Einstein-Podolsky-Rosen Correlations of Ultracold Atomic Gases

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We demonstrate that collective continuous variables of two species of trapped ultracold bosonic gases can be Einstein-Podolsky-Rosen-correlated (entangled) via *inherent* interactions between the species. We propose two different schemes for creating these correlations—a dynamical scheme and a static scheme analogous to two-mode squeezing in quantum optics. We quantify the correlations by using known measures of entanglement and study the effect of finite temperature on these quantum correlations.

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Einstein, Podolsky, and Rosen (EPR) pointed out [1] that correlations induced between quantum objects will persist after these objects have ceased to interact. Consequently, their joint continuous variables (CV), e.g., the difference of their positions and the sum of their momenta, may be specified, regardless of their distance, with arbitrary precision. EPR correlations have since been studied extensively and were shown to give rise to [2–6] inseparability (*entanglement*) of their quantum states.

In studying continuous variable entanglement, it is instructive to draw an analogy with the original EPR scenario [1], wherein two particles, 1 and 2, are defined through their position and momentum variables $x_{1,2}$ and $p_{1,2}$, respectively. EPR saw as a paradox that the observables of particle 2 (x_2 and p_2) are determined by the measurement choice on particle 1. Thus, the product of the variance of x_2 following a measurement of x_1, and of the variance of p_2 following a measurement of p_1, appears to be unbound by the Heisenberg relation $\Delta x_2 \Delta p_2 \leq 1/2$ (choosing $\hbar = 1$). The EPR state is deemed entangled in the continuous translational variables of the two particles. In quantum optics these variables are associated with the sum and difference of field quadratures of two light modes mixed by a symmetric beam splitter [6,7] [Fig. 1(a)]. EPR entanglement is a fundamental resource for quantum information [6,8,9] and for CV teleportation of light [10,11] and matter waves [12,13]. Lately, such entanglement has been demonstrated for *collective* CV of distant thermal-gas clouds, correlated by interaction with a common field [14,15].

Here we show that collective CV of two species of trapped ultracold bosonic gases can be EPR-correlated (entangled) via *inherent* interactions between the species. This paves the way to further quantum information and precision metrology applications of such systems and to analogous approaches in other related fields, such as quantum optics and superconducting Josephson junctions (JJs) [16,17]. Note the related recent work on other scenarios of EPR entanglement generation [18].

EPR criteria.—In order to qualify the EPR correlations, one may adopt two distinct criteria. The first criterion imposes an upper bound on the product of the variances of EPR-correlated *commuting* dimensionless operators, $\hat{x}_+$ and $\hat{p}_-$ or $\hat{x}_-$ and $\hat{p}_+$ [8,12]:

