Revealing Hidden Einstein-Podolsky-Rosen Nonlocality

S. P. Walborn,[1] A. Salles,[1,2] R. M. Gomes,[1] F. Toscano,[1] and P. H. Souto Ribeiro[1]

[1]*Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, RJ 21941-972, Brazil*

[2]*Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark*

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Steering is a form of quantum nonlocality that is intimately related to the famous Einstein-Podolsky-Rosen (EPR) paradox that ignited the ongoing discussion of quantum correlations. Within the hierarchy of nonlocal correlations appearing in nature, EPR steering occupies an intermediate position between Bell nonlocality and entanglement. In continuous variable systems, EPR steering correlations have been observed by violation of Reid's EPR inequality, which is based on inferred variances of complementary observables. Here we propose and experimentally test a new criterion based on entropy functions, and show that it is more powerful than the variance inequality for identifying EPR steering. Using the entropic criterion our experimental results show EPR steering, while the variance criterion does not. Our results open up the possibility of observing this type of nonlocality in a wider variety of quantum states.

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Since the early days of the theory, nonlocality was spotted as a salient feature of quantum mechanics. The debate started with the argument of Einstein, Podolsky, and Rosen in 1935 that the theory could not be considered complete if the assumptions of locality and realism (which they supported) were to hold [1]. They proposed the existence of some "hidden variables", which would complete quantum theory. Schrödinger, in turn, was troubled by the fact that according to quantum mechanics one party Alice (A) could seemingly "steer" a remote system of some other party Bob (B) into an eigenstate of some arbitrary observable. These counterintuitive nonlocal effects, or "spooky action at a distance" as EPR put it, were collectively dubbed "entanglement". In 1964, John S. Bell introduced his famous inequality for local hidden variable theories, which crucially brought the nonlocality debate to an experimentally testable form [2]. It was only with the recent advent of quantum information theory, however, that the various concepts packaged together under the "nonlocality" label would be sharply distinguished.

At present, essentially three distinct types of nonlocal correlations can be identified in nature: Bell nonlocality, Schrödinger's steerability, and entanglement [3,4]. Bell nonlocality, the strongest of the three, concerns the existence or not of a local hidden variable model for a set of correlations. If such a model does not exist, then the correlations violate some Bell-type inequality, and are termed Bell nonlocal. Bell nonlocality can be stated without any reference to quantum theory, and could thus be thought of as the most fundamental of the nonlocality types. This is reinforced by the fact that it is the most stringent to test experimentally, with a positive test for Bell nonlocality implying in nonlocality of the other two types as well.

On the lower end of the hierarchy is quantum entanglement, which was formally defined in 1989 by Werner who distinguished it from Bell nonlocality [5]. Entanglement has received a lot of attention as the underlying resource for quantum computation and certain quantum information tasks. A vast amount of criteria for entanglement detection and subsequent experimental tests have appeared in the literature [6,7].

The concept of steering has caught the least amount of attention. Schrödinger's intuition that one party can remotely prepare some ensemble of quantum states by local measurement on the other part of an entangled state has been formalized in the Hughston-Josza-Wootters theorem [8,9]. In 2007, Wiseman and collaborators cast steering in terms of a quantum information task, and showed that this class of nonlocality is distinct and strictly intermediate between the other two [3]. Interestingly, the three classes are identical for bipartite pure states, a fact which may be responsible for the pronounced delay in formally distinguishing among them. Experimental demonstration of this distinction has been performed for a pair of qubits [10]. We note that this framework has been extended to more than two parties [11], and refer the reader to [3,4] for a more in-depth discussion on the differences among the classes of nonlocality.

Steering has been proved to be the nonlocal property behind what came to be known as EPR inequalities, which are tests which exhibit the conflict between completeness of quantum theory and local realism—the EPR paradox [3,4,12,13]. We thus group these two terms together and refer to both types of inequalities as "EPR steering" criteria. The most famous EPR-steering criteria, formulated by Margaret Reid [12], was cast in terms of inferred variances of complementary observables. Reid's inequality has been violated for a number of continuous variable quantum systems [14,15], thus demonstrating "EPR steering" correlations.

