Direct Probes of Linearly Polarized Gluons inside Unpolarized Hadrons

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We show that linearly polarized gluons inside unpolarized hadrons can be directly probed in jet or heavy quark pair production in electron-hadron collisions. We discuss the simplest $\cos 2\phi$ asymmetries and estimate their maximal value, concluding that measurements of the unknown linearly polarized gluon distribution in the proton should be feasible in future Electron-Ion Collider or Large Hadron electron Collider experiments. Analogous asymmetries in hadron-hadron collisions suffer from factorization breaking contributions and would allow us to quantify the importance of initial- and final-state interactions.

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Although quarks and gluons are confined within hadrons, tests of their fundamental properties are possible through scattering processes. It has become clear that quarks are, in general, spin-polarized, even within unpolarized hadrons, with polarization directions and magnitudes that depend on their transverse momentum and flavor. This nontrivial feature of hadron structure shows itself through specific angular asymmetries in scattering processes [1–3] that have been studied in a number of experiments [4–7]. Quark spin polarization in unpolarized hadrons is also supported by first-principles lattice QCD calculations [8]. What has received much less attention is that gluons can exhibit a similar property; i.e., they can be linearly polarized inside an unpolarized hadron. In this Letter, we propose measurements which are directly sensitive to this unexplored gluon distribution. Its accurate measurement would allow one to take advantage of polarized scattering at colliders without polarized beams.

Thus far, experimental and theoretical investigations of gluons inside hadrons have focused on their momentum and helicity distributions. The gluon density $g(x)$ describing the distribution of unpolarized gluons with a collinear momentum fraction x in an unpolarized hadron, integrated over transverse momentum $\boldsymbol{p}_T$, has been extracted with considerable precision from measurements of high energy electron-proton collisions at HERA (DESY, Hamburg). This distribution enters the structure function F_L in inclusive deep inelastic scattering (DIS) at order α_s, and it drives the evolution of sea quark distributions at small values of x. The unintegrated gluon distribution $g(x, \boldsymbol{p}_T^2)$ enters less inclusive reactions where the transverse momentum of the gluons is taken into account, such as semi-inclusive deep inelastic scattering or dijet production in hadronic collisions. In these cases the gluons are not necessarily unpolarized, even if the parent hadron itself is unpolarized. In fact, because of their spin-orbit couplings, the gluons can obtain a linear polarization. This gives access to a different polarization mode compared to the helicity distribution $\Delta g(x)$, which is the distribution of circularly polarized gluons inside *polarized* nucleons.

Information on linearly polarized gluons in a hadron is formally encoded in the hadron matrix element of a correlator of the gluon field strengths $F^{\mu\nu}(0)$ and $F^{\nu\sigma}(\lambda)$ evaluated at fixed light-front time $\lambda^+ = \lambda \cdot n = 0$, where n is a lightlike vector conjugate to the parent hadron's four-momentum P. Specifically, the gluon content of an unpolarized hadron at leading twist (omitting gauge links) for a gluon momentum $p = xP + p_T + p^- n$ is described by the correlator [9]

$$\begin{aligned}\Phi_g^{\mu\nu}(x, \boldsymbol{p}_T) &= \frac{n_\rho n_\sigma}{(p\cdot n)^2} \int \frac{d(\lambda\cdot P) d^2\lambda_T}{(2\pi)^3} \\ &\quad \times e^{ip\cdot\lambda} \langle P | \mathrm{Tr}[F^{\mu\rho}(0) F^{\nu\sigma}(\lambda)] | P \rangle \rfloor_{\mathrm{LF}} \\ &= \frac{-1}{2x} \left\{ g_T^{\mu\nu} f_1^g - \left(\frac{p_T^\mu p_T^\nu}{M^2} + g_T^{\mu\nu} \frac{\boldsymbol{p}_T^2}{2M^2} \right) h_1^{\perp g} \right\}, \quad (1)\end{aligned}$$

with $p_T^2 = -\boldsymbol{p}_T^2$, $g_T^{\mu\nu} = g^{\mu\nu} - P^\mu n^\nu / P\cdot n - n^\mu P^\nu / P\cdot n$. This defines the transverse momentum dependent distribution functions (TMDs) $f_1^g(x, \boldsymbol{p}_T^2)$ representing the unpolarized gluon distribution $g(x, \boldsymbol{p}_T^2)$, at fixed light-front time, whereas $h_1^{\perp g}(x, \boldsymbol{p}_T^2)$ is the distribution of linearly polarized gluons in an unpolarized hadron. It is named $h_1^{\perp g}$, because of its resemblance to the transversely polarized quark distribution inside an unpolarized hadron $h_1^{\perp q}$ (also frequently referred to as the Boer-Mulders function) [1]. There are notable differences though: The T-odd distribution $h_1^{\perp q}$ for quarks is a chiral-odd distribution (chirality flip), and it is also odd in p_T (it enters as a rank 1 tensor). It is zero in the absence of initial- or final-state interactions