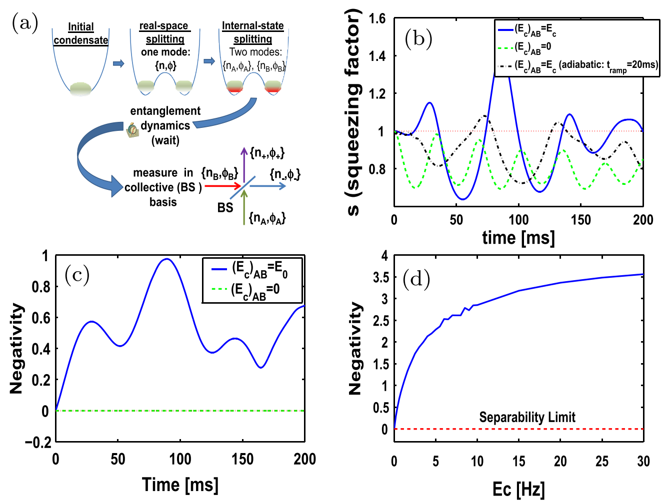

FIG. 1 (color online). (a) State-preparation scheme (see the text). (b) Dynamics of the entanglement defined by the two-mode squeezing criterion for both sudden and slow intermode coupling barrier ramping up, found through exact simulation of Eq. (1) in Ref. [25] [unless stated otherwise, the simulations are for $N_1 = N_2 = 20$, $J = 1$ kHz, and $(E_c)_{AA} = (E_c)_{BB} = (E_c)_{AB} = 1$ Hz]. The plot includes the case of a coupled system ($E_{c12} = E_c$, solid blue line), an uncoupled system ($E_{c12} = 0$, dashed green line), slow ramp-up (dash-dotted black line), and the classical limit (dotted red line). The trade-off between entanglement and nonlinear phase diffusion is better for the sudden coupling. (c) The same as (b) (except for slow barrier ramp-up) for the negativity entanglement measure (see the text). (d) Maximally achievable negativity (solid blue line) reached through the dynamics as a function of charging energy E_c, assuming all coefficients are equal: $(E_c)_{AA} = (E_c)_{BB} = (E_c)_{AB} \equiv E_c$ [32,33]. The separability limit is indicated by the red dashed line.

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$$\langle \Delta \hat{x}_{\pm}^2 \rangle \langle \Delta \hat{p}_{\mp}^2 \rangle \equiv \frac{1}{4s} \leq \frac{1}{4}. \quad (1)$$

Only one of the inequalities (upper or lower sign) can be simultaneously satisfied (otherwise, the uncertainty principle is violated). The EPR correlation is then measured by the two-mode squeezing factor $1 < s < \infty$. The second criterion is the inseparability (entanglement) for *Gaussian* states [9,15], related to the sum of the variances of the correlated observables $\epsilon \equiv \langle \Delta \hat{x}_{\pm}^2 \rangle + \langle \Delta \hat{p}_{\mp}^2 \rangle - 1 < 0$.

The criteria described above are commonly used and are experimentally accessible [15,19]. However, they are not well-posed entanglement measures. We thus employ a third entanglement criterion, the positivity of the partial transpose [20,21], which has been adapted for bipartite Gaussian CV [22–24]. The negativity measure, derived from positivity of the partial transpose, consists of transposing the density matrix with respect to one subsystem and summing the negative eigenvalues of the resulting matrix.

Scheme for "global" EPR correlations in bosonic JJs.— We first consider the correlation of the two species (two internal states of the atom), in the presence of tunnel coupling between the left (L) and right (R) wells. We assume that there is no population exchange between the internal states $|A\rangle$ and $|B\rangle$, and therefore the numbers of atoms N_A and N_B in these states are constants of motion. The Hamiltonian [25] can then be written in this basis in terms of the left-right atom-number differences in the two internal states, $\hat{n}_A = (\hat{a}_L^\dagger \hat{a}_L - \hat{a}_R^\dagger \hat{a}_R)/2$ and $\hat{n}_B = (\hat{b}_L^\dagger \hat{b}_L - \hat{b}_R^\dagger \hat{b}_R)/2$, and their canonically conjugate phase operators $\hat{\phi}_{A,B}$, obeying the commutation relations $[\hat{\phi}_\alpha, \hat{n}_{\alpha'}] = i\delta_{\alpha\alpha'}$ ($\alpha, \alpha' = A, B$). For simplicity we assume from now on that $N_A = N_B \equiv N$ and consider small interwell number differences such that $\langle \hat{n}_{A,B} \rangle \ll N$. The Hamiltonian [26] then becomes

$$\begin{aligned} H = {} & (E_c)_{AA}\hat{n}_A^2 + (E_c)_{BB}\hat{n}_B^2 + 2(E_c)_{AB}\hat{n}_A\hat{n}_B \\ & - JN(\cos\hat{\phi}_A + \cos\hat{\phi}_B) \\ & + \frac{2J}{N}(\hat{n}_A^2 \cos\hat{\phi}_A + \hat{n}_B^2 \cos\hat{\phi}_B). \end{aligned} \quad (2)$$

Here the nonlinearity coefficients ("charging" energies) $(E_c)_{AA}$, $(E_c)_{BB}$, and $(E_c)_{AB}$ are determined, respectively, by the intra- and interspecies s-wave scattering lengths. The tunneling energy J is the same for atoms in the internal states $|A\rangle$ and $|B\rangle$.

Equation (2) displays the full dynamics used in our numerics (Fig. 1), that of two quantum nonlinear pendula coupled via $2(E_c)_{AB}\hat{n}_A\hat{n}_B$. This coupling between the pendula associated with different species is the key to their global correlations (extending over both wells).