The Reid inequality relies on variances, and as a result is capable of detecting correlations that appear up to second-order in the tested observables. There are, however, a number of entangled continuous variable states (some even pure states), which do not violate the variance inequality. Since pure entangled states should display the EPR steering form of nonlocality, the correlations must be hidden in higher-order moments of the observables. A natural question is: do these states violate some higher-order EPR-steering inequality? Here we partially answer this question in the affirmative by introducing and experimentally testing a novel criterion for steering that is based on Shannon entropies. We exhibit the connection to the variance inequality [12], which arises from ours as a limiting case. This proves the new entropic criterion is more powerful in the sense that it detects EPR correlations that the variance inequality does not. This aspect is then confirmed experimentally using spatially entangled photons.

In Ref. [3], EPR steering was fashioned in terms of a task in which Alice sends a quantum system to Bob, with the goal of convincing Bob that she is sending him one part of an entangled state, as opposed to a state chosen from some ensemble. It was argued in [3] that EPR steering is thus demonstrated by the nonexistence of a local hidden state (LHS) model for their measurement outcomes. That is, we say that the correlations demonstrate EPR steering if the joint measurement probability cannot be written as

$$\mathcal{P}(r_A, r_B) = \sum_{\lambda} \mathcal{P}(\lambda)\mathcal{P}(r_A|\lambda)\mathcal{P}_Q(r_B|\lambda), \tag{1}$$

where r_A and r_B are the outcomes of measurements R_A and R_B. Here λ are predetermined (hidden) variables that specify an ensemble of states, $\mathcal{P}$ are general probability distributions and $\mathcal{P}_Q$ are probability distributions which correspond to measurement on the quantum state specified by λ. A bipartite state for which a LHS model like the one above can be written for any choice of measurements is called nonsteerable. Note that this definition is intrinsically asymmetric, since it encompasses the power of Alice to steer Bob's state.

Any constraint on the possible phenomena that stems from (1) is called an steering criterion [4]. Here we derive an EPR steering criteria using the Shannon entropy. First, we use (1) to write the conditional probability as $\mathcal{P}(r_B|r_A) = \sum_\lambda \mathcal{P}(r_B, \lambda|r_A)$, with $\mathcal{P}(r_B, \lambda|r_A) = \mathcal{P}(\lambda|r_A)\mathcal{P}_Q(r_B|\lambda)$. Consider now the non-negativity of the relative entropy [16] between $\mathcal{P}(r_B, \lambda|r_A)$ and $\mathcal{P}(\lambda|r_A)\mathcal{P}(r_B|r_A)$:

$$\sum_{\lambda}\int dr_B \mathcal{P}(r_B, \lambda|r_A) \ln\frac{\mathcal{P}(r_B, \lambda|r_A)}{\mathcal{P}(\lambda|r_A)\mathcal{P}(r_B|r_A)} \geq 0. \tag{2}$$

Rearranging terms, we arrive at

$$h(R_B|R_A = r_A) \geq \sum_{\lambda} \mathcal{P}(\lambda|r_A)h_Q(R_B|\lambda), \tag{3}$$

where $h(R) = -\int dr\mathcal{P}(r)\ln\mathcal{P}(r)$ is the Shannon entropy. Averaging with $\mathcal{P}(r_A)$ over r_A gives

$$h(R_B|R_A) \geq \sum_{\lambda} \mathcal{P}(\lambda)h_Q(R_B|\lambda), \tag{4}$$

where the conditional entropy is defined as [16]

$$h(R_B|R_A) = -\int dr_A \mathcal{P}(r_A)h(R_B|R_A = r_A). \tag{5}$$

Now let us consider that the measurements correspond to either position ($r = x$) or momentum ($r = p$) measurements. The x and p distributions of a quantum system must satisfy the entropic uncertainty relation [17]:

$$h_Q(X) + h_Q(P) \geq \ln\pi e, \tag{6}$$

which we can apply to each state marked by λ in (4) to arrive at an entropic steering criterion:

