Impact of a Global Quadratic Potential on Galactic Rotation Curves

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We present a conformal gravity fit to the 20 largest of a sample of 110 spiral galaxies. We identify the presence of a universal quadratic potential $V_\kappa(r) = -\kappa c^2 r^2/2$ with $\kappa = 9.54 \times 10^{-54}\text{cm}^{-2}$ induced by cosmic inhomogeneities. When $V_\kappa(r)$ is taken in conjunction with both a universal linear potential $V_{\gamma_0}(r) = \gamma_0 c^2 r/2$ with $\gamma_0 = 3.06 \times 10^{-30}\text{cm}^{-1}$ generated by the homogeneous cosmic background and the contribution generated by the local luminous matter in galaxies, the theory then accounts for the rotation curve systematics observed in the entire 110 galaxies, without the need for any dark matter whatsoever. Our study suggests that using dark matter may be nothing more than an attempt to describe global effects in purely local galactic terms. With $V_\kappa(r)$ being negative, galaxies can only support bound orbits up to distances of order $\gamma_0/\kappa = 100\text{kpc}$, with global physics imposing a limit on the size of galaxies.

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I. Introduction.—At the present time it is widely believed that on scales much larger than solar-system-sized ones astrophysical and cosmological phenomena are controlled by dark matter and dark energy, with luminous matter being only a minor contributor. However, given the lack to date of either direct detection of dark matter particles or of a solution to the cosmological constant problem, a few authors (see, e.g., [1] for a recent review) have ventured to suggest that the standard dark matter and dark energy picture may be incorrect, and that one instead needs to modify the standard Newton-Einstein gravitational theory that leads to that picture in the first place. In this Letter we study one specific alternative to Einstein gravity that has been advanced, namely, conformal gravity. We report here on the results of a conformal gravity study of the instructive 20 largest of a full sample of 110 galaxies, all of whose rotation curves we have been able to fit without the need for any dark matter whatsoever.

In seeking an alternative to Einstein gravity that is to address both the dark matter and dark energy problems, our strategy is to seek some alternate, equally metric-based theory of gravity that possesses all of the general coordinate invariance and equivalence principle structure of Einstein gravity, that yields a geometry that is described by the Ricci-flat Schwarzschild metric on solar-system-sized distance scales while departing from it on larger scales where the dark matter problem is first encountered, and that has a symmetry that can control the cosmological constant Λ. All of these criteria are met in the conformal gravity theory (see, e.g., [1]) that was first developed by Weyl. Specifically, as well as coordinate invariance, in addition one requires that the action be left invariant under local conformal transformations of the form $g_{\mu\nu}(x) \to e^{2\alpha(x)} g_{\mu\nu}(x)$ with arbitrary local phase $\alpha(x)$. Given this requirement, the gravitational action is then uniquely prescribed to be of the form $I_W = -2\alpha_g \int d^4x(-g)^{1/2}[R_{\mu\kappa}R^{\mu\kappa} - (1/3) \times (R^\alpha{}_\alpha)^2]$ where α_g is a dimensionless gravitational coupling constant. With the conformal symmetry forbidding the presence of any fundamental Λ term in I_W, conformal gravity has a control on Λ that is not possessed by Einstein gravity, and through this control conformal gravity is then able to solve the cosmological constant problem [2]. In addition, the conformal gravity equations of motion are given by [1]

$$4\alpha_g W^{\mu\nu} = T^{\mu\nu} \tag{1}$$

where $W^{\mu\nu}$ is a derivative function of $R^{\mu\nu}$. With $W^{\mu\nu}$ vanishing when $R^{\mu\nu}$ vanishes [1], Schwarzschild is thus a vacuum solution to conformal gravity, just as required [3].