We may, for didactic purposes, simplify (2) by expanding the cosine terms. In the lowest-order approximation $\cos\hat{\phi}_{A,B} \simeq 1 - \hat{\phi}_{A,B}^2/2$, the system is described by two *coupled* harmonic oscillators. This suggests that the system under study can indeed satisfy the entanglement or two-mode squeezing criteria, if the relevant collective variables in our system are mapped onto those of two field modes mixed by a symmetric beam splitter

$$\begin{aligned} \hat{n}_{\pm} &= \frac{1}{\sqrt{2}}(\hat{n}_A \pm \hat{n}_B) \leftrightarrow \hat{x}_{\pm}, \\ \hat{\phi}_{\pm} &= \frac{1}{\sqrt{2}}(\hat{\phi}_A \pm \hat{\phi}_B) \leftrightarrow \hat{p}_{\pm}. \end{aligned} \quad (3)$$

Using the collective variables defined in (3), we can rewrite Eq. (2) in the harmonic approximation, assuming $(E_c)_{AA} \simeq (E_c)_{BB} = E_c$, as

$$\begin{aligned} \hat{H} = {} & \left(E_c + (E_c)_{AB} + \frac{2J}{N}\right)\hat{n}_+^2 + \frac{JN}{2}\hat{\phi}_+^2 \\ & + \left(E_c - (E_c)_{AB} + \frac{2J}{N}\right)\hat{n}_-^2 + \frac{JN}{2}\hat{\phi}_-^2. \end{aligned} \quad (4)$$

Hence, the transformed Hamiltonian describes two *uncoupled* harmonic modes in the collective basis. The "+" mode corresponds to Josephson oscillations of the total atomic population (regardless of the internal state) between the two wells, such that the interspecies ratio in each well is constant (in-phase oscillations of the A and B species). The "−" mode corresponds to oscillations of the interspecies ratio between the two wells, such that the total population imbalance does not change (out-of-phase oscillations of the A and B species). These two modes have different fundamental frequencies $\omega_{\pm}$ (see [25]).

We now study the EPR correlation criteria using the uncoupled $\pm$ mode basis. For $(E_c)_{AB} > 0$ [Eqs. (1), (3), and (4)] the EPR criteria may be satisfied. The two-mode squeezing factor is $s = \{[2J/N + E_c + (E_c)_{AB}]\}/\{[2J/N + E_c - (E_c)_{AB}]\}$. We then obtain $s \gg 1$ for $E_c \simeq (E_c)_{AB} \gg 2J/N$ and for the ground states of both modes, approaching the *ideal* EPR limit $s \to \infty$ of full CV entanglement. Beyond the lowest-order approximation that has led to (4), there is parametric coupling of the collective modes that may induce nontrivial dynamics of CV wave packets conforming to the Born-Oppenheimer coupling regime (see [25]).

For *exact* calculation of the dynamics we must resort to the angular momentum operators that describe the two-pendula system [25]. The resulting entanglement criteria differ from those used for the number-phase operators only for significant nonlinear phase diffusion, which affects the state preparation that would yield the largest EPR correlations (see [25]).

In Fig. 1(d), we plot the maximally achievable negativity (from the positivity of the partial transpose criterion) as a function of the charging energy. We find that the entanglement grows with increasing charging energy and asymptotically reaches a maximal value for $NE_c \geq J$. We note, however, that as interactions become stronger, they cause significant nonlinear phase dispersion and therefore reduce the measurable EPR correlations.

The optimal *sudden* sequence for state preparation consists of [Fig. 1(a)] (a) filling the ground state of the original trap (single well) by a Bose-Einstein condensate (BEC) in internal state $|A\rangle$, (b) fast ramping up of the interwell potential barrier, thus creating a two-well symmetric superposition (in the two-mode approximation—see [25] for calculation of excitations created in this process), and (c) transforming state $|A\rangle$ into a symmetric superposition of $|A\rangle$ and $|B\rangle$ by a fast $\pi/2$ pulse. This sequence (which basically consists of $\pi/2$ transformations in both external and internal degrees of freedom) yields an initial coherent state in the two original modes. The EPR entanglement of these modes then builds up with time according to their coupled-pendula dynamics [Eq. (2)]. By contrast, *slower* ramping up of the barrier causes them to be exposed to both nonlinear phase diffusion and environment-induced dephasing (see below) much longer, thus potentially spoiling the experimentally measurable entanglement criteria [Fig. 1(b)].