(ISI/FSI) [10–12]. The T-even distribution $h_1^{\perp g}$ for gluons describes a $\Delta L = 2$ helicity-flip distribution, through a second rank tensor in the relative transverse momentum p_T (p_T-even). Since an imaginary phase is not required for T-even functions, it can in principle be nonzero in the absence of ISI or FSI. Nevertheless, as any TMD, $h_1^{\perp g}$ can receive contributions from ISI or FSI, leading to process-dependent gauge links in Eq. (1). Therefore $h_1^{\perp g}$ can be nonuniversal, and its extraction can be hampered in nonfactorizing cases.

Thus far, no experimental studies of the function $h_1^{\perp g}$ have been performed. It has been pointed out [13] that it contributes to the so-called dijet imbalance in hadronic collisions, which is commonly used to extract the average partonic intrinsic transverse momentum. Here it enters the observable as a convolution of two $h_1^{\perp g}$ functions, similarly to the double Boer-Mulders effect which leads to a large $\sin^2\theta\cos 2\phi$ term and the leading-twist violation of the Lam-Tung relation in Drell-Yan lepton pair production [2,3]. Although in principle it is possible to isolate the contribution from the $h_1^{\perp g}$ functions by appropriate weighting of the planar angular distribution, that is likely too difficult to do in practice. Moreover, it is unclear whether this weighted observable factorizes to begin with, because of factorization breaking effects such as discussed in Ref. [14].

Given its unique nature, it would be very interesting to obtain an extraction of $h_1^{\perp g}$ in a simple and theoretically safe manner. This turns out to be possible, since, unlike $h_1^{\perp q}$, it does not need to appear in pairs. In this Letter, we will discuss several new ways to probe the linear gluon polarization by using observables that involve only a single $h_1^{\perp g}$. The processes of interest, semi-inclusive DIS to two heavy quarks or to two jets, allow for TMD factorization and hence a safe extraction. Analogous processes in proton-proton collisions run into the problem of factorization breaking. A difference between the extractions will allow us to quantify the importance of ISI/FSI.

We first consider the electroproduction of heavy quarks, $e(\ell) + h(P) \to e(\ell') + Q(K_1) + \bar{Q}(K_2) + X$, where the four-momenta of the particles are given within brackets and the quark-antiquark pair in the final state is almost back-to-back in the plane perpendicular to the direction of the exchanged photon and hadron. The calculation proceeds along the lines explained in Refs. [13,15]. We obtain for the cross section integrated over the angular distribution of the backscattered electron $e(\ell')$:

$$\frac{d\sigma}{dy_1 dy_2 dy dx_B d^2\boldsymbol{q}_T d^2\boldsymbol{K}_\perp} = \delta(1 - z_1 - z_2)\frac{\alpha^2\alpha_s}{\pi s M_\perp^2}\frac{(1 + y x_B)}{y^5 x_B} \times \left[A + B\boldsymbol{q}_T^2\cos 2(\phi_T - \phi_\perp)\right]. \tag{2}$$

This expression involves the standard DIS variables: $Q^2 = -q^2$, where q is the momentum of the virtual photon, $x_B = Q^2/2P\cdot q$, $y = P\cdot q/P\cdot\ell$, and $s = (\ell + P)^2 = 2\ell\cdot P = 2P\cdot q/y = Q^2/x_B y$. Furthermore, we have for the jet momenta $K_{i\perp}^2 = -\boldsymbol{K}_{i\perp}^2$ and introduced the rapidities y_i for the heavy quark (HQ) or jet momenta (along the photon-target direction). We denote the heavy (anti)quark mass with M_Q. For the partonic subprocess we have $p + q = K_1 + K_2$, implying $z_1 + z_2 = 1$, where $z_i = P\cdot K_i/P\cdot q$. We introduced the sum and difference of the transverse HQ or jet momenta, $K_\perp = (K_{1\perp} - K_{2\perp})/2$ and $q_T = K_{1\perp} + K_{2\perp}$ with $|q_T| \ll |K_\perp|$. In that situation, we can use the approximate transverse HQ or jet momenta $K_{1\perp} \approx K_\perp$ and $K_{2\perp} \approx -K_\perp$ denoting $M_{i\perp}^2 \approx M_\perp^2 = M_Q^2 + \boldsymbol{K}_\perp^2$. The azimuthal angles of $\boldsymbol{q}_T$ and $\boldsymbol{K}_\perp$ are denoted by ϕ_T and $\phi_\perp$, respectively. The functions A and B, in general, depend on x_B, y, $z(\equiv z_2)$, $Q^2/M_\perp^2$, $M_Q^2/M_\perp^2$, and $\boldsymbol{q}_T^2$.

