

Long-Distance Dominance of the CP Asymmetry in $\bar{B} \rightarrow X_{s,d}\gamma$ Decays

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(Received 14 December 2010; published 4 April 2011)

We show that in the standard model the parametrically leading (by a factor $1/\alpha_s$) contribution to the inclusive CP asymmetry in $\bar{B} \rightarrow X_{s,d}\gamma$ decays arises from a long-distance effect in the interference of the electromagnetic dipole amplitude with the amplitude for an up-quark penguin transition accompanied by soft gluon emission. Using model estimates for the associated hadronic parameter $\tilde{\Lambda}_{17}^u$, we predict a value in the range $-0.6\% < \mathcal{A}_{X_s,\gamma}^{\text{SM}} < 2.8\%$. In view of current experimental data, a future precision measurement of the flavor-averaged CP asymmetry would signal the presence of new physics only if a value below -2% was found. A cleaner probe of new physics is offered by the difference of the CP asymmetries in charged versus neutral $\bar{B} \rightarrow X_s\gamma$ decays.

DOI: 10.1103/PhysRevLett.106.141801

PACS numbers: 13.20.He, 11.15.Tk, 11.30.Er, 12.39.Hg

The radiative decays $\bar{B} \rightarrow X_s\gamma$ and $\bar{B} \rightarrow X_d\gamma$ are important for testing the standard model (SM) and probing its possible extensions. Both the CP -averaged branching ratios and the CP asymmetries are useful in this context. Thanks to a vigorous effort, the experimental error on the $\bar{B} \rightarrow X_s\gamma$ branching ratio has been reduced below 10%, which is close to the (irreducible) theoretical uncertainty in the calculation of this observable. For the CP asymmetry, on the other hand, the experimental error is thought to be much larger than the theoretical one. The current world average is [1]

$$\mathcal{A}_{X_s,\gamma} = \frac{\Gamma(\bar{B} \rightarrow X_s\gamma) - \Gamma(B \rightarrow X_s\gamma)}{\Gamma(\bar{B} \rightarrow X_s\gamma) + \Gamma(B \rightarrow X_s\gamma)} = -(1.2 \pm 2.8)\%. \quad (1)$$

It is widely believed that the theoretical prediction for the CP asymmetry in the SM is short-distance dominated, leading to a tiny value of about 0.5% as a result of a combination of perturbative, CKM, and GIM suppression [2–4]. A dedicated analysis finds $\mathcal{A}_{X_s,\gamma}^{\text{SM}} = (0.44_{-0.10}^{+0.15} \pm 0.03_{-0.09}^{+0.19})\%$ [5], where the errors refer to uncertainties associated with the quark-mass ratio m_c/m_b , CKM parameters, and higher-order perturbative corrections. This suggests that finding an asymmetry outside the range $0 < \mathcal{A}_{X_s,\gamma} < 1\%$ would be a clean signal of new physics. Indeed, various extensions of the SM have been analyzed with regard to the constraints arising from the measured value of the CP asymmetry [3,6–15], and reducing the experimental error to the 1% level is considered an important goal of future superflavor factories.

In recent work [16], we have presented a new factorization formula for inclusive radiative B decays in the relevant region of large photon energy. In addition to the familiar “direct photon contributions”, in which the photon couples to a local operator mediating the weak decay in the effective low-energy theory, novel “resolved photon

contributions” appear. They account for the hadronic substructure of the photon, which is probed when it couples to light collinear partons. Importantly, even after integrating over the photon energy spectrum, the resolved photon contributions cannot be described using a local operator-product expansion. They give rise to first-order Λ_{QCD}/m_b corrections to the inclusive decay rate and CP asymmetry, which are proportional to B -meson matrix elements of nonlocal operators in heavy-quark effective theory (HQET). The analysis of [16] has shown that these nonperturbative effects lead to an irreducible uncertainty in the theoretical prediction for the CP -averaged $\bar{B} \rightarrow X_s\gamma$ branching ratio of about $\pm 5\%$.

An interesting feature of the resolved photon contributions is that they give rise to novel, calculable strong-interaction phases related to hard-collinear jet functions, which are convoluted with real, nonperturbative soft functions. It is interesting to explore the potential impact of these contributions on the inclusive CP asymmetries. While these effects are still of order Λ_{QCD}/m_b , we show that they give the parametrically leading contribution to the $\bar{B} \rightarrow X_{s,d}\gamma$ asymmetries in the SM, and that they can strongly influence the way in which new-physics effects might show up.