$$h(X_B|X_A) + h(P_B|P_A) \geq \ln\pi e. \tag{7}$$

The EPR steering criterion derived by Reid [12] can be obtained as a limiting case of (7). The Reid inequality is stated in terms of minimum variances $\Delta^2_{\min}(R_B)$ obtained when Alice infers the outcomes of Bob's measurement of the property R_B of system B given that she measured property R_A on system A. This is given by $\Delta^2_{\min}(R_B) = \int dr_A\mathcal{P}(r_A)\Delta^2(r_B|r_A)$, where $\Delta^2(r_B|r_A)$ is the variance of the conditional probability distribution $\mathcal{P}(r_B|r_A)$ [13]. The Shannon entropy of a probability distribution with variance $\Delta^2(r_B|r_A)$ is upper bounded by $\ln[2\pi e\Delta^2(r_B|r_A)]/2$ [16], which is an upper limit for Alice's ignorance about r_B given r_A. This upper bound, together with the definition of $\Delta_{\min}$ and the concavity of the logarithm function, leads to the upper bound for the left-hand side of Eq. (7):

$$\ln[2\pi e\Delta_{\min}(X_B)\Delta_{\min}(P_B)] \geq h(X_B|X_A) + h(P_B|P_A). \tag{8}$$

By combining inequalities (7) and (8), we arrive at the EPR steering criterion derived by Reid [12]:

$$\Delta^2_{\min}(X_B)\Delta^2_{\min}(P_B) \geq \tfrac{1}{4}. \tag{9}$$

Thus, criterion (9) emerges as a limiting case of criterion (7). The entropic criterion (7) is therefore more sensitive than (9), which implies that it should detect EPR steering in certain states that the variance criteria (9) does not.

As an example, consider the class of bipartite quantum states described by the wave function

$$\phi_n(x_A, x_B) = C_n\mathcal{H}_n\left(\frac{x_A + x_B}{\sqrt{2}\sigma_+}\right)e^{-((x_A+x_B)^2/4\sigma_+^2)}e^{-((x_A-x_B)^2/4\sigma_-^2)}, \tag{10}$$

where $\mathcal{H}_n$ is the nth-order Hermite polynomial and C_n a normalization constant. Numerical analysis ($n \leq 15$) shows that the variance criteria (9) identifies steering only when the ratio $\sigma_\pm/\sigma_\mp \gtrsim 1 + 1.5\sqrt{n}$, while the entropic criterion always identifies steering, except in the case when the state is indeed separable ($n = 0$ and $\sigma_+ = \sigma_-$). As another example, we tested the nonlocal cat state $|\psi_\phi\rangle$ that was experimentally produced in Ref. [18], using the experimental parameters reported therein. We found the violation $h(X_A|P_B) + h(P_A|X_B) \approx 2.10 < \ln \pi e$, and no violation of the variance inequality (9).

To further illustrate the utility of the entropic criterion (7), we experimentally tested it for a pair of spatially entangled photons. Spatial entanglement of photon pairs was shown experimentally in Ref. [15], and discussed at depth in a recent review paper [19]. FIG. 1 shows the experimental setup. The quantum state of the down-converted photons at the crystal is given to good approximation by [19,20]

$$|\psi\rangle = \iint dp_A dp_B v(p_A + p_B) s(p_A - p_B) |p_A\rangle |p_B\rangle, \quad (11)$$

where we consider only one spatial dimension for simplicity. Here, $v(p)$ is the angular spectrum of the pump beam, and p_A, p_B are the transverse wave vectors of the down-converted photons A and B, respectively. The function $s(p) \propto 4K \mathrm{sinc}(Lp^2/4K)$, where L is the length of the BBO crystal and K is the wave number of the pump beam. A number of steps were taken to engineer the wave function $\langle x_A, x_B|\psi\rangle$ corresponding to the state (11), so that it was similar to that of Eq. (10) with $n = 1$, as has been previously described elsewhere [21]. In our setup the pump beam was focused at the crystal face, so that σ_+ and

