Chiral Vacuum Fluctuations in Quantum Gravity

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We examine tensor perturbations around a de Sitter background within the framework of Ashtekar's variables and its cousins parameterized by the Immirzi parameter γ. At the classical level we recover standard cosmological perturbation theory, with illuminating insights. Quantization leads to real novelties. In the low energy limit we find a second quantized theory of gravitons which displays different vacuum fluctuations for right and left gravitons. Nonetheless right and left gravitons have the same (positive) energies, resolving a number of paradoxes suggested in the literature. The right-left asymmetry of the vacuum fluctuations depends on γ and the ordering of the Hamiltonian constraint, and it would leave a distinctive imprint in the polarization of the cosmic microwave background, thus opening quantum gravity to observational test.

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Loop quantum gravity is a promising scheme for quantizing the gravitational field [1–3]. At its core lies the idea that the connection (or its holonomies), rather than the metric, should be the central gravitational variable driving quantization. This permits borrowing a number of nonperturbative quantization techniques from non-Abelian gauge theories, notably the Wilson loop. The menace of nonrenormalizability can then be skirted, leading to a finite theory. Unfortunately the end product has not always made easy contact with the real world, with familiar concepts (such as smooth manifolds or gravitons) only recently finding a niche in the theory [4,5]. A reexamination of the theory from the perturbative viewpoint is in order, to establish whether it makes more pedestrian physical sense.

There have been a number of past attempts to stave off the above criticism. Loop quantum cosmology is a semiclassical scheme for deriving effective Hamiltonians [6]; however its links with the parent theory can be flimsy. Graviton states (and their loop representations) were identified early on in loop quantum gravity [7], but this work contained a number of technical deficiencies (spelled out in this Letter). More recently, following on from [7], Smolin proposed that fluctuations around the Kodama state (a well-known exact solution to the theory [8]) could provide well-defined representations for gravitons in a de Sitter background [9]. Witten claimed that such gravitons would be pathological because one of the helicities would have negative energy [10]. This was allegedly disproved in [11], but again in the shadow of technical errors.

In this Letter we reexamine the perturbative status of loop quantum gravity following a simple guiding principle: we never stray far from standard cosmological perturbation theory [12]. Clearly, well established *classical* results in cosmology must have exactly equivalent descriptions in Ashtekar's formalism; if they do not something has gone awry. Furthermore the loop quantization procedure should be mapped, in some approximation, onto the usual inflationary calculation of tensor vacuum quantum fluctuations. If differences arise one should understand their origin, and decide "who's at fault."

Crucial to this exercise are the reality conditions that supplement Ashtekar's formalism. In order for the central concept of duality to apply to a Lorentzian signature the geometry must be complexified. Additional constraints then ensure that "on shell" the geometry is real. This is implemented by the inner product with which the Hilbert space is endowed, and the implicit selection of physical (i.e., normalizable) states. In this Letter we show that physically sensible results can only be obtained if we include in all expansions both positive and negative frequencies. These should be associated with graviton and antigraviton states, to be identified only after reality conditions are imposed.

Once this simple point is recognized a number of mysteries evaporate. We reproduce Witten's negative energy gravitons [10], originally derived for Yang-Mills theories only. For example, for the self-dual (SD) connection we find that right-helicity (R) positive-frequency (+) and left-helicity (L) negative-frequency (−) modes have positive energy, whereas R− and L+ modes have negative energy. However we discover that the pathological modes are not normalizable under the inner product representing the reality conditions. Therefore they do not belong to the physical Hilbert space, and indeed these modes do not exist classically, i.e., by evaluating the SD connection using the equations of motion.

The only physical modes are the usual particles: right- and left-handed gravitons with a positive energy spectrum, albeit described chirally (the right graviton appears in the positive frequency of the SD connection, the left in its negative frequency). But a dramatic novelty creeps in. For a standard ordering of the Hamiltonian constraint only the negative frequency needs to be normal ordered. Thus a significant difference appears in the inflationary calculation for tensor vacuum fluctuations, using the SD connection: a (scale-invariant) spectrum is produced, but only for left gravitons. No right gravitons are produced.