Experiments measure the CP asymmetry $\mathcal{A}_{X_s,\gamma}(E_0)$ defined with a lower cut on the photon energy, $E_\gamma \geq E_0$, with E_0 ranging between 1.9 and 2.2 GeV. Detailed theoretical studies have shown that the dependence of the asymmetry on the value of E_0 is very mild [3]. We will assume for simplicity that the cutoff can be chosen sufficiently low, such that $\Delta \equiv m_b - 2E_0 = \text{few} \times \Lambda_{\text{QCD}}$ is in the perturbative domain. We will refer to the asymmetry defined with such a cut as “partially inclusive.” In this case the direct photon contributions to the CP asymmetry can be calculated in terms of local operator matrix elements using a combined expansion in powers of Δ/m_b and $\Lambda_{\text{QCD}}/\Delta$ [17].

The resolved photon contributions can still not be expressed in terms of local matrix elements. However, the relevant soft functions can be simplified in this limit, as described in [16].

The direct photon contributions to the partially inclusive CP asymmetry reduce to perturbative expressions depending on a cut parameter $\delta = \Delta/m_b$. At first nontrivial order in α_s , one obtains [3,18]

$$\begin{aligned} \mathcal{A}_{X_s\gamma}^{\text{dir}}(E_0) = & \alpha_s \left\{ \frac{40}{81} \text{Im} \frac{C_1}{C_{7\gamma}} - \frac{8z}{9} [v(z) \right. \\ & + b(z, \delta)] \text{Im} \left[(1 + \epsilon_s) \frac{C_1}{C_{7\gamma}} \right] - \frac{4}{9} \text{Im} \frac{C_{8g}}{C_{7\gamma}} \\ & + \frac{8z}{27} b(z, \delta) \frac{\text{Im}[(1 + \epsilon_s) C_1 C_{8g}^*]}{|C_{7\gamma}|^2} \\ & \left. + \frac{16z}{27} \tilde{b}(z, \delta) \left| \frac{C_1}{C_{7\gamma}} \right|^2 \text{Im} \epsilon_s \right\}, \end{aligned} \quad (2)$$

where $z = (m_c/m_b)^2$, and $\epsilon_s = (V_{ub}V_{us}^*)/(V_{tb}V_{ts}^*) = \lambda^2(i\bar{\eta} - \bar{\rho})/[1 - \lambda^2(1 - \bar{\rho} + i\bar{\eta})] + \mathcal{O}(\lambda^6)$ in terms of Wolfenstein parameters. The Wilson coefficients of the electromagnetic and chromomagnetic dipole operators in the effective weak Hamiltonian are denoted by $C_{7\gamma}$ and C_{8g} , while C_1 is the coefficient of the dominant current-current operator. Contributions from other operators are negligibly small in the SM. We suppress the scale dependence of α_s and C_i . Explicit expressions for the functions $v(z)$, $b(z, \delta)$, and $\tilde{b}(z, \delta)$ can be found in [3,19]. In the SM the Wilson coefficients are real, and only terms in (2) proportional to the imaginary part of ϵ_s contribute. The numerically most important contributions arise from the interference of charm- and up-quark penguin graphs with virtual or real gluon emission (first two diagrams in Fig. 1) with the leading electromagnetic dipole amplitude. The above expression is however more general, as it holds for all new-physics models in which the dominant nonstandard effects are described by additional (possibly complex) contributions to the dipole coefficients $C_{7\gamma}$ and C_{8g} . It would be straightforward to extend the analysis to include opposite-chirality dipole operators [3].

The above expression simplifies considerably if we adopt the power counting $m_c^2 = \mathcal{O}(m_b \Lambda_{\text{QCD}})$ for the charm-quark mass and expand in powers of z , $\delta = \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$. The terms proportional to $b(z, \delta)$ and $\tilde{b}(z, \delta)$ scale as $(\Lambda_{\text{QCD}}/m_b)^2$ and can be neglected to a good approximation, while the contribution proportional to $v(z)$

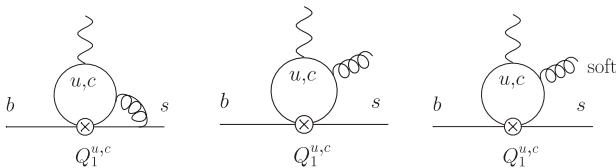


FIG. 1. Penguin diagrams with virtual and real gluon emissions. The gluons in the first two graphs are highly energetic.

