Anisotropy of Spatiotemporal Decorrelation in Electrohydrodynamic Turbulence

F. Carbone,[1,2] L. Sorriso-Valvo,[2] C. Versace,[1,2] G. Strangi,[1,2] and R. Bartolino[1,2]
[1]*Dipartimento di Fisica, Università della Calabria, Ponte Pietro Bucci 31C, I-87036 Rende (CS), Italy*
[2]*Liquid Crystal Laboratory, IPCF/CNR, Ponte Pietro Bucci 33B, I-87036 Rende (CS), Italy*
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Nonlinear straining and random sweeping spatiotemporal decorrelation properties, originally introduced as the main processes for turbulent fluctuations decorrelation in usual fluid flows, have been observed experimentally in anisotropic electroconvective turbulence generated in a nematic liquid crystal under the action of an external oscillating electric field. A transition between both processes occurs when the instability is driven toward states of increasing complexity, thus showing that decorrelation mechanisms in turbulent media are more universal than naively expected. A model for both decorrelation mechanisms is introduced, its comparison with experimental results providing an estimate of the characteristic sweeping velocity.

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A major aspect of the dynamics of turbulent flows is the rapid destruction of spatiotemporal field correlations [1–3]. Two physical mechanisms have been invoked to describe the phenomenon in turbulent fluids, the nonlinear straining of turbulent eddies, and the random sweeping generated by a large-scale random flow which transports small eddies past the observation point [3]. These effects have been investigating using numerical simulations of turbulence [4], while experiments have been mainly devoted to investigating decorrelation in time at a high Reynolds number [5]. In this Letter we will work out the conjecture that, in the presence of physical processes involving cross-scale dynamics, the decorrelation of a random field should be much more universal than naively expected. In particular, processes like straining and sweeping should represent a universal framework for a medium characterized by the superposition of "turbulent structures" on all dynamically interesting scales.

For a given space-time turbulent field $I(\mathbf{x}, t)$, information about its decorrelation properties is contained in the two-points, two-times autocorrelation function $R(\mathbf{r}, \tau) = \langle I(\mathbf{x}, t) I(\mathbf{x} + \mathbf{r}, t + \tau)\rangle$, computed at spatial and temporal lags $\mathbf{r}$ and τ, respectively (brackets indicating both spatial and temporal average). In the wave vector space this can be written as $R(\mathbf{k}, \tau) = S(\mathbf{k})\Gamma(\mathbf{k}, \tau)$, where $S(\mathbf{k})$ is the modal spectrum depending upon the wave vector $\mathbf{k}$ associated to the lag $\mathbf{r}$. The function Γ represents the dynamical decorrelation processes, by describing the time decay of information for each wave vector k [1,2]. One aspect of such an important study in real turbulence is the identification of a characteristic scaling time for $\Gamma(\mathbf{k}, \tau)$ which yields a universal form for the various time correlations at different wave vectors. In the Kolmogorov theory of isotropic fully developed turbulence [6], the hierarchy of fluctuations at all scales is generated by nonlinear straining, giving rise to an energy transfer toward smaller scales. The field correlation decay is then attributed to nonlinear interaction among triads of wave vectors [1,2]. Within this picture, the function Γ has the similarity form $\Gamma_{\rm nl}(\mathbf{k}, \tau) = \Gamma_{\rm nl}[\gamma_{\rm nl}(k)\tau]$, where $\gamma_{\rm nl}(k)$ is the scale dependent decorrelation rate of the field ($k = |\mathbf{k}|$), resulting in the exponential decay of the strain dominated decorrelation $\Gamma_{\rm nl} \sim \exp(-\gamma_{\rm nl}\tau)$, where $\gamma_{\rm nl} \sim k^{2/3}$ [4]. However, to our knowledge, such scaling has not been clearly observed in experimental data so far. When a large-scale random velocity field $\mathbf{u}$ is assumed to advectively transport fluctuations, field fluctuations at scale k decorrelate the field at frequency $\omega_0 = k|\mathbf{u}|$ [3]. When this random sweeping process dominates decorrelation, the dynamical function Γ depends on the velocity probability distribution $P(u/u_0)$, so that $\Gamma_{\rm sw} = \int d\omega e^{i\omega\tau} \int d(u/u_0) P(u/u_0) \delta(\omega - \omega_0)$ [2,7], where u is the velocity component along the lag $\mathbf{r}$ and u_0 is its characteristic rms value. At large scales, the sweeping velocity can be reasonably assumed to be a Gaussian random variable $P(u/u_0) = (\sqrt{2\pi})^{-1} \exp(-u^2/2u_0^2)$, from which $\Gamma_{\rm sw} = \exp[-(u_0\tau k)^2/2]$. In terms of the decorrelation rate, this result reads $\gamma_{\rm sw} \sim k$. This scaling has been observed in experimental and numerical flows [2,4,5], and is believed to be the main mechanism controlling decorrelation in fluid turbulence.