Had we employed the anti-SD (ASD) connection, the description would be reversed, leading to vacuum fluctuations containing only right-handed gravitons. More generally, Ashetkar's SD and ASD connections belong to a class of connections parametrized by the Immirzi parameter, γ. They hail from a canonical transformation applied to general relativity, resulting in equivalent classical descriptions, but inequivalent quantum theories. The main result in this Letter is a reflection of this fact at the perturbative level. We predict a γ dependent chirality in the gravitational wave background. The effects on the polarization of the cosmic microwave background are unique [13], opening up the doors to an observational test of quantum gravity.

As our starting point we take metric:

$$ds^2 = a^2[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j] \tag{1}$$

where h_{ij} is a transverse and traceless (TT) tensor. For definiteness the background is de Sitter (i.e., $a = -1/H\eta$, with $H^2 = \Lambda/3$ and $\eta < 0$) but what follows can be repeated with other backgrounds, and perturbations around Minkowski space-time can be recovered by setting $H = 0$. With a set of conventions fully spelled out in [14] (and following [3]) the connection is given by $A^i = \Gamma^i + \gamma\Gamma^{0i}$, with $\Gamma^i = -\frac{1}{2}\epsilon^{ijk}\Gamma^{jk}$. Here γ is the Immirzi parameter, best introduced by the Holst action [2,5,9]. The SD and ASD connections correspond to $\gamma = \pm i$. We then solve for the background using the Einstein-Cartan equations and expand the canonical variables as $A^i_a = \gamma H a \delta^i_a + \frac{a^i_a}{a}$ and $E^a_i = a^2\delta^a_i - a\delta e^a_i$, where E^a_i is the densitized inverse triad, canonically conjugate to A^i_a. Throughout this Letter we will adopt the following convention: we define δe^i_a via the triad $e^i_a = a\delta^i_a + \delta e^i_a$; we then raise and lower indices in all tensors with the Kronecker δ, possibly mixing group and spatial indices. This simplifies the notation and is unambiguous if it is understood that δe is originally the perturbation in the triad. It turns out that δe_{ij} is then proportional to the "v" variable beloved by cosmologists [12].

We now come to an important technical point. As in the usual cosmological treatment we subject the perturbations to Fourier and polarization expansions; however the Ashtekar formalism presents us with some subtleties. If reality conditions are yet to be enforced there must be graviton and antigraviton modes, so it is essential not to forget the negative frequencies in all expansions, and ensure that they are initially independent of the positive frequencies. Furthermore, for a clearer physical picture, it is convenient to use the quantum field theory convention stipulating that for free modes the spatial vector $\mathbf{k}$ points in the direction of propagation for both positive and negative frequencies. This is a simple point, but spurious couplings between $\mathbf{k}$ and $-\mathbf{k}$ modes otherwise come about, e.g., reality conditions constrain gravitons moving in opposite directions, which is physically nonsensical.

Bearing this in mind we adopt expansions:

$$\begin{aligned}\delta e_{ij} &= \int \frac{d^3k}{(2\pi)^{3/2}} \sum_r \epsilon^r_{ij}(\mathbf{k})\tilde{\Psi}_e(\mathbf{k},\eta)e_{r+}(\mathbf{k}) \\ &\quad + \epsilon^{r\star}_{ij}(\mathbf{k})\tilde{\Psi}^\star_e(\mathbf{k},\eta)e^\dagger_{r-}(\mathbf{k}) \\ a_{ij} &= \int \frac{d^3k}{(2\pi)^{3/2}} \sum_r \epsilon^r_{ij}(\mathbf{k})\tilde{\Psi}^{r+}_a(\mathbf{k},\eta)a_{r+}(\mathbf{k}) \\ &\quad + \epsilon^{r\star}_{ij}(\mathbf{k})\tilde{\Psi}^{r-\star}_a(\mathbf{k},\eta)a^\dagger_{r-}(\mathbf{k})\end{aligned} \tag{2}$$

