Measurement-Induced Nonlocality

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We interpret the maximum global effect caused by locally invariant measurements as measurement-induced nonlocality, which is in some sense dual to the geometric measure of quantum discord [Dakic, Vedral, and Brukner, Phys. Rev. Lett. **105**, 190502 (2010)]. We quantify measurement-induced nonlocality from a geometric perspective in terms of measurements, and obtain analytical formulas for any dimensional pure states and $2 \times n$ dimensional mixed states. We further derive a tight upper bound to measurement-induced nonlocality in general case. The physical significance of measurement-induced nonlocality is discussed in the context of correlations, entanglement, quantumness, and cryptographic communication.

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Nonlocality is a controversial, perplexing, and yet fundamental theme in the physical (e.g., gravitational and quantum-theoretical) descriptions of nature, and has many intriguing and subtle manifestations [1]. Classically, the principle of locality dictates that physical effect propagates by a knowable physical mechanism and is limited by the speed of light, and nonlocality usually refers to instantaneous influence of an object on a distant one in gravitation, resulting in violation of this principle. Quantum mechanically, nonlocality arises from at least two different scenarios: One is the Aharonov-Bohm effect related to quantum potential [2], and the other is quantum entanglement involving the Einstein-Podolsky-Rosen type correlations and the so-called "spooky-action-at-a-distance" [3]. Quantum nonlocality usually refers to correlations that cannot be described by any local hidden variable theory, and has been widely studied by means of Bell's inequalities [4–8]. It is intimately related to, but different from, other strange phenomena such as entanglement and quantumness [9].

In this Letter, by nonlocality, we will understand, in a most broad way, as some kind of correlations. This is more general than the conventionally mentioned quantum nonlocality related to entanglement or violation of Bell's inequalities. It is desirable to quantify nonlocality from as many aspects as possible in order to reveal its meaning and properties from different angles. As a particular approach to this program, we will try to quantify nonlocality from a geometric perspective based on von Neumann measurements.

One motivation for this investigation comes from the general consideration of exploiting nonlocality for the purpose of processing quantum information. Many quantum tasks such as superdense coding [10], teleportation [11], remote state preparation [12–15], etc., involve local measurements and comparison between the pre- and post-measurement states. Quantification of nonlocality may shed novel and deep insight into these tasks and their extensions. Moreover, since a bipartite state can be used as a quantum communication channel, and in order to study various capacities of such channels, it may be helpful to quantify the nonlocal resources therein.

Our intuitive setup for quantifying measurement-induced nonlocality is as follows. Consider a bipartite quantum state ρ shared by two parties a and b with respective system Hilbert spaces H^a and H^b. In order to probe the nonlocal feature in ρ, we perform local von Neumann measurements on party a, and investigate the difference between the overall pre- and post-measurement states. To capture the genuine nonlocal effect of measurements on the state, we require the measurements do not disturb the local state $\rho^a := \mathrm{tr}_b \rho$ (partial trace). Based on this idea, we may define the measurement-induced nonlocality (somewhat in contrast to the measurement-induced disturbance [16]) as

$$N(\rho) := \max_{\Pi^a} \|\rho - \Pi^a(\rho)\|^2, \tag{1}$$

where the max is taken over the von Neumann measurements $\Pi^a = \{\Pi^a_k\}$ *which do not disturb* ρ^a *locally*, that is, $\sum_k \Pi^a_k \rho^a \Pi^a_k = \rho^a$, and $\|\cdot\|^2$ may be any reasonable norm on states depending on particular applications and contexts. Here we take $\|X\|^2 := \mathrm{tr} X^\dagger X$ to be the Hilbert-Schmidt norm. This quantity is an indicator of the global effect caused by locally invariant measurements [17], which in turn is inspired and motivated by superdense coding consideration and related issues [10,18,19]. Our main purpose here is to illustrate this quantity and evaluate it for several important cases.