The explicit expression for the angular independent part A involves only f_1^g. We will focus here on the coefficient B of the $\cos 2(\phi_T - \phi_\perp)$ angular distribution, and we obtain

$$B^{eh\to eQ\bar{Q}X} = \frac{1}{M^2} e_Q^2 h_1^{\perp g}(x, \boldsymbol{q}_T^2)\mathcal{B}^{eg\to eQ\bar{Q}}, \tag{3}$$

with

$$\mathcal{B}^{eg\to eQ\bar{Q}} = \frac{1}{2}\frac{z(1-z)}{D^3}\left(1 - \frac{M_Q^2}{M_\perp^2}\right)a(y) \times \left\{[2z(1-z)b(y) - 1]\frac{Q^2}{M_\perp^2} + 2\frac{M_Q^2}{M_\perp^2}\right\}, \tag{4}$$

$D \equiv D(z, Q^2/M_\perp^2) = 1 + z(1-z)Q^2/M_\perp^2$, $a(y) = 2 - y(2-y)$, and $b(y) = [6 - y(6-y)]/a(y)$.

One observes that the magnitude B of the $\cos 2\phi$ asymmetry, where $\phi = \phi_T - \phi_\perp$, is determined by $h_1^{\perp g}$ and that if Q^2 and/or M_Q^2 are of the same order as $K_\perp^2$, the coefficient B is not power suppressed. Since $h_1^{\perp g}$ is completely unknown, we estimate the maximum asymmetry that is allowed by the bound:

$$|h_1^{\perp g(2)}(x)| \le \frac{\langle p_T^2\rangle}{2M^2} f_1^g(x), \tag{5}$$

that we derived from the spin density matrix given in Ref. [9] in the way presented in Ref. [16]. The superscript (2) denotes the $n = 2$ transverse moment. Transverse moments of TMDs are defined as $f^{(n)}(x) \equiv \int d^2\boldsymbol{p}_T(\boldsymbol{p}_T^2/2M^2)^n f(x, \boldsymbol{p}_T^2)$ (a suitably chosen regularization is understood, e.g., as discussed in Appendix B of Ref. [17]). If we select $Q^2 = M_Q^2 = K_\perp^2/4$, $y_1 = y_2$, the asymmetry ratio

$$\left|\frac{\int d^2\boldsymbol{q}_T\boldsymbol{q}_T^2\cos 2(\phi_T - \phi_\perp)d\sigma}{\int d^2\boldsymbol{q}_T\boldsymbol{q}_T^2 d\sigma}\right| = \frac{\int d\boldsymbol{q}_T^2\boldsymbol{q}_T^4|B|}{2\int d\boldsymbol{q}_T^2\boldsymbol{q}_T^2 A} \tag{6}$$

is maximally around 13%, which we view as encouraging.

If one keeps the lepton plane angle ϕ_ℓ, there are other azimuthal dependences such as a $\cos 2(\phi_\ell - \phi_T)$, but its bound is at least 6 times smaller than on $\cos 2(\phi_T - \phi_\perp)$.

The cross section for the process $eh \to e'$ jet jet X can be calculated in a similar way. The corresponding expressions can be obtained from Eqs. (3) and (4) with $M_Q = 0$. One can then also replace the rapidities of the outgoing particles, y_i, with the pseudorapidities $\eta_i = -\ln[\tan(\frac{1}{2}\theta_i)]$, θ_i being the polar angles of the final partons in the virtual photon-hadron center-of-mass frame. Note that A now also receives a contribution from $\gamma^* q \to gq$, leading to somewhat smaller asymmetries.

Since the observables involve final-state heavy quarks or jets, they require high energy colliders, such as a future Electron-Ion Collider or the Large Hadron electron Collider proposed at CERN. It is essential that the individual transverse momenta $K_{i\perp}$ are reconstructed with an accuracy $\delta K_\perp$ better than the magnitude of the sum of the transverse momenta $K_{1\perp} + K_{2\perp} = q_T$. Thus one has to satisfy $\delta K_\perp \ll |q_T| \ll |K_\perp|$.