where, in contrast with previous literature (e.g., [7,11]), e_{rp} and a_{rp} have two indices: $r = \pm 1$ for right and left helicities, and p for graviton ($p = 1$) and antigraviton ($p = -1$) modes. In a frame with direction $i = 1$ aligned with $\mathbf{k}$ the polarization tensors are:

$$\epsilon^{(r)}_{ij} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \pm i \\ 0 & \pm i & -1 \end{pmatrix}. \tag{3}$$

The base functions have form $\tilde{\Psi}(\mathbf{k},\eta) = \Psi(k,\eta)e^{i\mathbf{k}\cdot\mathbf{x}}$ and we impose boundary conditions $\Psi(k,\eta) \sim e^{-ik\eta}$ when $|k\eta| \gg 1$ for both $+\mathbf{k}$ and $-\mathbf{k}$ directions ($k = |\mathbf{k}| > 0$ throughout this Letter). Only then does $\mathbf{k}$ point in the direction of propagation, as required. This convention has the essential advantage of identifying the proper physical polarization (until we know in which sense the mode is moving we cannot assign to it a physical polarization). The functions Ψ_e and Ψ_a can in principle be anything, with the amplitudes e_{rp} and a_{rp} carrying the necessary time dependence. We may choose Ψ so that they carry the full time dependence. Hamilton's equations then merely confirm that the amplitudes are constant, but $\tilde{\Psi}^{rp}_a$ should have both r and p dependence. In these expansions we have already selected the physical degrees of freedom (i.e., the Gauss and diffeomorphism constraints have been implemented).

In order to canonically quantize the theory we need its Hamiltonian formulation. We will do this in detail elsewhere [14] but stress that we can read off the answer from cosmological perturbation theory [12]. Functions Ψ_e satisfy the same equation as the variable "v" used by cosmologists. Therefore, in a de Sitter background:

$$\Psi''_e + \left(k^2 - \frac{2}{\eta^2}\right)\Psi_e = 0, \tag{4}$$

where $'$ denotes derivative with respect to conformal time. This has solution:

$$\Psi_e = \frac{e^{-ik\eta}}{2\sqrt{k}}\left(1 - \frac{i}{k\eta}\right), \tag{5}$$

where the normalization ensures that the amplitudes e_{rp} become annihilation operators upon quantization. In addition, connection and metric are related by Cartan's torsion-free condition $T = de + \Gamma \wedge e = 0$, solved by

$$\delta\Gamma_i^0 = \frac{1}{a}\delta e'_{ij}dx^j \tag{6}$$

$$\delta\Gamma_{ki} = -\frac{2}{a}\partial_{[k}\delta e_{i]j}dx^j. \tag{7}$$

With the conventions given above the second of these equations implies $\delta\Gamma^i = \frac{1}{a}\epsilon^{ijk}\partial_j\delta e_{kl}dx^l$, so that

$$a_{ij} = \epsilon_{ikl}\partial_k\delta e_{lj} + \gamma\delta e'_{ij}. \tag{8}$$

Inserting decomposition (2) into this expression and using the relation $\epsilon_{nij}\epsilon^r_{il}k_j = irk\epsilon^r_{nl}$ we get:

$$\Psi_a^{rp} = \gamma p\Psi'_e + rk\Psi_e, \tag{9}$$

(we have assumed $a_{rp} = e_{rp}$). Inside the horizon ($|k\eta| \gg 1$) this has the important implication that $\Psi_a^{rp} = (r - ip\gamma)k\Psi_e$ leading to the result that the SD connection ($\gamma = i$) is made up of the right-handed positive frequency of the graviton and the left-handed negative frequency of the antigraviton. The ASD connection contains the other degrees of freedom (this result was derived long ago [15] but seems to have been forgotten in all subsequent work). For other values of γ this is shared differently, and as the modes leave the horizon ($|k\eta| \sim 1$) the classification breaks down.