The measurement-induced nonlocality $N(\rho)$ is fundamentally different from, and in some sense dual to, the geometric measure of quantum discord [20–22]

$$D(\rho) := \min_{\Pi^a} \|\rho - \Pi^a(\rho)\|^2,$$

which was first introduced in Ref. [20]. Here the min is over *all* local von Neumann measurements Π^a, in sharp contrast to the max over locally invariant ones used in defining the measurement-induced nonlocality $N(\rho)$ in Eq. (1). The geometric measure of quantum discord is itself motivated by the original quantum discord introduced as a measure of quantum correlations [23–25]. The relation between $N(\rho)$ and $D(\rho)$ is somewhat like the relation between the entanglement of assistance [26] and the entanglement of formation [27,28].

The measurement-induced nonlocality provides a novel classification scheme for bipartite states, and is a quantum resource quite different from entanglement. It may be useful in quantitative study of quantum state steering [29,30], remote state control [12–15,31], general quantum dense coding [32,33], cryptography, and may shed alternative light on these issues. It is particularly relevant to certain cryptographic communication. For example, consider the task that party a wants to send information to party b who is faraway. When they share a composite state ρ, party a can encode her messages by locally manipulating her part of the state, and then sends it to party b, who then decodes the message from the overall joint state. In order to exclude eavesdropping in the communication, party a chooses measurements that will not disturb her local state. In this scenario, it is natural for party a to choose a measurement which maximizes the difference between the pre- and post-measurement states in order for party b to detect the change of the joint state (thus the encoded messages) most reliably. The eavesdropper gets no information at all because he is always facing the same state ρ^a, which is left invariant. One may also consider the task of nonlocality-assisted manipulation of local states and its implications, just like the entanglement-assisted manipulation of states [34].

We now list some basic properties of the measurement-induced nonlocality. (i) $N(\rho) = 0$ for any product state $\rho = \rho^a \otimes \rho^b$. (ii) $N(\rho)$ is locally unitary invariant in the sense that $N((U \otimes V)\rho(U \otimes V)^\dagger) = N(\rho)$ for any unitary operators U and V acting on H^a and H^b, respectively. (iii) If ρ^a is nondegenerate with spectral decomposition $\rho^a = \sum_k \lambda_k |k\rangle\langle k|$, then $N(\rho) = \|\rho - \Pi^a(\rho)\|^2$ with $\Pi^a(\rho) = \sum_k (|k\rangle\langle k| \otimes \mathbf{1}^b)\rho(|k\rangle\langle k| \otimes \mathbf{1}^b)$. This is because in such a situation, the only von Neumann measurement that does not disturb ρ^a is $\Pi^a = \{\Pi^a_k = |k\rangle\langle k|\}$, and thus the max in Eq. (1) is not necessary. In particular, $N(\rho)$ vanishes for any classical-quantum state $\rho = \sum_k p_k |k\rangle\langle k| \otimes \rho^b_k$ whose marginal state $\rho^a = \sum_k p_k |k\rangle\langle k|$ is nondegenerate. (iv) $N(\rho)$ is strictly positive for any entangled state ρ because for any von Neumann measurement Π^a, the post-measurement state $\Pi^a(\rho)$ is a classical-quantum state and thus is separable. In particular, this implies that ρ and $\Pi^a(\rho)$ are always different. (v) For any Bell state, e.g., $|\Phi\rangle := \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, we have $N(|\Phi\rangle\langle\Phi|) = \frac{1}{2}$ (by Theorem 1) which achieves the maximal value of measurement-induced nonlocality in the class of two-qubit states. In contrast, for the classical state $\rho_c = \frac{1}{2}|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \otimes |1\rangle\langle 1|$, we have $N(\rho_c) = \frac{1}{4}$ (by Theorem 3). Thus $N(|\Phi\rangle\langle\Phi|) = 2N(\rho_c)$, which is reminiscent of the quantum mutual information relation $I(|\Phi\rangle\langle\Phi|) = 2I(\rho_c)$.

The measurement-induced nonlocality for any pure state can be evaluated as follows.

Theorem 1.—Let $|\Psi\rangle$ be a bipartite pure state with the Schmidt decomposition $|\Psi\rangle = \sum_i \sqrt{s_i}|\alpha_i\rangle \otimes |\beta_i\rangle$, then

$$N(|\Psi\rangle\langle\Psi|) = 1 - \sum_i s_i^2. \quad (2)$$

Interestingly, for any pure state ρ, the measurement-induced nonlocality $N(\rho)$ and the geometric measure of quantum discord $D(\rho)$ coincide. However, this is not necessary the case for mixed states.