II. Universal potentials from the rest of the Universe.—Since $W^{\mu\nu}$ is a derivative function of $R^{\mu\nu}$, it could potentially vanish even if the geometry is not Ricci flat, and the conformal theory could thus have non-Schwarzschild vacuum solutions as well. To identify such solutions, Mannheim and Kazanas solved for the metric associated with a static, spherically symmetric source, to find [4] that due to the underlying conformal symmetry one could bring the exact, all-order line element to the form $ds^2 = -B(r)dt^2 + dr^2/B(r) + r^2 d\Omega_2$. And with $3(W^0{}_0 - W^r{}_r)/B(r)$ then evaluating to $\nabla^4 B(r)$, the metric coefficient $B(r)$ is found to obey the remarkably simple and exact fourth-order derivative equation

$$\nabla^4 B(r) = f(r) \tag{2}$$

where $f(r) = 3(T^0{}_0 - T^r{}_r)/4\alpha_g B(r)$. For a local source of radius r_0 embedded in an empty vacuum (2) possesses an exterior solution of the form

$$B(r > r_0) = 1 - 2\beta/r + \gamma r. \tag{3}$$

Through the γr term the conformal gravity metric thus departs from the exterior Schwarzschild metric at large r alone, just as we want.

In conformal gravity a local gravitational source generates a gravitational potential

$$V^*(r) = -\beta^* c^2/r + \gamma^* c^2 r/2 \quad (4)$$

per unit solar mass, with β^* being given by the familiar $M_\odot G/c^2 = 1.48 \times 10^5$ cm, and with the numerical value of the solar γ^* needing to be determined by data fitting. In the theory the visible local material in a given galaxy would generate a net local gravitational potential $V_{\rm LOC}(r)$ given by integrating $V^*(r)$ over the visible galactic mass distribution. In disk galaxies luminous matter is typically distributed with a surface brightness $\Sigma(R) = \Sigma_0 e^{-R/R_0}$ with scale length R_0 and total luminosity $L = 2\pi\Sigma_0 R_0^2$, with most of the surface brightness being contained in the $R \leq 4R_0$ or so optical disk region. For a galactic mass to light ratio M/L, one can define the total number of solar mass units N^* in the galaxy via $(M/L)L = M = N^* M_\odot$. Then, on integrating $V^*(r)$ over this visible matter distribution, one obtains [1] the net local luminous contribution

$$\begin{aligned} v_{\rm LOC}^2 &= N^*\beta^* c^2 R^2 [I_0(x)K_0(x) - I_1(x)K_1(x)]/2R_0^3 \\ &\quad + N^*\gamma^* c^2 R^2 I_1(x)K_1(x)/2R_0 \end{aligned} \quad (5)$$

(where $x = R/2R_0$) for the velocities of particles orbiting in the plane of the galactic disk.

However, unlike the situation that obtains in standard second-order gravity, one cannot simply use (5) as is to fit galactic rotation curves, as there are two additional global effects coming from the rest of the material in the Universe that need to be taken into consideration as well. To understand why this is so, we recall that for the standard second-order Poisson equation $\nabla^2\phi(r) = g(r)$, the force associated with a general static, spherically symmetric source $g(r)$ is given by

$$\frac{d\phi(r)}{dr} = \frac{1}{r^2}\int_0^r dr' r'^2 g(r'). \quad (6)$$

As such, the import of (6) is that even though $g(r)$ could continue globally all the way to infinity, the force at any radial point r is determined only by the material in the local $0 < r' < r$ region. In this sense Newtonian gravity is local, since to explain a gravitational effect in some local region one only needs to consider the material in that region. Thus in Newtonian gravity, if one wishes to explain the behavior of galactic rotation curves through the use of dark matter, one must locate the dark matter where the problem is and not elsewhere, i.e., within the galaxies themselves.

However, this local character to Newtonian gravity is not a generic property of any gravitational potential. In particular for the fourth-order Poisson equation $\nabla^4\phi(r) = h(r) = f(r)c^2/2$ of interest to conformal gravity, the potential evaluates to

$$\begin{aligned} \phi(r) = &-\frac{r}{2}\int_0^r dr' r'^2 h(r') - \frac{1}{6r}\int_0^r dr' r'^4 h(r') \\ &- \frac{1}{2}\int_r^\infty dr' r'^3 h(r') - \frac{r^2}{6}\int_r^\infty dr' r' h(r'), \end{aligned} \quad (7)$$

so that this time we do find a global contribution to the force coming from material that is beyond the radial point of interest. Hence in conformal gravity one cannot ignore the rest of the Universe, with a test particle in orbit in a galaxy being able to sample both the local field due to the matter in the galaxy [viz. (5)] and the global field due to the rest of the Universe. This global field consists of two components, a cosmological background in which $W^{\mu\nu}$ and $\nabla^4 B(r)$ both vanish and the inhomogeneities in it that cause $W^{\mu\nu}$ and $\nabla^4 B(r)$ to be nonzero, with inhomogeneities in the Universe thus leading to integrals in (7) that can extend to very large distances.