In this Letter we investigate the two points spatiotemporal correlation function and scaling behavior of decorrelation effects in electroconvective turbulence, occurring in liquid crystals undergoing different instability regimes [8,9]. We show that both decorrelation processes, originally introduced for turbulent flows, are at work and an anisotropic transition between the two scaling relations is observed, as the instability is driven toward increasing complexity.

Particularly intriguing chaotic dynamics and irregular field patterns have been observed in nematic liquid crystals (NLC) electrohydrodynamic (EHD) instability typical of systems far from equilibrium [10]. NLC uniaxial molecules are locally oriented along an average direction called molecular director $\mathbf{n}(\mathbf{x})$, with consequent anisotropy of all physical properties. To explore the decorrelation properties of EHD turbulence we use a sample cell of MBBA [N-(4-methoxybenzyliden)-4-butylanilin] NLC film. The sample, $d = 50\ \mu$m thick, is planarly aligned, its elongated molecules lying in the plane $\mathbf{x} = (x, y)$. The NLC is stimulated by an oscillating electric field $\mathbf{E}$, directed along the $\hat{\mathbf{z}}$ axis, with amplitude $E_0 = (V_0/d)$. The frequency is fixed to $f_0 = 70$ Hz, while we use different values of V_0 to investigate different dynamical regimes. The sample is illuminated by a white light beam polarized along y, and the instantaneous transmitted light intensity $I(\mathbf{x}, t)$ is measured. The data set consists of eight space-time series taken at different V_0, each one about 9 sec long, of $L_x = 461\ \mu$m and $L_y = 432\ \mu$m snapshots. The (square) pixel size is $\Delta r = 0.9\ \mu$m, while the images sampling time is $\Delta t = 1/120$ sec. Figure 1 shows snapshots of the intensity field at four values of the applied voltage V_0, evidencing the increasing degree of complexity of the spatial patterns as V_0 increases. Each time series describes the space-time variation of the refractive index, which is in turn related to the average local distortion of the molecular director field $\mathbf{n}(\mathbf{x})$.

At the threshold voltage $V_{\rm th} = 7.5$ V a stationary pattern of convective rolls, called Williams domains (WD), with axis in the x direction, is observed (panel A of Fig. 1). The occurrence of WD is due to molecular reorientation resulting from the competition of a restoring dielectric torque, owing to the negative dielectric anisotropy of the NLC, and a force acting on the bulk fluid, due to the charge separation produced by the positive conductivity anisotropy [8,9]. By increasing V_0, the convective structures are stretched and broken, leading to formation of smaller and smaller structures [8] in an increasingly complex turbulent dynamics.