contributes at first order in Λ_{QCD}/m_b and can be simplified by expanding $v(z)$ to zeroth order in z . This yields the approximation

$$\begin{aligned} \mathcal{A}_{X_s\gamma}^{\text{dir}} = & \alpha_s \left\{ \frac{40}{81} \text{Im} \frac{C_1}{C_{7\gamma}} - \frac{4}{9} \text{Im} \frac{C_{8g}}{C_{7\gamma}} \right. \\ & \left. - \frac{40\Lambda_c}{9m_b} \text{Im} \left[(1 + \epsilon_s) \frac{C_1}{C_{7\gamma}} \right] + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^2}{m_b^2} \right) \right\}, \end{aligned} \quad (3)$$

which is independent of the cutoff E_0 . Here we have introduced the scale $\Lambda_c \sim \Lambda_{\text{QCD}}$ defined as (we use $m_b = 4.65$ GeV and $m_c = 1.13$ GeV as in [16])

$$\Lambda_c \equiv \frac{m_c^2}{m_b} \left(1 - \frac{2}{5} \ln \frac{m_b}{m_c} + \frac{4}{5} \ln^2 \frac{m_b}{m_c} - \frac{\pi^2}{15} \right) \approx 0.38 \text{ GeV}. \quad (4)$$

In the SM only the last term in (3) contributes, which exhibits the triple suppression by α_s , $\text{Im}(\epsilon_s) \sim \lambda^2$, and $(m_c/m_b)^2 \sim \Lambda_{\text{QCD}}/m_b$.

Using the factorization analysis of $\bar{B} \rightarrow X_s \gamma$ decay performed in [16], it is possible to derive for the first time the resolved photon contributions to the partially inclusive CP asymmetry. They arise from the interference of the electromagnetic dipole amplitude with resolved photon contributions involving up- and charm-quark penguin transitions or chromomagnetic dipole transitions. The result can be expressed in terms of three hadronic parameters $\tilde{\Lambda}_{ij}$ related to convolution integrals over two soft functions denoted by $h_{17}(\omega)$ and $h_{78}^{(1)}(\omega_1, \omega_2)$. At lowest order in α_s and $1/m_b$ we find

$$\begin{aligned} \mathcal{A}_{X_s\gamma}^{\text{res}} = & \frac{\pi}{m_b} \left\{ \text{Im} \left[(1 + \epsilon_s) \frac{C_1}{C_{7\gamma}} \right] \tilde{\Lambda}_{17}^c - \text{Im} \left[\epsilon_s \frac{C_1}{C_{7\gamma}} \right] \tilde{\Lambda}_{17}^u \right. \\ & \left. + \text{Im} \frac{C_{8g}}{C_{7\gamma}} 4\pi\alpha_s \tilde{\Lambda}_{78}^{\bar{B}} \right\}, \end{aligned} \quad (5)$$

where (omitting the scale dependence of the soft functions and $\tilde{\Lambda}_{ij}$ parameters)

$$\begin{aligned} \tilde{\Lambda}_{17}^u &= \frac{2}{3} h_{17}(0), \\ \tilde{\Lambda}_{17}^c &= \frac{2}{3} \int_{4m_c^2/m_b}^{\infty} \frac{d\omega}{\omega} f \left(\frac{m_c^2}{m_b \omega} \right) h_{17}(\omega), \\ \tilde{\Lambda}_{78}^{\bar{B}} &= 2 \int_{-\infty}^{\infty} \frac{d\omega}{\omega} [h_{78}^{(1)}(\omega, \omega) - h_{78}^{(1)}(\omega, 0)], \end{aligned} \quad (6)$$

with

$$f(x) = 2x \ln \frac{1 + \sqrt{1 - 4x}}{1 - \sqrt{1 - 4x}}. \quad (7)$$

The functions h_{ij} are defined in terms of B -meson matrix elements of nonlocal operators in HQET [16]. In light-cone gauge $n \cdot A = 0$, we have

$$h_{17}(\omega) = \int \frac{dt}{2\pi} e^{-i\omega t} \frac{\langle \bar{B} | \bar{h}(0) \bar{n} i \gamma_{\alpha}^{\perp} \bar{n}_{\beta} G^{\alpha\beta}(t\bar{n}) h(0) | \bar{B} \rangle}{2M_B}, \quad (8)$$