An analogous asymmetry arises in QED, in the "tridents" processes $\ell e(p) \to \ell \mu^+ \mu^- e'(p'$ or $X)$ or $\mu^- Z \to \mu^- \ell \bar{\ell} Z$ [18–21]. This could be described by the distribution of linearly polarized photons inside a lepton, proton, or atom. QCD adds the twist that for gluons inside a hadron, ISI or FSI can considerably modify the result depending on the process; for example, in HQ production in hadronic collisions: $pp \to Q\bar{Q}X$, which can be studied at Brookhaven National Laboratory's Relativistic Heavy Ion Collider and CERN's LHC, and $p\bar{p} \to Q\bar{Q}X$ at Fermilab's Tevatron. Since the description involves two TMDs, breaking of TMD factorization becomes a relevant issue; cf. [14] and references therein. The cross section for the process $h_1(P_1) + h_2(P_2) \to Q(K_1) + \bar{Q}(K_2) + X$ can be written in a way similar to the hadroproduction of two jets discussed in Ref. [13], in the following form:

$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{K}_{1\perp} d^2\boldsymbol{K}_{2\perp}} = \frac{\alpha_s^2}{sM_\perp^2}[A(\boldsymbol{q}_T^2) + B(\boldsymbol{q}_T^2)\boldsymbol{q}_T^2 \cos 2(\phi_T - \phi_\perp) + C(\boldsymbol{q}_T^2)\boldsymbol{q}_T^4 \cos 4(\phi_T - \phi_\perp)]. \tag{7}$$

Besides $\boldsymbol{q}_T^2$, the terms A, B, and C will depend on other, often not explicitly indicated, variables as z, $M_Q^2/M_\perp^2$, and momentum fractions x_1 and x_2 obtained from $x_{1/2} = (M_{1\perp}e^{\pm y_1} + M_{2\perp}e^{\pm y_2})/\sqrt{s}$.

In the most naive partonic description the terms A, B, and C contain convolutions of TMDs. Schematically,

$$A\colon f_1^q \otimes f_1^{\bar{q}}, f_1^g \otimes f_1^g, \qquad B\colon h_1^{\perp q} \otimes h_1^{\perp \bar{q}}, \frac{M_Q^2}{M_\perp^2} f_1^g \otimes h_1^{\perp g},$$
$$C\colon h_1^{\perp g} \otimes h_1^{\perp g}.$$

Terms with higher powers in $M_Q^2/M_\perp^2$ are left out. In Fig. 1, the origin of the factor $M_Q^2/M_\perp^2$ in the contribution of $h_1^{\perp g}$ to B is explained.

The factorized description in terms of TMDs is problematic, though. In Ref. [14], it was pointed out that for hadron or jet pair production in hadron-hadron scattering TMD factorization fails. The ISI/FSI will not allow a separation of gauge links into the matrix elements of the various TMDs. Only in specific simple cases, such as the single Sivers effect, can one find weighted expressions that do allow a factorized result but with, in general, different factors for different diagrams in the partonic subprocess [22,23]. Even if this applies to the present case for A and B as well, actually two different functions $h_1^{\perp g(2)}(x)$ [and $f_1^{g(1)}(x)$] will appear, corresponding to gluon operators with the color structures $f_{abe}f_{cde}$ and $d_{abe}d_{cde}$, respectively [23,24]. This is similar to what happens for single transverse spin asymmetries (A_N) in heavy quark production processes [25–29]. Because also there two different (f and d type) gluon correlators arise, the single-spin asymmetries in D and $\bar{D}$ meson production are found to be different. However, in the unpolarized scattering case considered in this Letter the situation is simpler, since only one operator contributes or dominates. In the $\gamma^* g \to Q\bar{Q}$ subprocess only the matrix element with the ff structure appears, while in the $gg \to Q\bar{Q}$ subprocess relevant for hadron-hadron collisions the dd structure dominates (the ff contribution is suppressed by $1/N^2$). A side remark on p_T broadening [30–32] is that, because of the two different four-gluon operators for $f_1^{g(1)}(x)$, we expect the broadening Δp_T^2 in semi-inclusive DIS, $(\Delta p_T^2)_{\rm DIS} \equiv \langle p_T^2 \rangle_{eA} - \langle p_T^2 \rangle_{ep}$, to be different from the one in hadron-hadron collisions, $(\Delta p_T^2)_{\rm hh} \equiv \langle p_T^2 \rangle_{pA} - \langle p_T^2 \rangle_{pp}$.