The theory can now be quantized from Poisson brackets $\{A_a^i(\mathbf{x}), E_j^b(\mathbf{y})\} = \gamma l_P^2\delta_a^b\delta_j^i\delta(\mathbf{x}-\mathbf{y})$. They imply commutation relations for the perturbative variables:

$$[a_a^i(\mathbf{x}), \delta e_j^b(\mathbf{y})] = -i\gamma l_P^2\delta_a^b\delta_j^i\delta(\mathbf{x}-\mathbf{y}). \tag{10}$$

These are valid before the Gauss and vector constraints are enforced and must be replaced by a TT projected δ function upon gauge fixing. Once this is done (details to be presented in [14], but see [16]) we have:

$$[\tilde{a}_{rp}(\mathbf{k}), \tilde{e}^\dagger_{sq}(\mathbf{k}')] = -i\gamma p\frac{l_P^2}{2}\delta_{rs}\delta_{p\bar{q}}\delta(\mathbf{k}-\mathbf{k}'), \tag{11}$$

where $\bar{q} = -q$ and $\tilde{a}_{rp} = a_{rp}\Psi_a^{rp}$ and $\tilde{e}_{rp} = e_{rp}\Psi_e$. In addition we must fix the inner product of the Hilbert space to implement the reality conditions. The reality of the metric ($\delta e_{ij} = \delta e^\star_{ij}$) implies $e_{r+}(\mathbf{k}) = e_{r-}(\mathbf{k})$, i.e., the graviton and antigraviton are identified, polarization by polarization, mode $\mathbf{k}$ by mode $\mathbf{k}$. This is eminently sensible. Reality conditions should never relate different polarizations, or modes $\mathbf{k}$ and $-\mathbf{k}$, since gravity waves are real (even if a complex notation is used). The presence of such spurious couplings in the literature [7,11] merely signals that the direction of motion for a given mode was not properly identified, and in consequence the polarization incorrectly assigned. This is avoided by using expansions (2).

For the connection, the reality and torsion-free conditions are combined: a_{ij} is allowed to be complex but only to the extent that's consistent with the metric being real, given the torsion-free condition. However in the Hamiltonian formalism we only need to impose $\mathrm{Re}A^i = \Gamma^i(E)$, leaving it for the dynamics to discover that $\mathrm{Im}A^i = |\gamma|\Gamma^{0i}$. Thus, $a_{ij} + \bar{a}_{ij} = 2a\delta\Gamma_{ij} = 2\epsilon_{ink}\partial_n\delta e_{kj}$, which in terms of expansion (2) becomes:

$$\tilde{a}_{r+}(\mathbf{k},\eta) + \tilde{a}_{r-}(\mathbf{k},\eta) = 2rk\tilde{e}_{r+}(\mathbf{k},\eta). \tag{12}$$

We defer the reality conditions' implementation via the inner product until after we have the Hamiltonian.

It is straightforward to repeat what follows for a general γ, but for clarity we will make our point by presenting calculations for $\gamma = \pm i$ only, which turn out to be the extreme cases. Then, the Hamiltonian reduces to:

$$\mathcal{H} = \frac{1}{2l_P^2}\int d^3xNE_i^aE_j^b\epsilon_{ijk}(F^k_{ab} + H^2\epsilon_{abc}E_k^c). \tag{13}$$

Expanding, and keeping only second order terms quadratic in first order perturbations leads to:

$$ {}^2_1\mathcal{H} = \frac{1}{2l_P^2}\int d^3x[-a_{ij}a_{ij} + 2\epsilon_{ijk}\delta e_{li}\partial_j a_{kl} - 2\gamma Ha\delta e_{ij}a_{ij} - 2H^2a^2\delta e_{ij}\delta e_{ij}]. \tag{14}$$