As regards the background, we note that we can add on to (7) any terms that would cause $W^{\mu\nu}$ to vanish, though such terms would only have content if they make $W^{\mu\nu}$ vanish nontrivially. Since the cosmological Robertson-Walker (RW) metric is homogeneous and isotropic, it is conformal to flat, and thus its geometry obeys $W^{\mu\nu} = 0$. For the cosmological background the vanishing of $W^{\mu\nu}$ entails that conformal cosmology be described by $T^{\mu\nu} = 0$. As discussed in [1] the equation $T^{\mu\nu} = 0$ can be satisfied nontrivially, and leads to a topologically open ($K < 0$) RW cosmology, with its contribution to $W^{\mu\nu}$ then vanishing nontrivially, just as desired.

To ascertain the impact of the cosmological background on rotation curves, we note that since cosmology is written in comoving Hubble flow coordinates while rotation curves are measured in galactic rest frames, one needs to transform the RW metric to static coordinates. As noted in [4], the transformation

$$\rho = 4r/[2(1+\gamma_0 r)^{1/2} + 2 + \gamma_0 r], \quad \tau = \int dt R(t) \quad (8)$$

effects the metric transformation

$$\begin{aligned} &-(1+\gamma_0 r)c^2 dt^2 + \frac{dr^2}{(1+\gamma_0 r)} + r^2 d\Omega_2 \\ &= e^{2\alpha(x)}\left[-c^2 d\tau^2 + \frac{R^2(\tau)}{[1-\gamma_0^2\rho^2/16]^2}(d\rho^2 + \rho^2 d\Omega_2)\right], \end{aligned} \quad (9)$$

where $e^{\alpha(x)} = (1+\gamma_0\rho/4)/(1-\gamma_0\rho/4)R(\tau)$. Since an RW geometry is conformal to flat and since it remains so under a conformal transformation, we thus see that in the rest frame of a comoving galaxy (i.e. one with no peculiar velocity with respect to the Hubble flow), a topologically open RW cosmology would look just like none other than a universal linear potential with cosmological strength $\gamma_0/2 = (-K)^{1/2}$.

In the conformal theory then we recognize not one but two linear potential terms, a local $N^*\gamma^*$ dependent one associated with the matter within a galaxy and a global cosmological one $\gamma_0 c^2 r/2$ associated with cosmological background. Thus in the weak gravity limit one can add the two potentials and replace (5) by [5]

$$v_{\rm TOT}^2 = v_{\rm LOC}^2 + \gamma_0 c^2 R/2. \tag{10}$$

In [5] (10) was used to fit the galactic rotation curve data of a sample of 11 galaxies (of which only NGC 2841 and NGC 3198 are in the sample considered here), and good fits were found, with the two universal linear potential parameters being fixed to the values $\gamma^* = 5.42 \times 10^{-41}\ {\rm cm}^{-1}$, $\gamma_0 = 3.06 \times 10^{-30}\ {\rm cm}^{-1}$. The value obtained for γ^* entails that the linear potential of the Sun is so small that there are no modifications to standard solar system phenomenology, with the values obtained for $N^*\gamma^*$ and γ_0 being such that one has to go to galactic scales before their effects can become as big as the Newtonian contribution. The value obtained for γ_0 shows that it is indeed of cosmological magnitude. In the fitting to the 110 galaxy sample these values do not change.