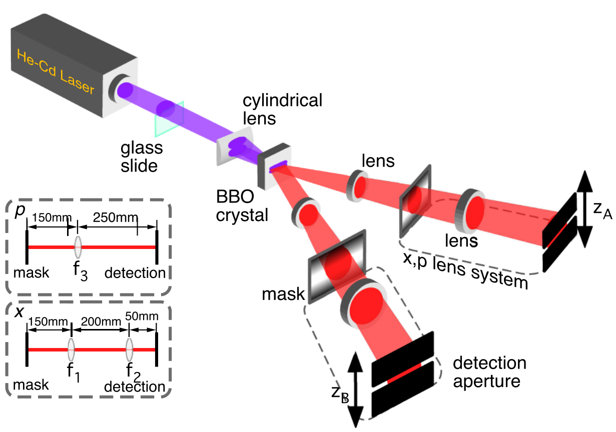

FIG. 1 (color online). Experimental setup. SPDC in a BBO crystal produces spatially entangled photon pairs ($\lambda = 884$ nm). A microscope slide is used to produce a first-order Hermite Gaussian beam, which is focused at the crystal face with a cylindrical lens. Lenses ($f = 100$ mm) are used to map the momentum distribution at the crystal face onto the output planes, where transmission masks with a Gaussian intensity profile are placed. The figure shows the setup for p measurements. Diagrams for the lens systems used for x and p measurements are shown in the bottom left. Here $f_1 = 150$ mm, $f_2 = 50$ mm and $f_3 = 250$ mm. The horizontal slit detection apertures are mounted directly onto Detectors D_A and D_B (not shown for clarity), and are scanned in the vertical direction.

σ_- were of the same order of magnitude. We measured $\sigma_+^2 \approx 0.566$ mm^2 and $\sigma_-^2 \approx 0.240$ mm^2 at the output planes.

An imaging lens system using two lenses (f_1 and f_2 in inset of FIG. 1) was used to measure the near field (x variable), so that the output plane of the source was imaged at the detection plane. The far-field (p variable) was measured by scanning in the focal plane of a second lens system (f_3 in inset of Fig. 1). The detectors were scanned in discrete steps z_{step} and coincidence counts were registered, resulting in two 2D tables of coincidence measurements $C_{xx}(z_A, z_B)$ and $C_{pp}(z_A, z_B)$, where z_A and z_B are the transverse positions of detectors D_A and D_B, respectively. The discrete joint probability distributions were then obtained by $\mathcal{P}_{rr}(z_A, z_B) = C_{rr}(z_A, z_B)/\sum_{z_A, z_B} C_{rr}(z_A, z_B)$, where $r = x, p$. The coincidence measurements are shown in Fig. 2.

We first tested the variance-product EPR criterion (9), using the experimental data to compute $\Delta^2_{\min}(R_i) = \gamma_r^2 \Delta^2_{\min}(Z_i)$, where $\Delta^2_{\min}(Z_i) = \sum_{z_j} \mathcal{P}(z_j)\Delta^2(z_i|z_j)$ for i, $j = A$, B. Here $\Delta^2(z_i|z_j)$ is the variance in z_i given result z_j and γ_r is the scaling factor, used to relate the detector positions z to the x and p variables. Explicitly, $\gamma_x = f_2/f_1$, due to the magnification factor of the lens system, and $\gamma_p = 2\pi/f_3\lambda$ for p measurements. We obtained $\Delta^2_{\min}(X_A) = 0.14 \pm 0.02$ mm^2, $\Delta^2_{\min}(P_A) = 3.1 \pm 0.2$ mm^{-2}, $\Delta^2_{\min}(X_B) = 0.15 \pm 0.02$ mm^2, and $\Delta^2_{\min}(P_B) = 3.4 \pm 0.2$ mm^{-2}. In all results reported here and below, the uncertainty in the experimental data was obtained by error propagation of the Poissonian count statistics. The variance EPR criterion (9) gives

$$\begin{aligned} \Delta^2_{\min}(X_A)\Delta^2_{\min}(P_A) &= 0.44 \pm 0.01 > \tfrac{1}{4}, \\ \Delta^2_{\min}(X_B)\Delta^2_{\min}(P_B) &= 0.51 \pm 0.01 > \tfrac{1}{4}. \end{aligned} \quad (12)$$

Thus, the variance inequality is satisfied, and EPR steering correlations are not detected.