In order to evaluate the measurement-induced nonlocality for general bipartite states, let us first recall some notations for operator Hilbert spaces. Let $L(H^a)$ be the Hilbert space of linear operators with the inner product $\langle X|Y\rangle := \mathrm{tr}X^\dagger Y$. In this space, a family of operators $\{X_i := i = 0, 1, \cdots, m^2 - 1\}$ is called an orthonormal operator base if $\langle X_i|X_j\rangle = \delta_{ij}$. Let $\{X_i: i = 0, 1, 2, \cdots, m^2 - 1\}$ and $\{Y_j: j = 0, 1, 2, \cdots, n^2 - 1\}$ be orthonormal Hermitian operator bases for $L(H^a)$ and $L(H^b)$, respectively, with $X_0 = \mathbf{1}^a/\sqrt{m}$ and $Y_0 = \mathbf{1}^b/\sqrt{n}$, then a general bipartite state ρ can always be represented as

$$\rho = \frac{1}{\sqrt{mn}} \frac{\mathbf{1}^a}{\sqrt{m}} \otimes \frac{\mathbf{1}^b}{\sqrt{n}} + \sum_{i=1}^{m^2-1} x_i X_i \otimes \frac{\mathbf{1}^b}{\sqrt{n}} + \frac{\mathbf{1}^a}{\sqrt{m}} \otimes \sum_{j=1}^{n^2-1} y_j Y_j + \sum_{i=1}^{m^2-1}\sum_{j=1}^{n^2-1} t_{ij} X_i \otimes Y_j. \quad (3)$$

Theorem 2.—For ρ represented as Eq. (3), we have

$$N(\rho) \le \sum_{i=1}^{m^2-m} \lambda_i, \quad (4)$$

where $\{\lambda_i: i = 1, 2, \cdots, m^2 - 1\}$ are the eigenvalues of the matrix TT^t listed in *decreasing* order, and $T := (t_{ij})$ is an $(m^2 - 1) \times (n^2 - 1)$ dimensional matrix, the superscript t denotes transpose of matrices. Furthermore, if $\rho^a := \mathrm{tr}_b \rho$ is nondegenerate with spectral projections $\{|k\rangle\langle k|\}$, then

$$N(\rho) = \mathrm{tr}TT^t - \mathrm{tr}ATT^tA^t. \quad (5)$$

Here, $A := (a_{ki})$ is an $m \times (m^2 - 1)$ dimensional matrix with

$$a_{ki} := \mathrm{tr}|k\rangle\langle k|X_i, \qquad i = 1, 2, \cdots, m^2 - 1. \quad (6)$$

For any $2 \times n$ dimensional system, due to the special structure of Bloch representations for the marginal qubit states, we have the following closed formula, which indicates that inequality (4) is tight.

Theorem 3.—Following the notations in Theorem 2, if $m = 2$, then

$$N(\rho) = \begin{cases} \mathrm{tr}TT^t - \frac{1}{\|\mathbf{x}\|^2}\mathbf{x}^t TT^t \mathbf{x} & \text{if } \mathbf{x} \neq \mathbf{0}, \\ \mathrm{tr}TT^t - \lambda_3 & \text{if } \mathbf{x} = \mathbf{0}. \end{cases} \quad (7)$$

Here TT^t is a 3×3 dimensional matrix with λ_3 being its minimum eigenvalue, and $\|\mathbf{x}\|^2 := \sum_i x_i^2$ with $\mathbf{x} = (x_1, x_2, x_3)^t$.

Nonlocality is often related to quantumness and one usually speaks about quantum nonlocality, but there is nonlocality without quantumness [35], just as there is nonlocality without entanglement [36–39]. The measurement-induced nonlocality is a different notion from quantumness of correlations. Even for classically correlated states [16], there may exist nonlocal effects. The underlying reason for this is that when a classically correlated state is expressed in some particular base, it is classical only with respect to this base, when other bases are involved, it will not be classical and thus may naturally contain nonlocality. "Quantumness" and "nonlocality" are relative notions.

