Spontaneous Electromagnetic Superconductivity of Vacuum in a Strong Magnetic Field: Evidence from the Nambu–Jona-Lasinio Model

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Using an extended Nambu–Jona-Lasinio model as a low-energy effective model of QCD, we show that the vacuum in a strong external magnetic field (stronger than 10^{16} T) experiences a spontaneous phase transition to an electromagnetically superconducting state. The unexpected superconductivity of, basically, empty space is induced by emergence of quark-antiquark vector condensates with quantum numbers of electrically charged rho mesons. The superconducting phase possesses an anisotropic inhomogeneous structure similar to a periodic Abrikosov lattice in a type-II superconductor. The superconducting vacuum is made of a new type of vortices which are topological defects in the charged vector condensates. The superconductivity is realized along the axis of the magnetic field only. We argue that this effect is absent in pure QED.

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Strong magnetic fields may lead to unusual effects such as magnetic catalysis in the (2 + 1)-dimensional Gross-Neveu (GN) model [1,2], in QED [3], and in QCD [4]. The strong field supports the chiral magnetic effect in hot quark-gluon plasma [5] and a metalliclike conductivity in a quarkless vacuum of lattice $SU(2)$ Yang-Mills theory [6]. Recently, we suggested in Ref. [7] that an interplay between strong and electromagnetic interactions in a background of a sufficiently strong magnetic field may turn the cold vacuum into an electromagnetic superconductor if the strength of the magnetic field exceeds

$$B_c = m_\rho^2/e \approx 10^{16}\ \mathrm{T}, \qquad (1)$$

where $m_\rho = 775.5$ MeV is the mass of the ρ meson and e is the elementary electric charge. Magnetic fields of such strength scale may emerge in the heavy-ion collisions at the Large Hadron Collider in CERN [8] and, presumably, in the early Universe.

Our idea is based on a very simple argument: the particle spectrum of QCD contains a charged vector resonance (a spin-triplet excitation), $\rho^\pm$ meson, which has a large magnetic dipole moment associated with an anomalous gyromagnetic ratio $g = 2$ of the ρ meson. If one treats the ρ meson as a free particle, then in a background of a uniform magnetic field B its ground state energy [corresponding to the lowest Landau level (LLL)] becomes a decreasing function of the magnetic field strength, $E^2_{\rho^\pm}(B) = m^2_{\rho^\pm} - eB$. The energy of $\rho^\pm$ vanishes when the magnetic field reaches the value (1). As the field strength increases further, the ground state energy $E_{\rho^\pm}$ becomes purely imaginary, indicating a tachyonic instability of the ground state towards condensation of the ρ mesons. Since $\rho^\pm$ are electrically charged, their condensation implies an electromagnetic superconductivity of the new ground state. Surprisingly, there is no Meissner effect in the $g = 2$ case. Moreover, the strong magnetic field makes the ρ mesons stable at QCD time scales [7].

The suggested vacuum superconductivity has at least two other analogues in particle physics: the Nielsen-Olesen instability of the gluonic vacuum in Yang-Mills theory [9] and the Ambjørn-Olesen condensation of the W bosons induced by a strong magnetic field in the standard electroweak model [10].

In condensed matter physics, a similar phenomenon is known as "reentrant" superconductivity [11]. Usually, an external magnetic field suppresses the superconductivity via pair breaking effects, so that in a strong magnetic field the superconductivity is lost. However, there are superconductors which may reenter the superconducting phase again at stronger magnetic fields, like, for example, in the uranium compound URhGe [12].

There are various material-dependent proposals to describe specific reentrant superconductors in the condensed matter physics. Our suggestion in QCD [7] is close to the idea of Refs. [11,13] that in a very strong magnetic field the Abrikosov flux lattice of a type-II superconductor may exhibit a reentrant quantum regime, characterized by the LLL dominance and the absence of the Meissner effect.

In Ref. [7] we suggested the existence of the new superconducting phase using effective bosonic electrodynamics of the ρ mesons of Ref. [14]. However, this model treats the ρ mesons as pointlike particles, and thus it may be inapplicable at strong magnetic fields (1) when the magnetic length becomes of the order of the size of the ρ meson. Here we use the much more general fermionic Nambu–Jona-Lasinio (NJL) model [15] as a low-energy effective theory of QCD in order to show the existence of the electromagnetic superconductivity induced by the strong magnetic fields in the vacuum.