To this one must add the boundary term: $\mathcal{H}_{\rm BT} = -\frac{1}{l_P^2}\int d\Sigma_a N\epsilon_{ijk}E_i^aE_j^bA_{bk}$ which perturbatively becomes: ${}^2_1\mathcal{H}_{\rm BT} = \frac{1}{l_P^2}\int d\Sigma_i\epsilon_{ijk}\delta e_{lj}a_{lk}$. Writing it as the volume integral we find for modes inside the horizon:

$$\mathcal{H}_{\rm eff} = \frac{1}{2l_P^2}\int d^3x[-a_{ij}a_{ij} - 2\epsilon_{ijk}(\partial_j\delta e_{li})a_{kl}], \tag{15}$$

to be identified with the Hamiltonian of the effective quantum field theory representing the theory perturbatively. It is easy to see that "on shell" [i.e., using (8)] this is the stress-energy tensor of gravitational waves, with the usual kinetic and gradient terms.

We proceed to find the quantum Hamiltonian for $k|\eta| \gg 1$. We assume an EEF ordering but what follows can be adapted to other orderings. Inserting expansions (2) into (15) we find:

$$\mathcal{H}_{\rm eff} = \frac{1}{l_P^2}\int d^3k\sum_r g_{r-}(\mathbf{k})g_{r+}(-\mathbf{k}) + g_{r-}(\mathbf{k})g^\dagger_{r-}(\mathbf{k}) + g^\dagger_{r+}(\mathbf{k})g_{r+}(\mathbf{k}) + g^\dagger_{r+}(\mathbf{k})g^\dagger_{r-}(-\mathbf{k}), \tag{16}$$

with:

$$g_{r+}(\mathbf{k}) = \tilde{a}_{r+}(\mathbf{k}) \tag{17}$$

$$g^\dagger_{r+}(\mathbf{k}) = -\tilde{a}^\dagger_{r-}(\mathbf{k}) + 2kr\tilde{e}^\dagger_{r-}(\mathbf{k}) \tag{18}$$

$$g_{r-}(\mathbf{k}) = -\tilde{a}_{r+}(\mathbf{k}) + 2kr\tilde{e}_{r+}(\mathbf{k}) \tag{19}$$

$$g^\dagger_{r-}(\mathbf{k}) = \tilde{a}^\dagger_{r-}(\mathbf{k}) \tag{20}$$

where we used $\epsilon^r_{ij}(\mathbf{k})\epsilon^{s\star}_{ij}(\mathbf{k}) = 2\delta^{rs}$ [note that with our conventions $\epsilon^r_{ij}(-\mathbf{k}) = \epsilon^{r\star}_{ij}(\mathbf{k})$]. We have identified (anti-)graviton creation and annihilation operators, $g^\dagger_{rp}$ and g_{rp}, as in [7]. From (11) they inherit the algebra:

$$[g_{rp}(\mathbf{k}), g^{\dagger}_{sq}(\mathbf{k}')] = -i\gamma l_P^2 (pr) k \delta_{rs}\delta_{pq}\delta(\mathbf{k}-\mathbf{k}'). \quad (21)$$

As in [10], half the particles are found to have negative energy (those with $i\gamma = pr$). The Hamiltonian also contains pathological particle production terms: the first and last of (16). These features are removed once the inner product is defined.

Notice first that the reality conditions amount to demanding that $g^{\dagger}_{rp}$ are indeed the Hermitian conjugates of g_{rp}. This fully fixes the inner product [7,17]. We work in a holomorphic representation for wave functions Φ which diagonalizes $g^{\dagger}_{rp}$, i.e.: $g^{\dagger}_{rp}\Phi(z) = z_{rp}\Phi(z)$ (z represents collectively all the $z_{rp}(\mathbf{k})$). Then, (21) implies:

$$g_{rp}\Phi = -i\gamma l_P^2 (pr) k \frac{\partial \Phi}{\partial z_{rp}}. \quad (22)$$