As regards the inhomogeneities in the cosmic background, we note that they would typically be in the form of clusters and superclusters and would be associated with distance scales between 1 Mpc and 100 Mpc or so. Without knowing anything other than that about them, we see from (7) that for calculating their effect on galactic distance scales (viz. scales much smaller than cluster scales themselves) the inhomogeneities would contribute constant and quadratic terms multiplied by integrals that are evaluated between end points that do not depend on the galaxy of interest, to thus be constants [6]. Thus, again up to peculiar velocity effects, we augment (10) to

$$v_{\rm TOT}^2 = v_{\rm LOC}^2 + \gamma_0 c^2 R/2 - \kappa c^2 R^2, \tag{11}$$

with asymptotic limit

$$v_{\rm TOT}^2 \to N^*\beta^* c^2/R + N^*\gamma^* c^2 R/2 + \gamma_0 c^2 R/2 - \kappa c^2 R^2. \tag{12}$$

It is thus (11) with its universal κ that we must use for fitting galactic rotation curves, and in making such fits the only parameter that can vary from one galaxy to the next is the galactic disk mass to light ratio as embodied in N^*. Our fits are thus highly constrained, one parameter per galaxy, fits (the fits also include the effect of HI gas, but for the gas the mass is known), with everything else being universal, and no dark matter being assumed.

III. Data fitting.—We recall that in [5] successful rotation curve fitting to an 11 galaxy sample was obtained using (10), and one would thus initially anticipate that even if the $-\kappa c^2 R^2$ term were to be present in principle, in practice it would be too small to have any effect. However, the sample we have studied now is altogether larger (110 galaxies) and it contains some very instructive galaxies whose data points extend to larger distances from galactic centers than had been the case for the earlier 11 galaxy sample. It is through fitting these highly extended galaxies that we are able to uncover a role for the $-\kappa c^2 R^2$ term and extract a value for κ given by $\kappa = 9.54 \times 10^{-54}\ {\rm cm}^{-2}$ [7]. And in the fitting to the full 110 galaxy sample to be reported elsewhere [8] (a varied sample of galaxies that includes high (HSB) and low (LSB) surface brightness galaxies and dwarfs) we are able to confirm that even with this now fixed value for κ, (11) fully accounts for the data. With κ being found to be of order $1/(100\ {\rm Mpc})^2$, it is indeed an inhomogeneous rather than a Hubble distance scale.

In Fig. 1 we present our fits to the 20 galaxy sample with all the relevant parameters being given in [8]. In Fig. 1 the rotational velocities and errors (in km sec^{-1}) are plotted as a function of radial distance (in kpc). For each galaxy we exhibit the contributions due to the luminous Newtonian term alone (dashed curve), the two linear terms alone (dash-dotted curve), the two linear terms and the quadratic terms combined (dotted curve), with the full curve showing the total contribution. As we see, without any need for dark matter, our fitting captures the essence of the data. Because the data go out to much further distances than had been the case for the sample studied in [5], the data are now sensitive to the rise in velocity associated with the linear potential terms, and it is here that the quadratic term acts to actually arrest the rise altogether (dotted curve) and cause all rotation velocities to ultimately fall. Moreover, since v^2 cannot be negative, beyond a distance $R \sim (N^*\gamma^* + \gamma_0)/2\kappa$, which would typically be of order 100 kpc or so, there could