To summarize, we have obtained the exact analytical formulas of measurement-induced nonlocality for any pure states, $2 \times n$ dimensional states, as well as any states with nondegenerate marginals. We have further established a tight upper bound for general states, and have indicated the physical significance of measurement-induced nonlocality in certain quantum tasks such as quantum communication. The measurement-induced nonlocality can be practically estimated by quantum tomography and its experimental investigation in the related setup of Ref. [40] may be interesting. Finally, we propose the challenging problems of finding an analytical formula of $N(\rho)$ for an arbitrary state and providing an operational interpretation for measurement-induced nonlocality.

Appendix.—Proof of Theorem 1 is in the supplemental material [41]. Proof of Theorem 2. We first observe that

$$\rho^a = \mathrm{tr}_b \rho = \frac{1}{m}\mathbf{1}^a + \sqrt{n}\sum_{i=1}^{m^2-1} x_i X_i.$$

Now suppose that $\Pi^a = \{\Pi_k^a\}$ is a von Neumann measurement leaving ρ^a invariant, that is, $\sum_k \Pi_k^a \rho^a \Pi_k^a = \rho^a$, then $\rho^a = \sum_k (\mathrm{tr}\Pi_k^a \rho^a)\Pi_k^a$ is a spectral decomposition of ρ^a. Noting that

$$\rho - \Pi^a(\rho) = \sum_{i=1}^{m^2-1}\sum_{j=1}^{n^2-1} t_{ij}\Big(X_i - \sum_k \Pi_k^a X_i \Pi_k^a\Big) \otimes Y_j,$$

if we put $a_{ki} = \mathrm{tr}\Pi_k^a X_i$, then

$$\begin{aligned}\|\rho - \Pi^a(\rho)\|^2 &= \mathrm{tr}\sum_{ii'jj'} t_{ij}t_{i'j'}\Big(X_i - \sum_k \Pi_k^a X_i \Pi_k^a\Big) \\ &\quad \times \Big(X_{i'} - \sum_k \Pi_k^a X_{i'} \Pi_k^a\Big) \otimes Y_j Y_{j'} \\ &= \sum_{ii'jj'} t_{ij}t_{i'j'}\Big(\mathrm{tr}X_i X_{i'} - \sum_k a_{ki}a_{ki'}\Big) \times \delta_{jj'} \\ &= \sum_{ij} t_{ij}^2 - \sum_{ii'jk} t_{ij}t_{i'j}a_{ki}a_{ki'} = \mathrm{tr}TT^t - \mathrm{tr}ATT^tA^t.\end{aligned}$$

If ρ^a is nondegenerate, then the only von Neumann measurement leaving the marginal state ρ^a invariant is $\Pi^a = \{\Pi_k^a = |k\rangle\langle k|\}$, and we obtain the desired Eq. (5).

When ρ^a is degenerate, we need to consider the optimization problem

$$\max_A(\mathrm{tr}TT^t - \mathrm{tr}ATT^tA^t) = \mathrm{tr}TT^t - \min_A \mathrm{tr}ATT^tA^t$$

subject to the constraint that A is defined via Eq. (6) with $\{|k\rangle\langle k|\}$ being any von Neumann measurement leaving ρ^a invariant. From Eq. (6) and putting $a_{k0} := \mathrm{tr}|k\rangle\langle k|X_0 = 1/\sqrt{m}$, we know that $\{a_{ki}: i = 0, 1, \cdots, m^2-1\}$ are the (real) coefficients for expanding the operator $|k\rangle\langle k|$ in the operator orthonormal base $\{X_i: i = 0, 1, \cdots, m^2-1\}$, and thus

$$\sum_{i=0}^{m^2-1} a_{ki}a_{k'i} = \mathrm{tr}|k\rangle\langle k|k'\rangle\langle k'| = \delta_{kk'}, \qquad k, k' = 1, 2, \cdots, m.$$