where $n^\mu = (1, 0, 0, 1)$ and $\bar{n}^\mu = (1, 0, 0, -1)$ are two lightlike vectors, and h are effective heavy-quark fields in HQET [20]. Similarly, $h_{78}^{(1)}(\omega_1, \omega_2)$ is given in terms of a matrix element of a four-quark operator [19]. The function h_{17} is an even function of ω , whose normalization is equal to the HQET parameter $2\lambda_2 \approx (0.5 \text{ GeV})^2$. No rigorous constraints are known for the function $h_{78}^{(1)}$. Note that the parameter $\tilde{\Lambda}_{78}^{\tilde{B}}$ in (5) depends on the flavor of the spectator quark inside the \tilde{B} meson. In the limit of SU(3) flavor symmetry, it can be shown that $\tilde{\Lambda}_{78}^{\tilde{B}} = e_{\text{spec}} \tilde{\Lambda}_{78}$ [16], where e_{spec} denotes the electric charge of the spectator quark in units of e ($e_{\text{spec}} = 2/3$ for B^- and $-1/3$ for \tilde{B}^0). Evaluating the hadronic matrix element of the corresponding nonlocal four-quark operator in the vacuum insertion approximation, we find that

$$\tilde{\Lambda}_{78}^{\tilde{B}} \approx e_{\text{spec}} \tilde{\Lambda}_{78} \approx e_{\text{spec}} \frac{2f_B^2 M_B}{9} \int_0^\infty d\omega \frac{[\phi_+^B(\omega)]^2}{\omega}, \quad (9)$$

where $f_B \approx 193 \text{ MeV}$ is the B -meson decay constant and ϕ_+^B its leading light-cone distribution amplitude [21].

At present, there does not exist any systematic theoretical approach to determine the hadronic parameters $\tilde{\Lambda}_{ij}$ from first principles. Numerical estimates must then be obtained by modeling the corresponding soft functions. Employing the models studied in [16], we obtain the ranges

$$\begin{aligned} -330 \text{ MeV} &< \tilde{\Lambda}_{17}^u < +525 \text{ MeV}, \\ -9 \text{ MeV} &< \tilde{\Lambda}_{17}^c < +11 \text{ MeV}, \\ 17 \text{ MeV} &< \tilde{\Lambda}_{78} < 190 \text{ MeV}. \end{aligned} \quad (10)$$

All three estimates are very uncertain, but we observe that $\tilde{\Lambda}_{17}^u$ and $\tilde{\Lambda}_{78}$ are expected to be of order Λ_{QCD} . The slight preference for positive values of $\tilde{\Lambda}_{17}^q$ is due to the normalization constraint on the function h_{17} mentioned above. Note that in the formal limit $m_c \rightarrow m_u = 0$ the values of $\tilde{\Lambda}_{17}^u$ and $\tilde{\Lambda}_{17}^c$ coincide. However, we predict a strong GIM violation owing to the fact that the integral in the second relation in (6) starts at $4m_c^2/m_b \approx 1.1 \text{ GeV}$, at which the soft function is expected to take already rather small values, since it is governed by nonperturbative dynamics and must vanish for $\omega \rightarrow \infty$.

The complete theoretical result for the partially inclusive CP asymmetry in $\tilde{B} \rightarrow X_s \gamma$ decay is obtained by adding the direct and resolved contributions (2) and (5). In order to understand better the structure of the result, we now present some approximate formulae obtained by using expression (3) for the direct contribution. Our numerical results will always be derived using the exact expression. For the CP asymmetry in the SM, we obtain

$$\begin{aligned} \mathcal{A}_{X_s \gamma}^{\text{SM}} &\approx \pi \left| \frac{C_1}{C_{7\gamma}} \right| \text{Im} \epsilon_s \left(\frac{\tilde{\Lambda}_{17}^u - \tilde{\Lambda}_{17}^c}{m_b} + \frac{40\alpha_s}{9\pi} \frac{\Lambda_c}{m_b} \right) \\ &= \left(1.15 \frac{\tilde{\Lambda}_{17}^u - \tilde{\Lambda}_{17}^c}{300 \text{ MeV}} + 0.71 \right) \%. \end{aligned} \quad (11)$$