With ansatz $\langle \Phi_1 | \Phi_2 \rangle = \int dz d\bar{z} e^{\mu(z,\bar{z})} \bar{\Phi}_1(\bar{z})\Phi_2(z)$, condition $\langle \Phi_1 | g^{\dagger}_{rp} | \Phi_2 \rangle = \overline{\langle \Phi_2 | g_{rp} | \Phi_1 \rangle}$ therefore requires:

$$\mu(z,\bar{z}) = \int d\mathbf{k} \sum_{rp} \frac{pr}{i\gamma k l_P^2} z_{rp}(\mathbf{k}) \bar{z}_{rp}(\mathbf{k}), \quad (23)$$

fixing $\langle \Phi_1 | \Phi_2 \rangle$. Integrating $g_{rp}\Phi_0 = 0$ leads to the vacuum $\Phi_0 = \langle z | 0 \rangle = 1$. Particle states are monomials in the respective variables, $\Phi_n = \langle z | n \rangle \propto (g^{\dagger}_{rp})^n \Phi_0 = z^n_{rp}$. With the inner product just derived these are not normalizable for $i\gamma = pr$. Therefore such modes should be excluded from the physical Hilbert space, and this removes all pathologies found in the Hamiltonian. We stress that the quantum modes we have disqualified do not exist classically [see discussion after (9)]. For example for $\gamma = i$ the only physical modes are $G_R = g_{R+}$ and $G_L = g_{L-}$.

We therefore regain the usual physical Hamiltonian but with one major difference. For $\gamma = i$, for example, $\mathcal{H}^{\rm phy}_{\rm eff} \approx \frac{1}{l_P^2}\int d\mathbf{k}(G_L G_L^{\dagger} + G_R^{\dagger} G_R)$ and so only the left-handed graviton needs to be normal ordered. Following the standard inflationary calculation (extrapolating the vacuum expectation value V_r of a mode from $|k\eta| \gg 1$ to $|k\eta| \ll 1$) we discover a scale-invariant spectrum with left gravitons only. Repeating this calculation (see [14], for details) for general γ shows that:

$$\mathcal{H}^{\rm phy}_{\rm eff} \approx \frac{1}{2l_P^2}\int d\mathbf{k} \sum_r [G_r G_r^{\dagger}(1 + ir\gamma) + G_r^{\dagger} G_r (1 - ir\gamma)]$$

so, after normal ordering, right and left particles are exactly symmetric, but a chiral V_r is found with:

$$\frac{V_R - V_L}{V_R + V_L} = i\gamma. \quad (24)$$

Strictly speaking this calculation only covers imaginary γ in the range $-i \le \gamma \le i$, but an extension for all γ (including real) will be presented elsewhere [14]. For standard Palatini gravity $\gamma = 0$ and no effect is predicted.

In a longer paper [14] we will spell out the various steps of this calculation and generalize its scope. The relation with other work [4,8,11,18] will also be examined. We note that in [4] a chiral contribution was found for the graviton propagator. The relation with our results should not pass unnoticed but remains tantalizing, since [4] employed a Euclidean signature and a real γ. A generalized formula (24), combining γ with ordering prescriptions will be presented in [14] (note that *FEE* ordering reverses the above argument; *EFE* ordering produces no chirality at all). In any quantum mechanical theory ordering issues are ultimately resolved by their experimental consequences. Quantum gravity is no exception. We have provided one such experimental probe.

In the meantime we have shown how a perturbative reexamination of quantum gravity can be fruitful. We hope to have cleared up a few misconceptions and paradoxes. Above all, we derived a striking prediction for the theory, which could be tested in upcoming cosmic microwave background polarization experiments. There are other mechanisms to generate gravitational chirality (e.g., [18,19]), but the one pointed out in this Letter is by far the simplest. As explained in [13], even moderate chirality in the gravitational wave background would render its detection easier. “Catching two pigeons with one stone” was the expression used in [13] to qualify the ensuing state of affairs.

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