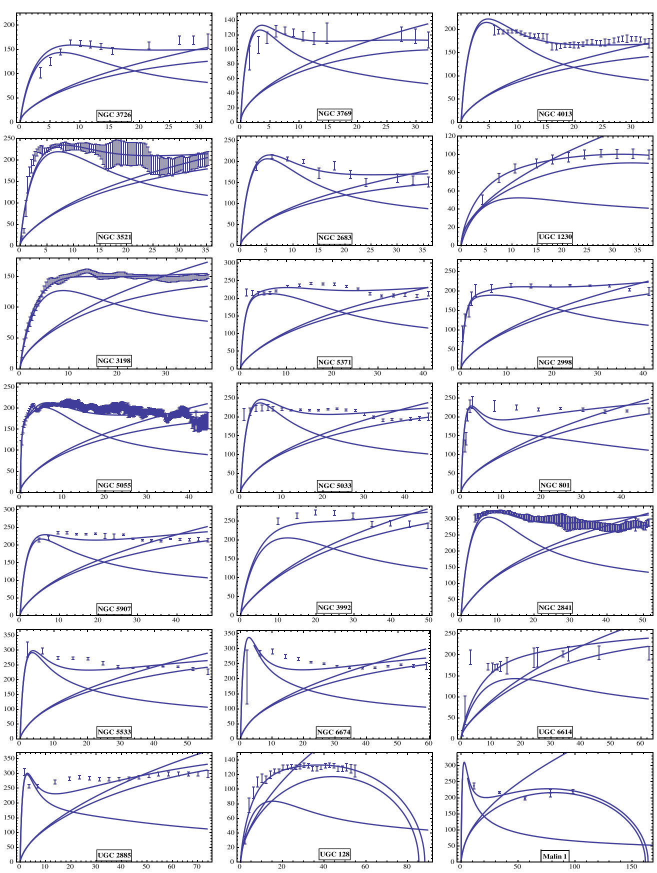

FIG. 1 (color online). Fitting to the rotational velocities of the large galaxy sample.

no longer be any bound orbits, with galaxies thus having a natural way of terminating, and with global physics thus imposing a natural limit on the size of galaxies. To illustrate this we plot the rotation curve for UGC 128 over an extended range [9]. The fits presented here and in [8] are noteworthy since the universal γ_0 and κ terms have no dependence on individual galactic properties whatsoever and yet have to work in every single case.

It is important to appreciate that the fits provided by conformal gravity (and likewise by other alternate theories such as the MOND and MSTG [10] theories) are predictions. Specifically, for all these theories the only input one needs is the optical data, and the only free parameter is the M/L ratio for each given galaxy, with rotation velocities then being determined [11]. It is important to emphasize that the fits are predictions since dark matter fitting to galactic data works very differently. There one first needs to know the velocities so that one can then ascertain the needed amount of dark matter; i.e., in its current formulation dark matter is only a parametrization or postdiction of the velocity discrepancies that are observed and is not a prediction of them. Dark matter theory has yet to develop to the point where one is able to predict rotation velocities given a knowledge of the luminous distribution alone. Thus dark matter theories, and, in particular, those theories that produce dark matter halos in the early Universe, are currently unable to make an *a priori* determination as to which halo is to go with which particular luminous matter distribution, and need to fine-tune halo parameters to luminous parameters galaxy by galaxy. No such shortcoming appears in conformal gravity, and if standard gravity is to be the correct description of gravity, then a universal formula akin to the one given in (11) would need to be derived by dark matter theory. However, since our study establishes that global physics does indeed influence local galactic motions, the invoking of dark matter in galaxies could potentially be nothing more than an attempt to describe global effects in purely local galactic terms.

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Note added.—We are indebted to a referee who kindly informed us that the galaxy Malin 1 goes out even further (to a mammoth 98.0 kpc) than any of the galaxies in our sample. As such it provides an immediate test of our ideas. Malin 1 has been studied by Pickering *et al.* and by Lelli *et al.* [12]. For our fit we use the first four points of Lelli *et al.* and the fifth point of Pickering *et al.* (as then adjusted to the 38° inclination given in Lelli *et al.*). For Malin 1 $D_L = 338.4$ Mpc, $D_A = D_L/(1+z)^2 = 288.9$ Mpc, $L_B = 7.9 \times 10^{10} L_\odot^B$, $M_{\mathrm{HI}} = 5.4 \times 10^{10} M_\odot$, and $(v^2/c^2R)_{\mathrm{last}} = 1.77 \times 10^{-30}$ cm^{-1}, just as we want [11]. The galaxy has a disk with $R_0(\mathrm{disk}) = 84.2$ kpc and a bulge, and with $R_0(\mathrm{gas}) = 4R_0(\mathrm{disk})$ we obtain the fit shown in Fig. 1, with $M_{\mathrm{disk}} = 1.0 \times 10^{10} M_\odot$, $M_{\mathrm{bulge}} = 9.5 \times 10^{10} M_\odot$, $R_0(\mathrm{bulge}) = 1.0$ kpc, $(M_{\mathrm{disk}} + M_{\mathrm{bulge}})/L_B = 1.32 M_\odot/L_\odot^B$. As we see, the fit is quite acceptable, with the required ultimate fall in velocity beginning to set in shortly beyond the last data point.

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[7] With our data sample being so big (110 galaxies) that it includes galaxies that go out to radial distances that are large enough for the quadratic potential to matter, our study underscores the value of having a large data set.
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