We fix the photon cut at $E_0 = 1.9 \text{ GeV}$ and use $\lambda = 0.2254$, $\bar{\rho} = 0.144$, $\bar{\eta} = 0.342$ for the Wolfenstein parameters. Our choice $\mu = 2 \text{ GeV}$ for the factorization scale (for which $\alpha_s(\mu) = 0.307$, and $C_1(\mu) = 1.204$, $C_{7\gamma}(\mu) = -0.381$, $C_{8g}(\mu) = -0.175$ at leading order) is motivated by the fact that the strong phases required for a nonzero CP asymmetry arise either from GIM violations related to charm-quark loops (for which $\mu \sim 2m_c$), or from cut hard-collinear propagators (for which $\mu \sim \sqrt{m_b \Lambda_{\text{QCD}}}$). In the resolved photon term we keep the contribution of $\tilde{\Lambda}_{17}^c$ in order to make explicit that the CP asymmetry vanishes in the formal limit $m_c = m_u$ due to the GIM mechanism. In practice, however, this contribution can be safely neglected. The dominant contribution arises from the up-quark penguin graph with emission of a soft gluon, in which the quark loop is probed at a lightlike distance away from the heavy quarks (last diagram in Fig. 1). We do not show the dependences of our results on variations of the input parameters, which are much smaller than the uncertainties associated with the resolved photon contributions. Our central value 0.71% for the direct photon term is larger than that obtained in [5] since we use smaller values for μ and m_c .

The resolved photon term proportional to $\tilde{\Lambda}_{17}^u$ in (11) is parametrically larger than the direct photon term, which contains an additional α_s suppression. Numerically, this term dominates as long as $|\tilde{\Lambda}_{17}^u|$ is larger than about 200 MeV. Using the model estimates shown in (10) we find the range $-0.6\% < \mathcal{A}_{X_s \gamma}^{\text{SM}} < 2.8\%$, which covers most of the experimentally allowed range (1). Only a value of the asymmetry below -2% could be interpreted as a sign of new physics, as in this case $\tilde{\Lambda}_{17}^u < -700 \text{ MeV}$ would be much larger in magnitude than our model expectations.

In extensions of the SM, in which the dipole coefficients $C_{7\gamma}$ and C_{8g} receive new CP -violating contributions, additional terms arise. Using the approximation (3), we find

$$\begin{aligned} \frac{\mathcal{A}_{X_s \gamma}}{\pi} &\approx \left[\left(\frac{40}{81} - \frac{40}{9} \frac{\Lambda_c}{m_b} \right) \frac{\alpha_s}{\pi} + \frac{\tilde{\Lambda}_{17}^c}{m_b} \right] \text{Im} \frac{C_1}{C_{7\gamma}} \\ &\quad - \left(\frac{4\alpha_s}{9\pi} - 4\pi\alpha_s e_{\text{spec}} \frac{\tilde{\Lambda}_{78}}{m_b} \right) \text{Im} \frac{C_{8g}}{C_{7\gamma}} \\ &\quad - \left(\frac{\tilde{\Lambda}_{17}^u - \tilde{\Lambda}_{17}^c}{m_b} + \frac{40}{9} \frac{\Lambda_c}{m_b} \frac{\alpha_s}{\pi} \right) \text{Im} \left(\epsilon_s \frac{C_1}{C_{7\gamma}} \right). \end{aligned} \quad (12)$$

For the first two terms the resolved photon contributions give rise to power corrections to the direct photon terms, which are numerically significant since $\alpha_s/\pi \sim \Lambda_{\text{QCD}}/m_b$. For the third term, which is the only one present in the SM, the resolved photon contribution is likely to be more important than the direct photon term.

To illustrate the impact of the resolved photon terms, we consider the class of new-physics models in which the dominant nonstandard effects are encoded in the values of the dipole operators, which we parameterize in

the form $C_{7\gamma}/C_1 = (C_{7\gamma}/C_1)^{\text{SM}} r_7 e^{i\theta_7}$ and $C_{8g}/C_1 = (C_{8g}/C_1)^{\text{SM}} r_8 e^{i\theta_8}$. Using that $\arg(-\epsilon_s) \approx -\gamma$ in the SM, we then obtain for our default parameters

$$\begin{aligned} \mathcal{A}_{X_s\gamma}[\%] = & \left(10.12 + 2.14 \frac{\tilde{\Lambda}_{17}^c}{10\text{MeV}}\right) \frac{1}{r_7} \sin\theta_7 \\ & + \left(1.26 \frac{\tilde{\Lambda}_{17}^u - \tilde{\Lambda}_{17}^c}{300\text{MeV}} + 0.74\right) \frac{1}{r_7} \sin(\gamma + \theta_7) \\ & + \left(6.27 - 11.98 e_{\text{spec}} \frac{\tilde{\Lambda}_{78}}{100\text{MeV}}\right) \frac{r_8}{r_7} \sin(\theta_7 - \theta_8) \\ & + 0.18 \frac{r_8}{r_7^2} \sin\theta_8 + \frac{0.037}{r_7^2} \sin\gamma \\ & - 0.004 \frac{r_8}{r_7^2} \sin(\gamma + \theta_8). \end{aligned} \quad (13)$$