We consider an extended two-flavor ($N_f = 2$) Nambu–Jona-Lasinio model with three colors ($N_c = 3$):

$$\mathcal{L}(\psi, \bar\psi) = \bar\psi(i\partial\!\!\!/ + \hat{Q}\mathcal{A}\!\!\!/ - \hat{M}^0)\psi + \mathcal{L}_S^{(4)} + \mathcal{L}_V^{(4)}, \quad (2)$$

where the light quarks are represented by the doublet $\psi = (u, d)^T$ and $\hat{M}^0 = \mathrm{diag}(m_u^0, m_d^0)$ is the corresponding bare mass matrix [16]. The uniform magnetic field background $\vec{B} = (0, 0, B)$ is encoded in the Abelian gauge field $\mathcal{A}^\mu \equiv (\mathcal{A}^0, \vec{\mathcal{A}}) = (0, -Bx_2/2, Bx_1/2, 0)$, and the electric charges of the quarks, $q_u = +2e/3$ and $q_d = -e/3$, are combined into the matrix $\hat{Q} = \mathrm{diag}(q_u, q_d)$. The hat over a symbol indicates a 2×2 matrix in the flavor space.

The last two terms in Eq. (2) represent the scalar and vector four-quark interactions, respectively:

$$\mathcal{L}_S^{(4)} = \frac{G_S^{(0)}}{2}[(\bar\psi\psi)^2 + (\bar\psi i\gamma^5\vec\tau\psi)^2], \quad (3)$$

$$\mathcal{L}_V^{(4)} = -\frac{G_V^{(0)}}{2}\sum_{i=0}^{3}[(\bar\psi\gamma_\mu\tau^i\psi)^2 + (\bar\psi\gamma_\mu\gamma_5\tau^i\psi)^2], \quad (4)$$

where $G_S^{(0)}$ and $G_V^{(0)}$ are corresponding bare couplings, and $\vec\tau = (\tau^1, \tau^2, \tau^3)$ are the Pauli matrices

We follow the standard approach [16] and introduce the following bosonic fields corresponding to the quark-antiquark bilinears: one scalar field $\sigma \sim \bar\psi\psi$, the triplet of three pseudoscalar fields $\vec\pi \sim \bar\psi\gamma^5\vec\tau\psi$ [made of the electrically neutral, $\pi^0 \equiv \pi^3$, and electrically charged, $\pi^\pm = (\pi^1 \mp i\pi^2)/\sqrt{2}$, pions], four vector fields $V_\mu^i \sim \bar\psi\gamma_\mu\tau^i\psi$, and four axial fields $A_\mu^i \sim \bar\psi\gamma^5\gamma_\mu\tau^i\psi$,

$$\hat{V}_\mu \equiv \sum_{i=0}^{3}\tau^i V_\mu^i = \begin{pmatrix} \omega_\mu + \rho_\mu^0 & \sqrt{2}\rho_\mu^+ \\ \sqrt{2}\rho_\mu^- & \omega_\mu - \rho_\mu^0 \end{pmatrix}, \quad (5)$$

$$\hat{A}_\mu \equiv \sum_{i=0}^{3}\tau^i A_\mu^i = \begin{pmatrix} f_\mu + a_\mu^0 & \sqrt{2}a_\mu^+ \\ \sqrt{2}a_\mu^- & f_\mu - a_\mu^0 \end{pmatrix}. \quad (6)$$

The vector-meson matrix (5) is composed of the singlet (in the flavor space) vector (in the coordinate space) ω-meson field ω_μ, while $\rho_\mu^0 \equiv \rho_\mu^3$ and $\rho_\mu^\pm = (\rho_\mu^1 \mp i\rho_\mu^2)/\sqrt{2}$ represent, respectively, electrically neutral and charged components of the ρ-meson triplet. The light axial mesons are encoded in the matrix (6): the fields f_μ and $(a_\mu^0, a_\mu^\pm)$ represent, respectively, the singlet axial f_1 meson and the $\vec{a}_1$ triplet of the axial mesons, respectively.