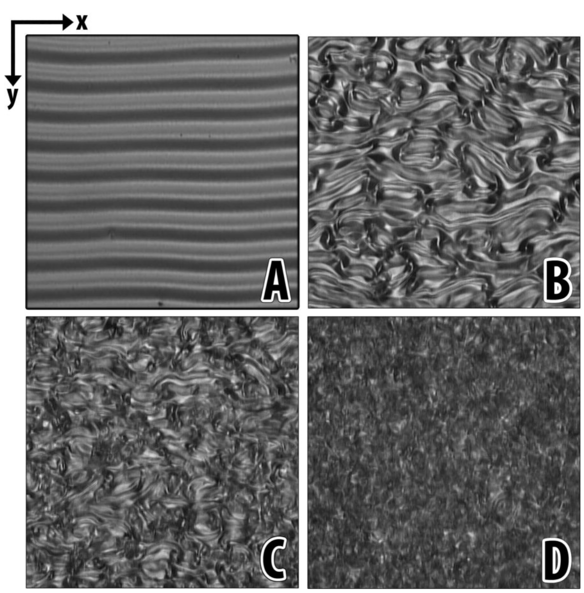

FIG. 1. Four snapshots of the intensity field $I(\mathbf{x}, t^*)$ of EHD turbulence taken at a fixed time $t^* = 1$ sec, for different values of ϵ: (a) $\epsilon = 1$, (b) $\epsilon = 4.67$, (c) $\epsilon = 6$, (d) $\epsilon = 6.67$.

The scaling behavior observed for EHD [11–13] has been attributed to the existence of a stochastic fragmentation process [13]. This is phenomenologically similar to the energy cascade responsible for the generation of small scale structures within fluid turbulence. The regimes of instability observed under different experimental conditions (e.g., for different aspect ratio of the cell) can be compared by using the reduced voltage $\epsilon = V_0/V_{\rm th}$. However, investigations of the chaotic dynamics of the system [14], show that the transition to chaos is not universal. This is reflected, for example, in the role of the parameter ϵ with respect to the Reynolds number. In fact, when ϵ is larger than unity the system is driven out of equilibrium, thus displaying increasing complexity as it increases. However, even a slight value $\epsilon > 1$ is enough to trigger a state of weak turbulence. Owing to the above mentioned statistical similarities, a nonlinear straining phenomenology could describe the decay of correlation in EHD. On the other hand, a sweeping effect could also be present in EHD, due to the persistent random motion of large-scale structures, observed up to high voltages as subleading contribution. A remarkable difference with respect to fluid dynamics is the intrinsic anisotropy of EHD turbulence. This arises both because NLCs are anisotropic fluids even at large ϵ, and because of the large-scale oriented Williams structure. It is thus necessary to study the field properties separately in two directions, parallel $r_\parallel$ and perpendicular $r_\perp$ to the Williams domain orientation, correspondingly, $\mathbf{k} = (k_\parallel, k_\perp)$. In strongly anisotropic flows, as, for example, in magnetohydrodynamics with large-scale mean magnetic field, turbulence is usually confined to planes perpendicular to the anisotropy axis [7]. In these conditions, the Kolmogorov decorrelation effect is expected to act perpendicularly with respect to the anisotropy direction, leading to $\gamma_{\rm nl} \sim k_\perp^{2/3}$.

From each data set we have calculated the two-point, two-times correlation function $R(\mathbf{r}, \tau)$, thus obtaining $\Gamma(\mathbf{k}, \tau)$ for different wave vectors $\mathbf{k}$ and different values of ϵ. These are shown in Fig. 2 as a function of the time lag τ, for five wave vectors $k_\parallel = n(2\pi/L_x)$, in the range $1 \le n \le 50$. From the decay of Γ, it is evident that at small wave vectors the field is coherent for long times, while at large wave vectors decorrelation is faster. Similar results were found in the perpendicular direction $k_\perp = n(2\pi/L_y)$ (not shown). To investigate the dependence of γ on the wave vector $\mathbf{k}$, decorrelation rates have been estimated from data as $1/\gamma(k_\alpha) = \int_0^\infty \Gamma(k_\alpha, \tau) d\tau$ [1], and have been fitted with the power-law $\gamma(k_\alpha) \sim k_\alpha^\mu$, where α indicates the parallel or perpendicular component and μ is a free parameter. In Fig. 3 we show the scaling behavior of both $\gamma(k_\parallel)$ and $\gamma(k_\perp)$, for three different values of ϵ. A power-law scaling range is found up to wave vectors of the order of $k \simeq 2\ \mu$m^{-1}. The scaling exponents μ, computed from the fit of the decorrelation rates, are plotted in Fig. 4, as a function of ϵ. In the perpendicular direction the

