Efficient Decoherence-Free Entanglement Distribution over Lossy Quantum Channels

Rikizo Ikuta, Yohei Ono, Toshiyuki Tashima, Takashi Yamamoto, Masato Koashi, and Nobuyuki Imoto
Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka 560-8531, Japan
(Received 27 December 2010; published 14 March 2011)

We propose and demonstrate a scheme for boosting the efficiency of entanglement distribution based on a decoherence-free subspace over lossy quantum channels. By using backward propagation of a coherent light, our scheme achieves an entanglement-sharing rate that is proportional to the transmittance T of the quantum channel in spite of encoding qubits in multipartite systems for the decoherence-free subspace. We experimentally show that highly entangled states, which can violate the Clauser-Horne-Shimony-Holt inequality, are distributed at a rate proportional to T.

DOI: 10.1103/PhysRevLett.106.110503 PACS numbers: 03.67.Hk, 03.67.Pp, 42.50.Ex

The distribution of photonic entangled states among remote parties is an important issue in order to realize quantum information processing, such as quantum key distribution [1–3], quantum teleportation [4], and quantum computation [5]. In practice, however, the quantum states are disturbed by fluctuations during the transmission. One of the possible schemes to overcome this problem is to encode the quantum states into a decoherence-free subspace (DFS) in multipartite systems. In photonic systems, several proposals and experimental demonstrations have been done to show the robustness of quantum states in a DFS against collective fluctuations [6–12]. Furthermore, the capability of faithful quantum-state transmission and entanglement distribution through an optical fiber has been demonstrated [13–15].

A serious drawback of all photonic DFS schemes is that the photon losses in the quantum channel severely limit the transmission rate of quantum states, since all the photons forming the DFS must reach the receiver. When the quantum channel delivers a photon to the receiver with transmittance T, one can transmit a quantum state of interest only with a rate proportional to T^n using an n-photon system with previous DFS schemes [6–15]. For the realization of robust long-distance quantum communication systems, it is thus desirable to improve the channel-transmission dependence of DFS schemes. In this Letter, we propose and experimentally demonstrate a two-photon DFS scheme for sharing entangled photon pairs, which boosts the efficiency to be proportional to T from T^2 of the previous protocols in Refs. [13–15].

We first introduce our DFS scheme against collective phase fluctuations, as shown in Fig. 1. At step (a), the sender Alice generates a maximally entangled photon pair A and B in the state $|\phi^+\rangle_{AB} \equiv (|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B)/\sqrt{2}$ and transmits photon B to Bob, where $|H\rangle$ and $|V\rangle$ represent horizontal (H) and vertical (V) polarization states of a photon, respectively. Meanwhile, the receiver Bob prepares an ancillary photon R in the state $|D\rangle_R \equiv (|H\rangle_R + |V\rangle_R)/\sqrt{2}$ and sends photon R to Alice. After transmission of the photons, the states are transformed to $e^{i\phi_H}|HH\rangle_{AB} + e^{i\phi_V}|VV\rangle_{AB}$ and $e^{i\phi'_H}|H\rangle_R + e^{i\phi'_V}|V\rangle_R$ by the phase fluctuations in the channel, where $\phi_{H(V)}$ and $\phi'_{H(V)}$ represent phase shifts to the H (V) components of photon B and R in the channel, respectively. Assuming that the difference between the phase shifts $\phi_{H(V)}$ and $\phi'_{H(V)}$ is negligibly small, the state of the three photons at the end of step (a) becomes

$$|\phi^+\rangle_{AB}|D\rangle_R \to \tfrac{1}{2}[e^{i(\phi_H+\phi_V)}(|H\rangle_B|HV\rangle_{AR} + |V\rangle_B|VH\rangle_{AR}) + e^{2i\phi_H}|H\rangle_B|HH\rangle_{AR} + e^{2i\phi_V}|V\rangle_B|VV\rangle_{AR}]. \quad (1)$$

In this scheme, the pair of photons B and R that went through the collective noises end up being split between Alice and Bob. Nonetheless, Eq. (1) can be interpreted as if the photons A and R, both of which are possessed by Alice, had gone through the collective noises. This comes from an important property of an entangled photon pair that a disturbance on one half of the photon pair is equivalent to a similar disturbance on the other half of the photon pair [16,17]. We find that the first two terms in Eq. (1) are invariant under phase fluctuations. At step (b), by performing quantum parity checking on photons A and R [18], Alice extracts the state in the DFS spanned by