We rewrite the four-quark interactions (3) and (4) via Gaussian integrals over the bosonic fields σ, $\vec\pi$, $\hat{V}_\mu$, $\hat{A}_\mu$, and integrate over the quarks in the partition function:

$$\mathcal{Z} = \int D\bar\psi D\psi e^{i\int d^4x\mathcal{L}} = \int D\sigma D\pi DV DA e^{iS[\sigma,\vec\pi,V,A]},$$

where the effective bosonic action $S = S[\sigma, \vec\pi, V, A]$ is

$$S = S_\psi + \int d^4x\Big[-\frac{1}{2G_S^{(0)}}(\sigma^2 + \vec\pi^2) + \frac{1}{2G_V^{(0)}}(V_\mu^k V^{k\mu} + A_\mu^k A^{k\mu})\Big], \quad (7)$$

$$S_\psi = -iN_c\,\mathrm{Tr}\ln(i\mathcal{D}), \quad (8)$$

$$i\mathcal{D} = i\partial\!\!\!/ + \hat{Q}\mathcal{A}\!\!\!/ - \hat{M}^0 + \hat{V\!\!\!/}_\mu + \gamma^5\hat{A\!\!\!/} - (\sigma + i\gamma^5\vec\pi\vec\tau). \quad (9)$$

Next, we calculate the effective action (7) in the strong magnetic field background in the mean field approach. We use simplified notations for the expectation values of the fields, $\langle\sigma\rangle = \sigma$, etc. In the absence of the external magnetic field the expectation values of the fields $\vec\pi$, V, and A are zero [16], while the expectation value of σ plays a role of the constituent quark mass, $m_q = \sigma \sim 300$ MeV.

In order to simplify our calculations, we notice that the presence of the external magnetic field breaks the flavor symmetry down to its diagonal subgroup, so that the diagonal chiral rotations $\Omega = e^{i\alpha_5\tau^3\gamma_5}$ can still be used to eliminate the neutral pion condensate π^0. We also neglect the mass matrix M^0 because $m_{u,d}^0 \ll \sigma$.

The operator (9) can be represented as the sum $i\mathcal{D} = i\mathcal{D}_0 + \hat{W}$ of the tree-level operator $i\mathcal{D}_0 = i\partial\!\!\!/ + \hat{Q}\mathcal{A}\!\!\!/ - \sigma$ and the contribution $\hat{W}$ from the "exotic" condensates,

$$\hat{W} = \hat{V\!\!\!/}_\mu + \gamma^5\hat{A\!\!\!/} - i\gamma^5(\pi^1\tau^1 + \pi^2\tau^2), \quad (10)$$

At low magnetic fields $\hat{W} \equiv \langle\hat{W}\rangle = 0$. Let us assume that, at a certain strong magnetic field $B = B_c^{\mathrm{NJL}}$, the expectation value of the condensate (10) is nonzero. Let us advance slightly into the new phase taking $B \gtrsim B_c^{\mathrm{NJL}}$, so that the magnitude of the suspected condensate is still small, $0 < |\hat{W}| \ll \sigma$. Then the effective action (7) can be expanded in powers of the $\hat{W}$ field, and the fact of the emergence of the new condensate should be seen as a tachyonic instability of the effective potential at $\hat{W} = 0$.

The tree-level propagator $S^{(0)} \equiv \mathcal{D}_0^{-1}$ of the fermion doublet in the strong magnetic field has the following form: $S^{(0)}(x, y) = \mathrm{diag}[S_u^{(0)}(x, y), S_d^{(0)}(x, y)]$, where S_f is the propagator of the fth quark species.

The ρ-meson condensation and, consequently, the induced superconductivity are the LLL phenomena [7]. Thus, it is natural to restrict ourselves to the LLL approximation which usually gives a dominant contribution to nonperturbative low-energy quantities in the limit of the strong magnetic field [3,4,17].