In case weighting does allow for factorized expressions, we present here the relevant expressions for $B = \mathcal{B}^{q\bar{q}\to Q\bar{Q}} + (M_Q^2/M_\perp^2)\mathcal{B}^{gg\to Q\bar{Q}}$, where

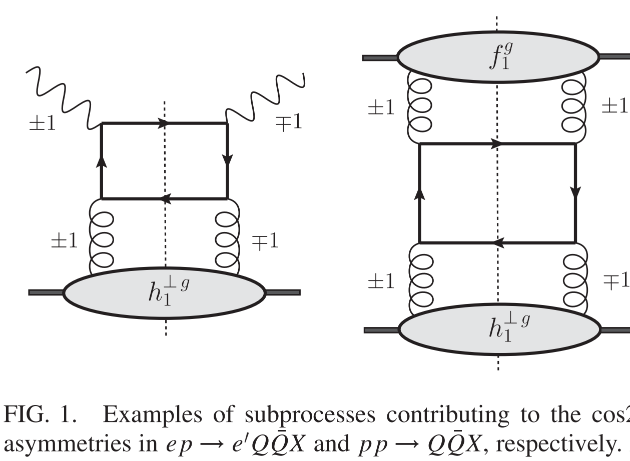

FIG. 1. Examples of subprocesses contributing to the $\cos 2\phi$ asymmetries in $ep \to e'Q\bar{Q}X$ and $pp \to Q\bar{Q}X$, respectively. As the helicities of the photons and gluons indicate, the latter process requires helicity flip in quark propagators resulting in an $M_Q^2 = M_\perp^2$ factor.

$$\mathcal{B}^{q\bar{q}\to Q\bar{Q}} = \frac{N^2-1}{N^2} z^2(1-z)^2\left(1-\frac{M_Q^2}{M_\perp^2}\right)[\mathcal{H}^{q\bar{q}}(x_1,x_2,\boldsymbol{q}_T^2) + \mathcal{H}^{\bar{q}q}(x_1,x_2,\boldsymbol{q}_T^2)],$$
$$\mathcal{B}^{gg\to Q\bar{Q}} = \frac{N}{N^2-1}\mathcal{B}_1\mathcal{H}^{gg}(x_1,x_2,\boldsymbol{q}_T^2), \tag{8}$$

with N being the number of quark colors and

$$\mathcal{B}_1 = z(1-z)\left(z^2+(1-z)^2-\frac{1}{N^2}\right)\left(1-\frac{M_Q^2}{M_\perp^2}\right). \tag{9}$$

The weighted integrals which appear in the $\boldsymbol{q}_T^2/M^2$-weighted cross section (cf. [13]) for $M_1 = M_2 = M$ are

$$\pi\int d\boldsymbol{q}_T^2\left(\frac{\boldsymbol{q}_T^2}{M^2}\right)\boldsymbol{q}_T^2\mathcal{H}^{q\bar{q}}(x_1,x_2,\boldsymbol{q}_T^2) = 8\sum_{\text{flavors}} h_1^{\perp q(1)}(x_1)h_1^{\perp\bar{q}(1)}(x_2), \tag{10}$$

already discussed in [13], and

$$\pi\int d\boldsymbol{q}_T^2\left(\frac{\boldsymbol{q}_T^2}{M^2}\right)\boldsymbol{q}_T^2\mathcal{H}^{gg}(x_1,x_2,\boldsymbol{q}_T^2) = 4[h_1^{\perp g(2)}(x_1)f_1^g(x_2) + f_1^g(x_1)h_1^{\perp g(2)}(x_2)]. \tag{11}$$

Whether $gg\to Q\bar{Q}$ is more important than $q\bar{q}\to Q\bar{Q}$ depends strongly on the values of x_i and $M_Q^2/M_\perp^2$ and on whether one deals with pp or $p\bar{p}$. In $p\bar{p}$ collisions and for $K_\perp^2$ not too large compared to M_Q^2, the contribution from $h_1^{\perp g}$ is expected to be the dominant one. The importance of the contribution from $h_1^{\perp q}$ can be assessed through a comparison to photon-jet production [15].

In summary, measurements of the azimuthal asymmetry of jet or heavy quark pair production in ep and in pp or $p\bar{p}$ collisions (and possibly also in diphoton or even Higgs production [33,34]) can directly probe the distribution of linearly polarized gluons inside unpolarized hadrons. From a theoretical viewpoint, this asymmetry in the ep process is among the simplest TMD observables. Breaking of TMD factorization is expected in the pp case and may lead to uncontrolled corrections. A comparison between extractions from these two types of processes would therefore be very interesting. The relative simplicity of the proposed measurements (polarized beams are not required) suggests a promising prospect for the extraction of this gluon distribution in the future and for the study of its potential process dependence.

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