi.org/10.1103/PhysRevD.74.054034)