In the LLL regime the propagator $S_f^{(0)}$ factorizes into the B-transverse and B-longitudinal parts which depend, separately, on the B-transverse, $x^\perp = (x^1, x^2)$, and B-longitudinal, $x^\parallel = (x^0, x^3)$, coordinates [3]:

$$S_f^{(0),\mathrm{LLL}}(x, y) = P_f^\perp(x^\perp, y^\perp)S_f^\parallel(x^\parallel - y^\parallel) \quad (11)$$

[below we omit the superscripts "(0)" and "LLL"]. Here,

$$P_f^\perp(x^\perp, y^\perp) = \frac{|q_f B|}{2\pi}e^{(i/2)q_f B\varepsilon_{ab}x^a x^b - (1/4)|q_f B|(x^\perp - y^\perp)^2} \quad (12)$$

is the transverse projector onto the LLL states and q_f is the electric charge of the fth quark.

The longitudinal part of the fermion propagator (11), $S_f^\parallel \equiv S_{\mathrm{sgn}(q_f B)}^\parallel$, is, basically, a fermion propagator in the

$1+1$ dimensions (we always take $eB>0$),

$$S_f^{\|}(k_\|) = \frac{i}{\gamma^\| k_\| - m} P_f^\|, \qquad P_f^\| = \frac{\mathbb{1} - if\gamma^1\gamma^2}{2}, \tag{13}$$

and the matrix $P_f^\|$ (we use $f=\pm 1$ for, respectively, $f=u, d$) is the spin projector operator onto the fermion states with the spin polarized along (for u quarks) or opposite (for d quarks) to the magnetic field. The operator $P_f^\|$ projects the original four $3+1$ fermionic states onto two $(1+1)$-dimensional fermionic states, so that fermions can move only along the axis of the magnetic field. The projector (12) satisfies the relation $P_f^\perp \circ P_f^\perp = P_f^\perp$, where "$\circ$" is the convolution operator in the B-transverse space, $A\circ B \equiv \int d^2y^\perp A(\ldots, y^\perp)B(y^\perp, \ldots)$.

For a coordinate-independent condensate σ, the zero-order (in powers of $\hat{W}$) contribution to the effective action (8) gives us the potential $V(\sigma) = V_\psi^{(0)}(\sigma) + \sigma^2/(2G_S^{(0)})$ related to the action as $S = -\int d^4x V$, with

$$V_\psi^{(0)} = iN_c \operatorname{Tr}\ln i\mathcal{D}_0 = \frac{|eB|N_c}{8\pi^2}\left[\sigma^2 \ln\frac{\sigma^2}{\mu^2} - \left(\frac{1}{\bar{\epsilon}} + 1\right)\sigma^2\right],$$

where $1/\bar{\epsilon} = 1/\epsilon - \gamma_E + \log 4\pi$, $\gamma_E \approx 0.577\,22$ is Euler's constant, and μ is a renormalization mass scale. In order to regularize the divergent contributions of the $(1+1)$-dimensional fermions, we implemented the dimensional regularization in $d = 2-2\epsilon$ dimensions. The renormalization of the NJL coupling constant in the $\overline{\mathrm{MS}}$ scheme, $1/G_S = 1/G_S^{(0)} - N_c|eB|/(4\pi^2\bar{\epsilon})$, resembles the renormalization of the $(1+1)$-dimensional GN model [1] with the identification $G_S \equiv 2\pi G_{\mathrm{GN}}/(N_c|eB|)$ [3].

The minimum $\sigma = \sigma_{\min}$ of the renormalized potential,

$$V(\sigma) = \frac{1}{2G_S}\sigma^2 + \frac{|eB|N_c}{8\pi^2}\left(\ln\frac{\sigma^2}{\mu^2} - 1\right)\sigma^2,$$

provides us with the B-dependent quark mass

$$m_q(B) = \sigma_{\min}(B) = \mu \exp\{-2\pi^2/(G_S N_c |eB|)\}. \tag{14}$$

In the LLL approximation to the NJL model the scale μ is not fixed as it is related to the B-longitudinal $1+1$ motion of the quarks. Beyond the LLL approach the scale may perhaps be set as $\mu^2 \propto |eB|$ following Ref. [4].