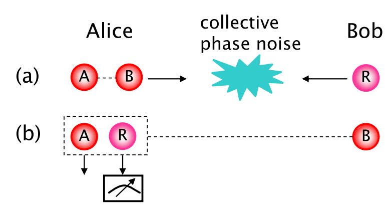

FIG. 1 (color online). The concept of our DFS scheme. At step (a), Alice prepares a maximally entangled photon pair A and B and sends photon B to Bob's side. On the other hand, Bob sends an ancillary photon R to Alice's side. At step (b), Alice extracts the DFS by quantum parity checking on photons A and R and decodes back the initial entangled state from the DFS. For boosting the efficiency, we use a coherent light instead of a single photon R.

$\{|HV\rangle_{AR}, |VH\rangle_{AR}\}$ from the state in Eq. (1). Then, the decoding back of the state into $|\phi^+\rangle_{AB}$ is done by a projective measurement $\{|D\rangle\langle D|, |\bar{D}\rangle\langle\bar{D}|\}$ on photon R and a feedforward operation on photon A, where $|\bar{D}\rangle \equiv (|H\rangle - |V\rangle)/\sqrt{2}$. When the transmittance of the quantum channel is T, the efficiency of this scheme is proportional to T^2 because both photons B and R must pass through the channel.

Our strategy for enhancing the efficiency from $\mathcal{O}(T^2)$ to $\mathcal{O}(T)$ is to replace the single-photon state in mode R by a coherent state. Suppose that the average photon number in mode R when received by Alice is μ; namely, Bob initially prepares a coherent state of mean photon number $\mu_B \equiv \mu T^{-1}$. With a probability of $\mathcal{O}(\mu T)$, Alice finds exactly one photon in mode R, and Bob also receives the photon B from Alice. The protocol then works exactly the same as was described before, leading to shared state $|\phi^+\rangle_{AB}$. On the other hand, the use of the coherent state also produces unwanted events where two or more photons arrive at Alice, and a usual setup for quantum parity checking with linear optics and imperfect photon detectors cannot fully discriminate such events from the desired ones. Since these unwanted events occur with probability $\mathcal{O}(\mu^2 T)$, the condition $\mu \ll 1$ is needed to have a good fidelity of the final state. This condition is independent of T, which means that, given a target value of the fidelity, we may use a constant value of μ (and hence μ_B proportional to T^{-1}) to reach the target for any value of T. This scheme thus gives a rate proportional to T instead of T^2 in the previous two-photon DFS schemes.

In our scheme, the counterpropagations of photons B and R are essential. If Bob prepares all the pulses A, B, and R and sends A and R to Alice, the desired events occur with the same probability of $\mathcal{O}(\mu T)$, but the unwanted events occur with a larger probability of $\mathcal{O}(\mu^2)$, which makes the requirement on μ too stringent. Although the counterpropagation setup requires the phase fluctuations to be much slower than the propagation time, such a requirement has been experimentally shown to be met up to ~100 km in fiber-based quantum cryptography systems [19].

The detail of our experimental setup is shown in Fig. 2. We use a mode-locked Ti:sapphire (Ti:S) laser (wavelength: 790 nm; pulse width: 90 fs; repetition rate:

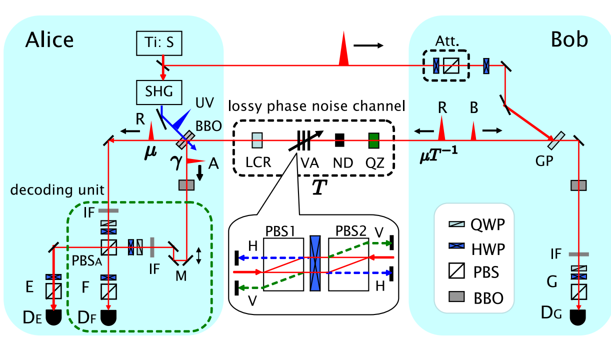

FIG. 2 (color online). Our experimental setup.