Experimentally, this entanglement can be measured through the variances of the collective number and phase variables of Eq. (3), from which the squeezing and the separability can be calculated. Measuring these variables amounts to counting the atoms in each well and in each internal state (via state-dependent absorption imaging) and measuring the phase of each species through time-of-flight interference (see [19] for details).

In Fig. 1(c), we plot the negativity as a function of time for the above sequence. The observed dynamics is seen to be inseparable and follows qualitatively the dynamics of the two other criteria [Fig. 1(b)].

We note that it is not advantageous in this scheme to redistribute the conjugate variances of the initial state by creating a single-mode squeezed state in each well. Intuitively, this is due to the fact that such squeezing does not translate into correlations between the wells and, thus, does not induce reduced variances of the two-mode observables (for details, see [25]).

Scheme for local-mode correlations in bosonic JJs.—We now present an approach based on correlations of two squeezed *local* (left- and right-well) modes [Fig. 2(a)]. The system is initialized in the left well (L) of a double-well potential, in a single internal state (A). Then, the barrier is suddenly dropped in order to create a coherent superposition of the ground vibrational state $|g\rangle$ and first-excited state $|e\rangle$ of the new (single-well) potential (see [25] for the validity of the two-mode approximation). Next, a $\pi/2$ pulse creates a coherent superposition of the internal states A and B of the atoms.

To lowest order the cross coupling between $|g\rangle$ and $|e\rangle$ is neglected, and therefore the number of particles in each vibrational state is conserved. This conservation allows us to rewrite the Hamiltonian in terms of the internal-state number difference operator in each vibrational state, $\hat{n}_g = (\hat{n}_g)_A - (\hat{n}_g)_B$ and $\hat{n}_e = (\hat{n}_e)_A - (\hat{n}_e)_B$. The Hamiltonian then becomes (see [25])

$$\hat{H} = [(E_c)_{AA} + (E_c)_{BB} - 2(E_c)_{AB}](\hat{n}_g^2 + \hat{n}_e^2). \quad (5)$$

Thus, the system evolves *separately* in $|g\rangle$ and $|e\rangle$, each undergoing dynamical single-mode squeezing in the internal-state basis [27,28]. Such internal-state squeezing was demonstrated recently [19].

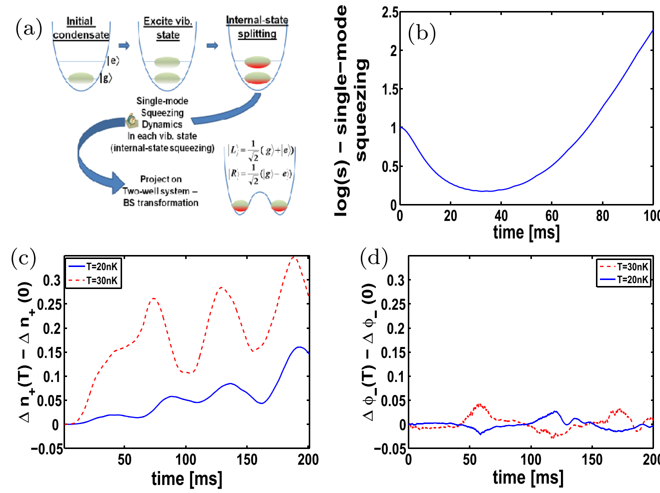

FIG. 2 (color online). (a) Schematic sequence for the creation of "nonlocal" two-mode entanglement in analogy with the BS approach. (b) Single-mode squeezing dynamics as a function of time, for $N = 100$. Decoherence effects: the variance of n_+ (c) and of ϕ_- (d) as a function of time, in the presence of proper dephasing. We subtract the variance of the Hermitian dynamics to single out the effect of dephasing. The coherence time is estimated to be ~100 ms at 20 nK.

Having accumulated maximal internal-state squeezing, we raise the barrier quickly to create two separate symmetric wells, denoted L (left) and R (right). This sudden projection creates a beam splitter (BS)-like transformation to the basis $|L\rangle = (1/\sqrt{2})(|g\rangle + |e\rangle)$, $|R\rangle = (1/\sqrt{2})(|g\rangle - |e\rangle)$. Local measurements may now be done in the internal-state basis in each well separately, resulting in *nonclassical* correlations between observables pertaining to the spatially separated wells.