Recall that $a_{k0} = 1/\sqrt{m}$ for $k = 1, 2, \cdots, m$, we obtain

$$\sum_{i=1}^{m^2-1} a_{ki}^2 = \frac{m-1}{m}, \qquad k = 1, 2, \cdots, m; \tag{8}$$

$$\sum_{i=1}^{m^2-1} a_{ki}a_{k'i} = -\frac{1}{m}, \qquad k \neq k' \tag{9}$$

Equations (8) and (9) can be compactly expressed as

$$AA^t = \frac{1}{m}\begin{pmatrix} m-1 & -1 & \cdots & -1 \\ -1 & m-1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \vdots \\ -1 & -1 & \cdots & m-1 \end{pmatrix}, \tag{10}$$

which is a real matrix with eigenvalues 0 and 1 (of multiplicity $m-1$), and can be diagonalized as $AA^t = UDU^t$ with U a *real* unitary matrix (thus $U^\dagger = U^t = U^{-1}$), and

$$D = \begin{pmatrix} \mathbf{1}_{m-1} & 0 \\ 0 & 0 \end{pmatrix}$$

is a matrix of order $m \times m$, where $\mathbf{1}_{m-1}$ denotes the unit matrix of order $m-1$.

Put

$$B := U^tA, \tag{11}$$

then the constraint Eq. (10) is equivalent to

$$BB^t = \begin{pmatrix} \mathbf{1}_{m-1} & 0 \\ 0 & 0 \end{pmatrix}. \tag{12}$$

Consequently, B can be written as

$$B = \begin{pmatrix} C \\ 0 \end{pmatrix} \tag{13}$$

and Eq. (12) is equivalent to $CC^t = \mathbf{1}_{m-1}$. Now

$$N(\rho) = \mathrm{tr}TT^t - \min_A \mathrm{tr}ATT^tA^t = \mathrm{tr}TT^t - \min_C \mathrm{tr}CTT^tC^t, \tag{14}$$

where the optimization is over C arising from Eqs. (6), (10), (11), and (13). In particular, C satisfies $CC^t = \mathbf{1}_{m-1}$. If we only consider this latter constraint, then

$$\min_{C:\ CC^t=\mathbf{1}_{m-1}} \mathrm{tr} CTT^tC^t = \sum_{i=m^2-m+1}^{m^2-1} \lambda_i.$$

Therefore, we conclude that

$$N(\rho) \leq \mathrm{tr}TT^t - \sum_{i=m^2-m+1}^{m^2-1} \lambda_i = \sum_{i=1}^{m^2-m} \lambda_i,$$

which is the desired inequality (4).

Proof of Theorem 3. The result follows from the proof of Theorem 2. First, noting that $\rho^a = \frac{1}{2}\mathbf{1}^a + \sqrt{n}\sum_{i=1}^3 x_i X_i$ is nondegenerate if and only if $\mathbf{x} \neq \mathbf{0}$, and in this instance, its eigenprojections are

$$\Pi_1^a = \frac{\mathbf{1}^a}{2} + \frac{\sum_{i=1}^3 x_i X_i}{\sqrt{2}||\mathbf{x}||}, \qquad \Pi_2^a = \frac{\mathbf{1}^a}{2} - \frac{\sum_{i=1}^3 x_i X_i}{\sqrt{2}||\mathbf{x}||}.$$

Therefore,

$$a_{1i} = \mathrm{tr}\Pi_1^a X_i = \frac{x_i}{\sqrt{2}||\mathbf{x}||} = -a_{2i}, \qquad i = 1, 2, 3.$$

Combining these with Eq. (5), we obtain the first equation in (7). If $\mathbf{x} = 0$, then ρ^a is degenerate, but since $a_{2i} = -a_{1i}$, and $\rho^a = \frac{1}{2}\mathbf{1}^a + \sqrt{n}\sum_{i=1}^3 x_i X_i$ is an operator representing a pure state *if and only if* $||\mathbf{x}||^2 = \frac{1}{2n}$ (this is not the case for higher dimensional Bloch representation [42,43]), the optimization problem (14) reduces to $\min_{C:\ CC^t=1} \mathrm{tr}CTT^tC^t = \lambda_3$, from which the second equation in (7) follows.

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