82 MHz) as a light source, which is divided into two beams. One beam is frequency doubled (wavelength: 395 nm; power: 75 mW) by second harmonic generation (SHG) and then pumps a pair of type I phase-matched 1.5-mm-thick β-barium borate (BBO) crystals to prepare the entangled photon pair A and B through spontaneous parametric down-conversion (SPDC). The difference between the group velocities of H- and V-polarized photons is compensated by BBO crystals in each path of photon A and B. Photon A goes to Alice's decoding unit, while photon B enters a lossy phase noise channel and goes to the detector D_G after passing through a glass plate (GP) (reflectance ~5%). The other beam from the laser is used to prepare a coherent light pulse R at Bob's side. After adjusting the intensity of the coherent light pulse by an attenuator (Att.) composed of a half-wave plate (HWP) and a polarization beam splitter (PBS), we set its polarization to D by rotating a HWP. The coherent light pulse R is then reflected toward Alice's side by the GP and enters the lossy phase noise channel. After that, it goes to Alice's decoding unit.

To see the T dependence of the rate, we use the lossy phase noise channel composed of a liquid crystal retarder (LCR), a polarization-independent variable attenuator (VA), a neutral-density filter (ND) of transmittance 0.1, and a quartz plate (QZ). The LCR provides a phase shift between $|H\rangle$ and $|V\rangle$ according to the applied voltage. For simulating the collective random phase fluctuations, we slowly switched among eight values of phase shifts, $n\pi/4$ $(n = 0, \ldots, 7)$, such that pulses A and R undergo the same fluctuations. This simulates the cases where the phase fluctuations are much slower than the propagation time of pulses A and R in fiber-optic communication. The VA is composed of a HWP sandwiched with two calcite PBSs, which deflect V-polarized photons, as shown in a subfigure inserted in Fig. 2. By rotating the HWP, we can vary transmission T continuously for both polarizations. The QZ compensates an additional group delay introduced by PBS1 and PBS2. In addition to the variable loss, the VA also swaps the H- and V-polarization components of light. This effect would be removed by inserting another HWP, but in our experiment, we cancel it by a proper relabeling of polarizations in Bob's apparatus.

Alice's detection unit carries out quantum parity checking for extracting the DFS and decoding for recovering the entangled state [18], when each of modes A and R has a single photon. After receiving the pulse R, Alice inverts its polarization by a HWP before PBS_A. Adjusting a temporal delay by mirrors (M) on a motorized stage, Alice mixes the pulses A and R at PBS_A and postselects later the cases where there is at least one photon in each mode E and F. This operation is the quantum parity checking, which discards the cases where the input state of photons A and R was $|HH\rangle_{AR}$ or $|VV\rangle_{AR}$. In mode F, Alice selects the cases where the photon is projected onto $|D\rangle_F$ by the detector D_F with a HWP and a PBS. The final state of the shared photon pair E and G, which should be $|\phi^+\rangle_{EG}$

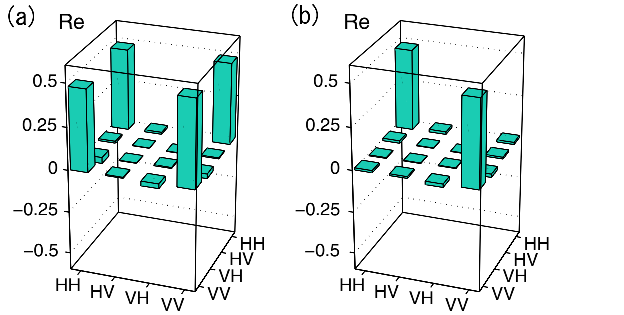

FIG. 3 (color online). The real parts of (a) ρ_{AB} and (b) ρ'_{AB}.

TABLE I. The observed visibilities (V_Z and V_X) and a lower bound $F_{\mathrm{low}} = (V_Z + V_X)/2$ on the shared state for channel transmittance T.

T	V_Z	V_X	F_{low}
0.1	0.88 ± 0.02	0.82 ± 0.03	0.85 ± 0.02
0.03	0.91 ± 0.02	0.79 ± 0.03	0.85 ± 0.02
0.01	0.88 ± 0.02	0.77 ± 0.03	0.82 ± 0.02
0.005	0.82 ± 0.03	0.72 ± 0.04	0.77 ± 0.02
0.003	0.74 ± 0.03	0.66 ± 0.04	0.70 ± 0.03

ideally, is then analyzed by projecting the photons E and G to various polarizations H, V, D, and $\bar{D}$. The collected data are thus composed of the rates of triple coincidence events among D_E, D_F, and D_G for different rotation angles of HWPs in front of D_E and D_G. The spectral filtering of the photons for all detectors is performed by narrow-band interference filters (IF) (wavelength: 790 nm; bandwidth: 2.7 nm). All the detectors D_E, D_F, and D_G are silicon avalanche photodiodes which receive photons through single-mode optical fibers.