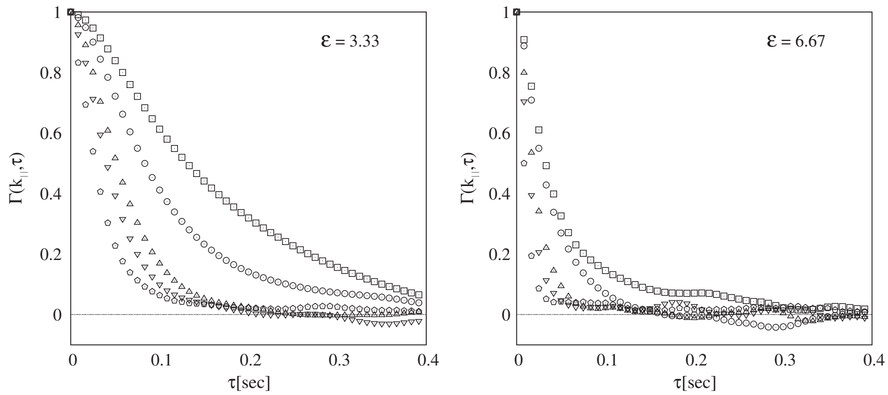

FIG. 2. The function $\Gamma(k_\parallel, \tau)$ for two different values of ϵ. Symbols refer to different wave vectors $k_\parallel = (2\pi/L_x)n$, namely: $n = 1$ (squares), $n = 10$ (circles), $n = 20$ (up triangles), $n = 30$ (down triangles), $n = 50$ (diamonds).

measured scaling exponent is compatible with the value $\mu = 2/3$ at any voltage. This represents experimental evidence of Kolmogorov-like scaling, suggesting that decorrelation is dominated by a fragmentation mechanism, similar to that generating the nonlinear energy cascade in turbulence. On the other hand, the scaling exponent for parallel decorrelation increases from the Kolmogorov value, towards values more compatible with a Kraichnan exponent $\mu \simeq 1$, the kink happens at about $\epsilon_c \simeq 3.5$, thus suggesting that a process similar to random sweeping is at work in the parallel direction.

It is remarkable that a smooth transition between the two theoretical values is observed in the parallel direction. In fact, a random motion of fragmented large-scale WD structures is expected in the transverse direction. We can conjecture that, in the parallel direction, the random sweeping is responsible for the decorrelation in the system thus obtaining

$$\Gamma(k_\parallel, \tau) = e^{-(u_0 \tau k_\parallel)^2/2}. \quad (1)$$

The large-scale rms velocity u_0 represents the level of fluctuations in the EHD convective process, therefore determining the intensity of the sweeping effect. Since velocity measurements are not available for the sample under study, the fit of the experimental curves with relation (1) provides an estimate of the characteristic u_0 as a function of ϵ. Results of the fit are shown in Fig. 5. The characteristic velocity responsible for the sweeping increases linearly with the voltage. A change of slope is observed around $\epsilon_c \simeq 3.5$, separating a low voltage regimes, where decorrelation is described by $\mu \simeq 2/3$, from the region where the action of the sweeping effect is much more evident. The kink indicates a change of regime due to increasing amplitude of the velocity fluctuations which amplifies the sweeping effect, in agreement with the behavior of μ (Fig. 4). Direct measurements of velocity in an independent similar system have been reported in the past [15,16]. The same double linear increase of velocity was observed (but not explained), with a kink located at about $\epsilon \simeq 3.5$ [15], in good agreement with our results. This confirms that the values of u_0 obtained here are a reliable proxy for the typical velocity fluctuations of the convective flow. Our results allow us to interpret the change of slope of velocity, observed in EHD, as due to a kind of threshold setup where the sweeping effect becomes important in the decorrelation processes.