Next we tested the entropic steering criterion (7). The discrete entropies of the coincidence count distributions were calculated using $H(Z) = -\sum_z \mathcal{P}(z) \ln \mathcal{P}(z)$, and $H(Z_A, Z_B) = \sum_{z_A, z_B} \mathcal{P}(z_A, z_B) \ln \mathcal{P}(z_A, z_B)$. The differential entropies $h(Z_A, Z_B)$ and $h(Z)$ of the continuous variables

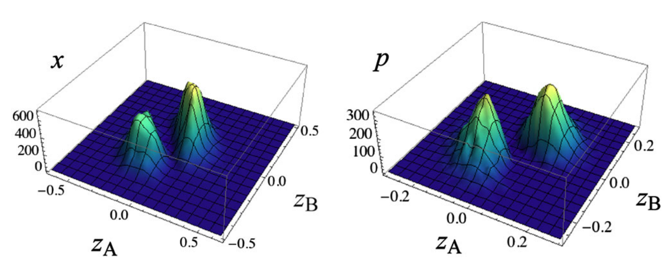

FIG. 2 (color online). Coincidence counts for x and p measurements used to calculate the probability distributions $\mathcal{P}(x_A, x_B)$ and $\mathcal{P}(p_A, p_B)$.

can be calculated from the discrete entropies $H(Z_A, Z_B)$ and $H(Z)$:

$$h(Z_A, Z_B) \approx H(Z_A, Z_B) + \ln(z_{\text{step}}^2), \tag{13a}$$

$$h(Z) \approx H(Z) + \ln(z_{\text{step}}). \tag{13b}$$

Here, z_{step} appears due to the discretization of the continuous distribution [16]. For x measurements $z_{\text{step}} = 0.02$ mm, while $z_{\text{step}} = 0.05$ mm for p measurements. Finally, the entropy of the probability distributions for the $R = X$, P variables are calculated from the experimental data using $h(R) = h(Z) + \ln\gamma_r$ and $h(R_A, R_B) = h(Z_A, Z_B) + \ln\gamma_r^2$. Using these experimental results, the conditional entropies of the corresponding continuous probability distributions can be calculated using $h(R_i|R_j) = h(R_i, R_j) - h(R_j)$. We found $h(X_A) = 0.56 \pm 0.01$, $h(P_A) = 2.35 \pm 0.01$, $h(X_B) = 0.58 \pm 0.01$, $h(P_B) = 2.37 \pm 0.01$, $h(X_A, X_B) = 0.73 \pm 0.02$, and $h(P_A, P_B) = 4.17 \pm 0.03$. Finally, we calculated

$$h(X_A|X_B) + h(P_A|P_B) = 1.94 \pm 0.04, \tag{14}$$

$$h(X_B|X_A) + h(P_B|P_A) = 1.99 \pm 0.04. \tag{15}$$

Both of these equations are less than $\ln\pi e \approx 2.145$ by more than 3 standard deviations, indicating violation of inequality (7). Using the wave function (10), the sum of conditional entropies was calculated to be 1.91, showing reasonably good agreement between theory and experiment. Thus, EPR steering correlations, which go undetected under the variance criterion (9), are revealed through test of the entropic criterion (7).

We have introduced a novel inequality for identification of EPR steering correlations. Our theoretical and experimental results agree that an EPR steering criterion based on entropic uncertainty relations is capable of detecting nonlocal correlations which remain hidden to the criteria based on variances. This should encourage further investigation and characterization of EPR steering and its relation to the other forms of nonlocality, as well as application to quantum information tasks such as quantum cryptography. We note that our criteria has recently been applied to correlations in angular position and momentum [22].

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