In our experiments, we use SPDC with a photon pair generation rate γ as the entangled photon source. In this case, an additional condition between μ and γ is required to reduce false triple coincidences caused by the multiple photon pair generation from SPDC, whose probability is $\mathcal{O}(\gamma^2 T)$. Since the true coincidences occur at probability $\mathcal{O}(\mu\gamma T)$, the condition $\gamma \ll \mu$ is required. Therefore, to achieve a high fidelity, we need to satisfy $\gamma \ll \mu \ll 1$. In the following experiments, we set $\gamma \approx 3.0 \times 10^{-3}$ and $\mu \approx 1.1 \times 10^{-1}$.

As preliminary experiments, we characterized the two-photon state from SPDC by recording the coincidence events between D_E and D_G without sending the photons in mode R. We chose $T = 0.1$ and performed quantum-state tomography by rotating the quarter-wave plate (QWP) and the HWP before PBS_A at Alice's side and rotating the QWP and the HWP in mode G at Bob's side [20]. Without the phase fluctuations, the density operator ρ_{AB} of the two-photon state is reconstructed as in Fig. 3(a). The iterative maximum likelihood method was used for the reconstruction [21,22]. The observed fidelity of ρ_{AB} to the maximally entangled state $|\phi^+\rangle_{AB}$ was 0.98 ± 0.01, which implies that the photon pair prepared by Alice was in a highly entangled state. Figure 3(b) shows the state ρ'_{AB} with the phase fluctuations. We see that the off-diagonal elements vanished as expected, indicating that the phase noises by the LCR effectively simulated the random phase noise channel. The observed fidelity of ρ'_{AB} was 0.51 ± 0.01.

We then performed our DFS scheme. The quality of the shared entangled state was evaluated by determining two visibilities $V_Z \equiv \langle Z_E Z_G\rangle$ and $V_X \equiv \langle X_E X_G\rangle$ from the observed coincidence rates, where $Z \equiv |H\rangle\langle H| - |V\rangle\langle V|$ and $X \equiv |D\rangle\langle D| - |\bar{D}\rangle\langle\bar{D}|$. A lower bound F_{low} of the fidelity is then given by $F_{\mathrm{low}} = (V_Z + V_X)/2$ [23]. $F_{\mathrm{low}} > 1/\sqrt{2} \sim 0.707$ implies that the observed two photons are strongly entangled and can violate the Clauser-Horne-Shimony-Holt inequality. The experimental value of F_{low} at $T = 0.1$ was 0.85 ± 0.02, which shows our DFS scheme well protects the quantum correlations against phase fluctuations. Next, we demonstrated our DFS scheme for various values of T ranging from 0.1 to 0.003. We chose the intensity of the coherent light pulse R at Bob's side to be proportional to T^{-1}, such that μ would be a constant. Table I shows the results of observed visibilities and the derived values of F_{low}. Visibility V_Z is generally better than V_X since only the latter is affected by mode mismatch between pulses A and R at PBS_A. We plot the relationship between T and F_{low} in Fig. 4(a), which implies that the shared states between Alice and Bob were highly entangled for $T \geq 0.005$. The sharing rate of output states at each T is shown in Fig. 4(b). We clearly see that the sharing rate is proportional to T. A broken line in Fig. 4(b), which is proportional to T^2, is the rate expected when Bob uses an ideal single photon for mode R. By comparison, we see that our scheme is favorable for smaller values of T as long as the observed values of F_{low} are acceptable.

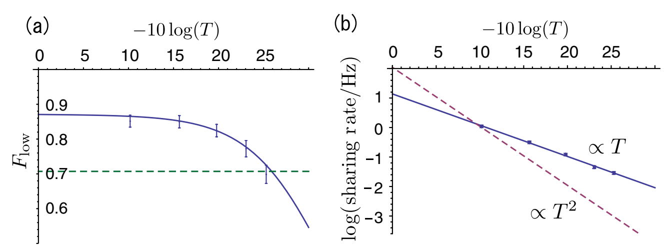

FIG. 4 (color online). (a) The dependence of F_{low} on the transmittance T in decibels. Dots with error bars are the derived values of F_{low} from V_X and V_Z in Table I. The solid curve is obtained by theoretical calculation with $V_{\mathrm{sp}} \approx 0.90$ and experimental parameters. The broken line indicates the lower bound of the fidelity to a maximally entangled state to see the violation of the Clauser-Horne-Shimony-Holt inequality. (b) The experimental sharing rate of output states. The slope of the solid line fitted to the experimental data is 1.06 ± 0.04, which clearly shows that the sharing rate is proportional to T. The broken line describes where an ideal single photon is used for mode R instead of the coherent light pulse, whose rate is proportional to T^2. The two lines are expected to intersect at $T = \mu$.