For the flavor-averaged CP asymmetry, $\frac{1}{2}(\mathcal{A}_{X_s^-\gamma} + \mathcal{A}_{X_s^0\gamma})$, we must replace $e_{\text{spec}} \rightarrow \frac{1}{6}$. Then the resolved photon contributions are subdominant except for the second term, which is already present in the SM. In principle very large asymmetries are possible from the first and third terms [3], which however are already ruled out by the data. Once we assume that the effects of new physics are at most a few percent in magnitude, it will be difficult to disentangle them from the hadronic uncertainty due to the $\tilde{\Lambda}_{17}^u$ parameter.

A nontrivial feature of our analysis is that the resolved photon contributions induce a flavor-dependent term in the CP asymmetry already at order Λ_{QCD}/m_b . In the SM such effects are suppressed, compared with (11), by at least one additional factor of Λ_{QCD}/m_b and are bound to be negligible. We thus propose a future precision measurement of the CP asymmetry difference

$$\begin{aligned} \Delta \mathcal{A}_{X_s\gamma} & \equiv \mathcal{A}_{X_s^-\gamma} - \mathcal{A}_{X_s^0\gamma} \approx 4\pi^2 \alpha_s \frac{\tilde{\Lambda}_{78}}{m_b} \text{Im} \frac{C_{8g}}{C_{7\gamma}} \\ & \approx 12\% \frac{\tilde{\Lambda}_{78}}{100 \text{ MeV}} \frac{r_8}{r_7} \sin(\theta_8 - \theta_7) \end{aligned} \quad (14)$$

as a sensitive probe for flavor physics beyond the SM. Even though it will be difficult to determine the value of the hadronic parameter $\tilde{\Lambda}_{78}$ with any reasonable accuracy, we observe that if either the electromagnetic or the chromomagnetic dipole coefficients (or both) receive a sizable CP -violating new-physics phase, the difference $\Delta \mathcal{A}_{X_s\gamma}$ can easily reach the level of 10% in magnitude. It is important in this context that the Wilson coefficient of the chromomagnetic operator can be much enhanced with regard to its SM value, so that $r_8/r_7 \sim$ a few is possible [3].

All of the above expressions also apply to the CP asymmetry in $\bar{B} \rightarrow X_d\gamma$ decay, once we replace ϵ_s by $\epsilon_d = (V_{ub}V_{ud}^*)/(V_{tb}V_{td}^*) = (\tilde{\rho} - i\tilde{\eta})/(1 - \tilde{\rho} + i\tilde{\eta})$. As a result, the CP asymmetry for $\bar{B} \rightarrow X_d\gamma$ decay in the SM differs by that in $\bar{B} \rightarrow X_s\gamma$ decay by a factor $\text{Im}(\epsilon_d)/\text{Im}(\epsilon_s) \approx -22$. With the parameter values shown in (10), we obtain the

range $-62\% < \mathcal{A}_{X_d\gamma}^{\text{SM}} < 14\%$. It is worth emphasizing that in the endpoint region of large photon energy there do exist contributions to the photon energy spectrum which could affect the CP asymmetries in the decays $\bar{B} \rightarrow X_s\gamma$ and $\bar{B} \rightarrow X_d\gamma$ in a way different from the above rescaling. They are sensitive to four-quark soft functions with flavor content $\bar{b}s\bar{s}b$ and $\bar{b}d\bar{d}b$, respectively, whose matrix elements between B_d meson states can be different even in the flavor SU(3) limit. However, as shown in [16] these effects integrate to zero in the partially inclusive CP asymmetries at order Λ_{QCD}/m_b . As a result, the untagged CP asymmetry for $\bar{B} \rightarrow X_{s+d}\gamma$ decays vanishes in the SM (up to tiny U-spin breaking corrections [2,3,22]) even after the resolved photon terms are taken into account.

We are grateful to Tobias Hurth for useful discussions. The research of M.B. and M.N. is supported by BMBF grant 05H09UME and by the Research Centre of Excellence Elementary Forces and Mathematical Foundations. The work of G.P. is supported by the DOE grant DE-FG02-90ER40560.

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