The effective bosonic model [7] suggests that the possible superconducting ground state should exhibit an inhomogeneous behavior in the B-transverse plane. Thus, we assume that the exotic condensates may be $x^\perp$ dependent, $\hat{W} = \hat{W}(x^\perp)$, and calculate the corresponding quadratic contribution to the effective action (8),

$$S_\psi^{(2)} = -\int d^4x V_\psi^{(2)} = \frac{iN_c}{2}\operatorname{Tr}\frac{1}{i\mathcal{D}_0}W\frac{1}{i\mathcal{D}_0}W. \tag{15}$$

We find that the potential (15) involves only the B-transverse components of the vector and axial mesons,

$$\int d^2x^\perp V_\psi^{(2)} = -\frac{4N_c|eB|}{9\pi^2}\left[\left(\frac{1}{\bar{\epsilon}} - \ln\frac{\sigma^2}{\mu^2}\right)(\phi^* \circ P_e \circ \phi) + \left(\frac{1}{\bar{\epsilon}} - \ln\frac{\sigma^2}{\mu^2} - 2\right)(\xi^* \circ P_e \circ \xi)\right], \tag{16}$$

where $\phi = (\rho_1^+ + i\rho_2^+)/2$ and $\xi = (a_1^+ + ia_2^+)/2$. The B-transverse projector for the unit charged particle, $P_e^\perp(x^\perp, y^\perp) = (9\pi/|eB|)P_u^\perp(x^\perp, y^\perp)P_d^\perp(y^\perp, x^\perp)$, is given by Eq. (12) with the replacement $q_f \to e$.

The unstable tachyonic mode of the potential (7) and (16) turns out to be an inhomogeneous eigenstate of the charge-1 projection operator P_e,

$$(P_e \circ \phi)(x^\perp) = \phi(x^\perp). \tag{17}$$

The solution is a general Abrikosov-like configuration [18]

$$\phi = \phi_0 K(\bar{z}/L_B), \qquad L_B = \sqrt{2\pi/|eB|}, \tag{18}$$

$$K(z) = e^{-(\pi/2)(|z|^2+z^2)} \sum_{n=-\infty}^{+\infty} c_n e^{-\pi n^2 + 2\pi n z}, \tag{19}$$

where ϕ_0 and c_n are arbitrary complex parameters and $z = x^1 + ix^2$ (and similarly for the axial vector field ξ).

The solution (18) represents a (periodic) flux-tube structure similar to the Abrikosov lattice which is realized in a mixed state of a type-II superconductor subjected to a near-critical external magnetic field [18]. Generally, the coefficients c_n can be fine-tuned by a complicated minimization procedure if the full potential is known [18]. Here we follow Refs. [7,10] and set $c_n = 1$ so that the solution (18) represents a square lattice with the quantized area $2\pi/|eB| \equiv L_B^2$ given by the magnetic length L_B.

The quadratic potential, evaluated at the solution (18),

$$V^{(2)} = \sqrt{2}\left[\frac{1}{G_B}(|\phi_0|^2 + |\xi_0|^2) - \frac{2N_c|eB|}{9\pi^2}(|\phi_0|^2 - |\xi_0|^2)\right],$$

is unstable towards a spontaneous creation of the B-transverse $\rho^\pm$ condensates with the tachyonic mode $\rho_1^+ = i\rho_2^+ = \phi$ if the magnetic field exceeds

$$B_c^{\mathrm{NJL}} = \frac{9\pi^2}{2eN_c G_B}, \qquad \frac{1}{G_B} = \frac{1}{G_V} - \frac{8}{9G_S}, \tag{20}$$

with $1/G_V = 1/G_V^{(0)} - N_c|eB|/(9\pi^2\bar{\epsilon}')$ and $1/\bar{\epsilon}' \equiv 1/\bar{\epsilon} - 1$. Since the phenomenological values of the parameters $G_{S,V}$ vary in a broad region [19], we can only give an approximate estimation of the critical field: $eB_c \sim 1\ \mathrm{GeV}^2$ or $B_c \sim 10^{16}$ T.

The quartic correction to the potential in Eq. (8),

$$V_\psi^{(4)} = C_0 \frac{|eB|N_c}{2\pi^2 m^2}|\phi_0|^4, \tag{21}$$

allows us to find the condensate at $B \geq B_c^{\mathrm{NJL}}$:

$$\phi_0(B) = e^{i\theta_0} C_\phi m_q(B)(1 - B_c^{\mathrm{NJL}}/B)^{1/2}, \tag{22}$$

where θ_0 is a constant phase, $C_0 \approx 1.2$, $C_\phi \approx 0.51$, and the quark mass m_q is given in Eq. (14). At $B < B_c^{\mathrm{NJL}}$ the condensate (22) is zero. The phase transition at $B = B_c$ is of the second order with the critical exponent $1/2$.