The actual ensemble measurements consist of counting the number of atoms in each species in each well in every experimental realization, either directly or following a rotation of the internal states by using microwave pulses (to map the phase variables into number variables). These techniques are described in detail in Ref. [29].

The scheme presented above is analogous to the quantum optics scheme [6], in which two independent single-mode squeezed states are injected into the input ports of a BS, thereby creating pairs of entangled modes at the output ports of the BS. However, the *intrinsic* nonlinearity of $|g\rangle$ and $|e\rangle$ in the BEC causes unwarranted mixing of these single-mode analogs even *before* the BS-like transformation, causing fidelity loss (see [25]). The best fidelity is achieved by modifying the barrier fast compared to the change of the plasma frequency (nonadiabatic) but slow compared to the local intrawell trapping frequencies.

In this sequence we can immediately use the maximal internal-state squeezing factor s of a single "mode" [6], to calculate the BEC analog of optical two-mode squeezed variances

$$\langle \Delta \hat{n}_+^2 \rangle = \langle \Delta(\hat{n}_L + \hat{n}_R)^2 \rangle = \langle \Delta(n_+^{(0)})^2 \rangle / s,$$
$$\langle \Delta \hat{\phi}_-^2 \rangle = \langle \Delta(\hat{\phi}_L - \hat{\phi}_R)^2 \rangle = \langle \Delta(\phi_-^{(0)})^2 \rangle / s; \quad (6)$$

namely, here the two-mode squeezing parameter is equal to that of single-mode squeezing. This squeezing parameter now characterizes the knowledge obtained about variables in one well having measured their counterparts in the other well [Fig. 2(b)].

Decoherence effects.—We now turn to the effect of environment-induced decoherence on the robustness of EPR entanglement in this system. We assume proper dephasing created by independently fluctuating (stochastic) energy shifts of atoms in each internal state and well, caused by the thermal atomic or electromagnetic environment. Because of the spectroscopic similarity of the two BEC species, we reduce the number of independent stochastic energy shifts $\epsilon_{A(B)L(R)}$ by setting $\epsilon_{AL}/\epsilon_{BL} = \epsilon_{AR}/\epsilon_{BR} = (1-\xi)/(1+\xi)$ and assuming a *"symmetrized environment,"* i.e., $\xi \ll 1$. In order to calculate the effect of these fluctuations on the variance of the number-phase operators, we proceed with a linear expansion around the Josephson regime solution and use the Wiener-Khinchin theorem to get

$$\langle \Delta \hat{n}_+^2 \rangle = \langle \Delta \hat{n}_+^2 \rangle|_{t=0} + \langle \Delta_+^2 \rangle_{\text{gr}} \zeta \mathcal{T}(\omega_+) t,$$
$$\langle \Delta \hat{\phi}_-^2 \rangle = \langle \Delta \hat{\phi}_-^2 \rangle|_{t=0} + 4N^{-1} \xi^2 \mathcal{T}(\omega_-) t. \quad (7)$$

Here $\zeta = [1 - \xi(N_A - N_B)/N^2]^2$ and $\mathcal{T}(\omega) = \int d\omega' \mathcal{S}_c(\omega') \sin[(\omega - \omega')t]/(\omega - \omega')$, $\mathcal{S}_c(\omega)$ being the power spectrum of the fluctuating energy imbalance. Because of the small value of ξ, the variance of $\hat{\phi}_-$ almost does not change (in either the global or local scheme), while the variance of $\hat{n}_+$ increases linearly and is responsible for the growing loss of entanglement (see [25]). Hence, we may manipulate the system as we see fit *within* the coherence time [see Figs. 2(c) and 2(d)].

Discussion.—We have addressed EPR effects in an ultracold-atom analog of two coupled JJs: a *two-species* BEC, each species corresponding to a different sublevel of the atomic internal ground state [30], trapped in a tunnel-coupled double-well potential [Fig. 1(a)]. We have shown that such bosonic coupled JJs can induce EPR entanglement of appropriate combinations of collective variables. Alternatively, it can dynamically realize the analog of what is known in optics as mixing by a beam splitter of two squeezed modes [31]. This entanglement has been shown to be resilient to environmental noise (decoherence).

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