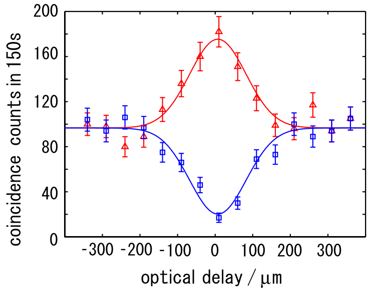

FIG. 5 (color online). The observed quantum interference by mixing photon A in the state $|R\rangle_A$ and the coherent light pulse R with D polarization, where $|R\rangle \equiv |H\rangle + i|V\rangle$. The triangles and squares show the coincidence counts measured on the bases $|R\rangle_E|D\rangle_F$ and $|L\rangle_E|D\rangle_F$, respectively. Here $|L\rangle \equiv |H\rangle - i|V\rangle$. The visibility at the zero delay is 0.83 ± 0.04.

In order to see the reason of the degradation of $F_{\rm low}$ for small T, we constructed a simple theoretical model which regards each pulse as a single mode but takes into account multiphoton emission events and the mode matching $V_{\rm sp}$ between modes A and R. We used the following experimental parameters in the model: $\gamma \approx 3.0 \times 10^{-3}$, $\mu\eta \approx 1.4 \times 10^{-2}$, $\eta \approx 0.13$, $\eta_G \approx 0.09$, and $d \approx 1.5 \times 10^{-6}$. Here, η is the quantum efficiency of D_E and D_F, η_G is the quantum efficiency of D_G, and d is the dark-count rate of D_G. The value of $V_{\rm sp}$ was then determined to be 0.90 by requiring that the model should correctly predict the observed value of $V_X = 0.82$ at $T = 0.1$. With no other adjustable parameters, the theory predicts the solid curve in Fig. 4(a), which is in good agreement with the observed values. In the theoretical model, the degradation of $F_{\rm low}$ for small T is mainly caused by the relative increase of the contribution from the dark counts of Bob's detector D_G. Hence, the degradation of the fidelity will be avoided by using low dark-count detectors, such as the superconducting single-photon detectors used in quantum key distribution experiments [24].

We note that the interference occurring at PBS_A is robust against timing mismatch between photon A and the coherent light pulse R. Figure 5 shows observed quantum interference as a function of the optical delay introduced by moving the mirrors M in Fig. 2. The FWHM is calculated as ~ 180 μm. This value is over 200 times larger than the photon wavelength 790 nm, which implies that wavelength-order precision of the control is not required in our scheme. While we have derived the coherent light pulse of Bob from the pump laser sitting on Alice's side for simplicity of our experiment, the robustness against timing fluctuations suggests that the coherent light pulse can be independently prepared by Bob. Such two-photon interference experiments using independently prepared pump lasers have been demonstrated in Refs. [25,26].

We have proposed and demonstrated an efficient decoherence-free entanglement-sharing scheme with a rate proportional to the transmittance of the quantum channel. In our scheme, the property of an entangled photon pair, that a phase disturbance on one half can be cancelled at the other side, enables us to use counterpropagations of the two photons. This permits us to use a coherent light pulse with the prepared intensity inversely proportional to the transmittance of the channel as an ancillary system, which leads to boosting up of the efficiency of entanglement distribution. Because the phase-cancellation property holds true for any state of the form $\alpha|HH\rangle + \beta|VV\rangle$, our DFS scheme is applicable to distribution of any unknown single qubit $\alpha|H\rangle + \beta|V\rangle$ by encoding it into $\alpha|HH\rangle + \beta|VV\rangle$ using quantum parity checking [18]. We believe that the proposed scheme is useful for realizing stable long-distance quantum communication [27].

We thank Tsuyoshi Kitano and Şahin K. Özdemir for helpful discussions. This work was supported by the Funding Program for World-Leading Innovative R&D on Science and Technology (FIRST), MEXT Grant-in-Aid for Scientific Research on Innovative Areas 20104003 and 21102008, JSPS Grant-in-Aid for Scientific Research (C) 20540389, and the MEXT Global COE Program.

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