In conclusion, we have investigated the spatiotemporal decorrelation processes in EHD turbulence. The scaling

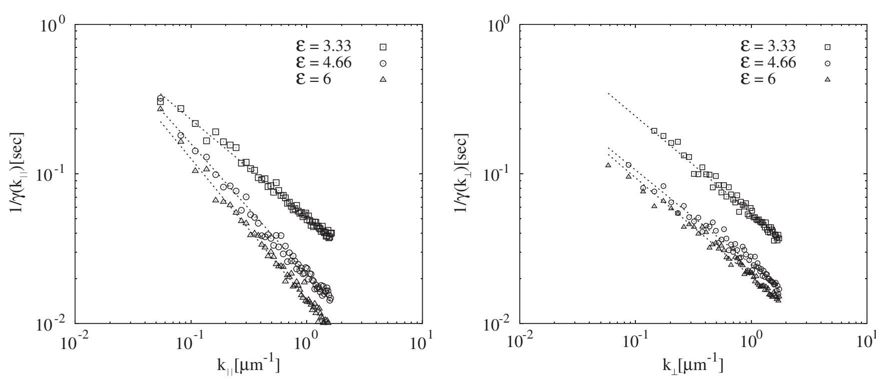

FIG. 3. Scaling behavior of the decorrelation rates $1/\gamma(k_\parallel)$ (top panel) and $1/\gamma(k_\perp)$ (bottom panel), for three different values of the reduced voltage ϵ. Dashed lines represent power-law fits.

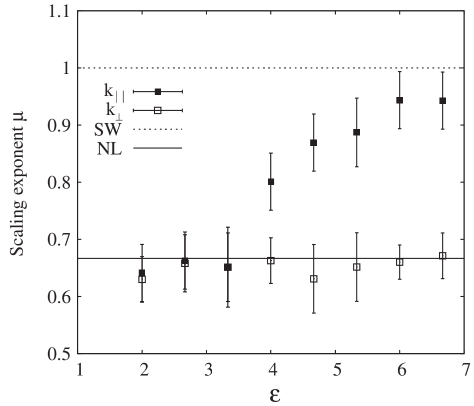

FIG. 4. The scaling exponents μ versus the reduced voltage ϵ. Symbols refer to $k_\parallel$ (black squares) and $k_\perp$ (white square). Full line represents the model value $\mu = 2/3$, predicted for straining decorrelation (NL), while the dashed line represents the model value $\mu = 1$, typical of sweeping decorrelation process (SW).

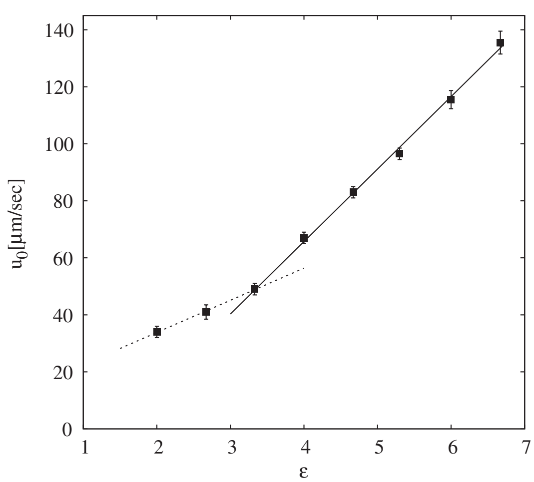

FIG. 5. Large-scale characteristic velocity u_0 as a function of the reduced voltages ϵ. The straight lines indicate linear fit. The kink location is evaluated at the intersection of the fitting straight lines, for $\epsilon \simeq 3.5$.

law $\gamma \sim k_\perp^{2/3}$ was found for the decorrelation rate in the perpendicular direction, suggesting a straining process resulting from fragmentation of large-scale structures. In the direction parallel to the large-scale structures a transition occurs at $\epsilon = \epsilon_c \simeq 3.5$. At lower voltages we recover the scaling law $\gamma \sim k_\parallel^{2/3}$, while at $\epsilon > \epsilon_c$ an increasingly relevant role of the sweeping effect is recorded, and the scaling exponent approaches $\mu \simeq 1$ at higher voltages. Our conjecture on the role of decorrelation process provides an estimate of the characteristic random velocity u_0, responsible for the sweeping. The velocity linearly increases with V_0, with a slope change at ϵ_c, in good agreement with independent direct measurements of the characteristic speed previously found in similar systems. The anisotropic crossover can be related to the transition from a weaker to a stronger turbulent regime in EHD. Finally, our results show that processes as the decorrelation properties of fluctuations, usually related to turbulent fluid flows, are universal. We expect that other physical systems can share the same properties.

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