Thus, the magnetic field induces the quark condensate

$$\langle \bar{u}\gamma_1 d\rangle = -i\langle \bar{u}\gamma_2 d\rangle = \rho_0(B) K\left(\frac{x_1 + ix_2}{L_B}\right) \equiv \rho(x^\perp), \quad (23)$$

where $\rho_0(B) = \phi_0(B)/G_V$. Using known (see, e.g., Ref. [18]) general properties of the function $K(z)$, Eq. (19), we conclude that the ground state should be given by a periodic (in general) lattice of a new type of topological vortices which are parallel to the magnetic field. The phase of the condensate (23) winds around the center of each vortex where the absolute value of $\rho(x^\perp)$ vanishes.

The condensate (23) breaks the local $U(1)_{\mathrm{e.m.}}$ transformations by locking them with the global $O(2)_{\mathrm{rot}}$ rotations of the coordinate space about the magnetic field axis [7,20]: $U(1)_{\mathrm{e.m.}} \times O(2)_{\mathrm{rot}} \to G_{\mathrm{lat}}$, where G_{lat} is a discrete symmetry group of rotations of the ρ-vortex lattice.

The new vacuum state is superconducting. One can show that there is no B-transverse current, $J^1 = J^2 = 0$, so that the electric current flows along the magnetic field axis only. In a very weak (test) electric field $\vec{E} = (0, 0, E_z)$ with $E_z \ll B$, the induced electric current in the new vacuum state (23) in a linear-response approximation is (we use the retarded Green functions)

$$J^\mu(x) = \sum_{f=u,d} q_f \langle \bar{\psi}_f \gamma^\mu \psi_f\rangle \equiv -\mathrm{Tr}[\gamma^\mu \hat{Q} S(x,x)]. \quad (24)$$

We average the current (24) over the B-transverse plane and, in the leading order in powers of ρ, we get

$$\frac{\partial \mathcal{Q}}{\partial z} + \frac{\partial \mathcal{J}}{\partial t} = \frac{2C_q}{(2\pi)^3} e^3 (B - B_c^{\mathrm{NJL}}) E_z, \quad (25)$$

where $\mathcal{Q}$ is the plane-averaged electric charge density J^0, $\mathcal{J}$ is the plane-averaged current J^z, and $C_q \approx 1$ [21]. At $B < B_c$ the right-hand side of Eq. (25) is zero. Apart from prefactors, the transport laws in the NJL model (25) and in the ρ-meson electrodynamics [7] are identical.

The linear-response law (25) can be rewritten in a Lorentz-covariant form, $\partial^{[\mu,} J^{\nu]} = \gamma \cdot (F, \tilde{F}) \tilde{F}^{\mu\nu}$, via the invariants $(F, \tilde{F}) = 4(\vec{B}, \vec{E})$ and $(F, F) = 2(\vec{B}^2 - \vec{E}^2)$. Here $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}/2$ and γ is a function of (F, F) [20].

Equation (25) is a London equation for an anisotropic superconductivity. Thus, we have just shown that the strong magnetic field induces the new electromagnetically superconducting phase of the vacuum if $B > B_c$. An empty space becomes an anisotropic superconductor.

The superconductivity of the vacuum is a new effect which is realized at the QCD-QED interface. This mechanism should not work in the pure QED since electrically charged spin-1 bound states are absent there.

On general grounds one can expect that increase in temperature T (which, in general, should be of a hadronic scale) should lead to an evaporation of the ρ condensate with a loss of the superconductivity. The suggested low-T part of the B-T phase diagram is shown in Fig. 1.

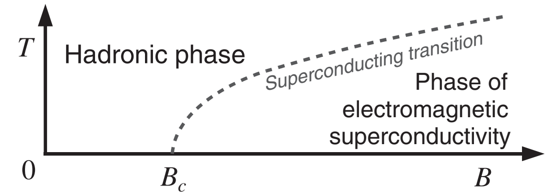

FIG. 1. Low-temperature part of the QCD phase diagram.

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