

Entanglement between the Future and the Past in the Quantum Vacuum

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We note that massless fields within the future and past light cone may be quantized as independent systems. The vacuum is shown to be a nonseparable state of these systems, exactly mirroring the known entanglement between the spacelike separated Rindler wedges. This leads to a notion of timelike entanglement. We describe an inertial detector which exhibits a thermal response to the vacuum when switched on at $t = 0$, due to this property. The feasibility of detecting this effect is discussed, with natural experimental parameters appearing at the scale of 100 GHz.

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A basic and far-reaching property of the quantum vacuum is that it is an entangled state—a fact underlying an impressive number of theoretical insights and predictions [1]. In the case of flat Minkowski space-time, this is typically shown in the context of the Unruh effect [2–4]. There, the vacuum state of the field can be written as an entangled state between two sets of modes, respectively, spanning two space-time wedges, known as the Rindler wedges (see Fig. 1). A uniformly accelerated observer sees only one set of Rindler modes. The tracing out of the unobserved modes leads to the prediction that such an accelerated observer sees a thermalized vacuum.

Having been predicted over 30 years ago, the Unruh effect remains unobserved. Its validity, though widely accepted, is sometimes debated on theoretical grounds [5–7]. The small scale of the effect motivates a search for related phenomena that can be tested experimentally.

Here, our main result is to demonstrate that the same entanglement exists between massless fields within the future and past light cone (F and P) as between the left and right Rindler wedges (L and R), and that the Unruh effect can be mapped onto an equivalent thermal effect for an inertial observer interacting with the field only in the future or the past. We will show the explicit form of this timelike entanglement for a massless scalar field in $2-d$ space-time, and the detector effect in $4-d$ space-time. Dimensional analysis suggests that observation of this effect may be within range of current technology.

This Letter is organized as follows: We first note that massless fields in F and P may be quantized as independent systems, and then describe our coordinatization of space-time, and the mode functions living in each quadrant. We then express the state of the Minkowski vacuum restricted to F and P in terms of these modes, and note entanglement. An Unruh-DeWitt detector is then described, which shows a thermal response to these modes in F (or P). The feasibility of an experimental observation of this effect is discussed. We then offer some conclusions.

Future-past as independent systems.—The concept of entanglement between the left and right Rindler wedges

rests on the fact that the fields within may be quantized as independent systems. This is expressed through the vanishing of the Pauli-Jordan function, $i\Delta(x - y) = [\hat{\phi}(x), \hat{\phi}^\dagger(y)]$ for spacelike intervals. This general feature holds for both massive and massless fields.

In the case of massless fields, however, the Pauli-Jordan function $\Delta(x - y)$ vanishes for all but lightlike intervals, $(x - y)^2 = 0$ [8]. In particular, it vanishes for timelike intervals. This will allow us to regard the fields in F and P as independent systems.

In what follows, we assume a massless, noninteracting field for which $\hat{\phi}(x_F)$ and $\hat{\phi}(x_P)$ commute. It is important to note that the concept of independent systems also remains valid as an approximation when the commutator is small but nonvanishing, as in the case of an arbitrarily small but nonvanishing mass. This ensures that the concept of timelike entanglement we develop here remains stable under small deviations from the ideal case.

Coordinates.—We now break space-time into quadrants F , P , R , L , and introduce coordinates for each. Each of these coordinate systems will be used to define a set of field modes, complete in each region. We emphasize that these modes are not all independent from one another; the modes

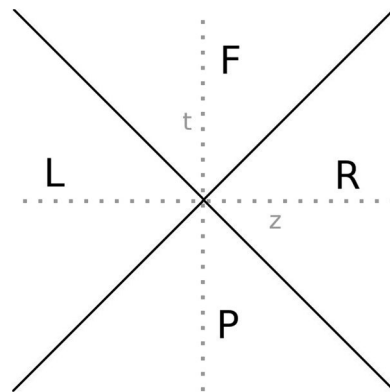


FIG. 1. Space-time divided into quadrants consisting of regions contained by the future and past light cones (F and P), and the right and left Rindler wedges (R and L).

in F are independent from modes in P , but they are not independent of the modes in R and L .

In F , we adopt the coordinates η and ζ :

$$t = a^{-1} e^{a\eta} \cosh(a\zeta), \quad (1)$$

$$z = a^{-1} e^{a\eta} \sinh(a\zeta). \quad (2)$$

In P , we will use the coordinates $\bar{\eta}$ and $\bar{\zeta}$:

$$t = -a^{-1} e^{a\bar{\eta}} \cosh(a\bar{\zeta}), \quad (3)$$

$$z = -a^{-1} e^{a\bar{\eta}} \sinh(a\bar{\zeta}). \quad (4)$$

These are to be compared with the usual Rindler coordinates τ and ϵ , in R :

$$t = a^{-1} e^{a\epsilon} \sinh(a\tau), \quad (5)$$

$$z = a^{-1} e^{a\epsilon} \cosh(a\tau). \quad (6)$$

As well as $\bar{\tau}$ and $\bar{\epsilon}$, in L :

$$t = -a^{-1} e^{a\bar{\epsilon}} \sinh(a\bar{\tau}), \quad (7)$$

$$z = -a^{-1} e^{a\bar{\epsilon}} \cosh(a\bar{\tau}). \quad (8)$$

In each of these coordinate systems the metric is conformally Minkowski, and owing to the conformal invariance of the massless wave equation in two dimensions, the same equation holds separately in the four coordinate systems (see Eq. 2.46 of [4]), namely,

$$\left(\frac{\partial^2}{\partial \eta^2} - \frac{\partial^2}{\partial \zeta^2}\right)_F \phi = 0, \quad \left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \epsilon^2}\right)_R \phi = 0, \quad (9)$$

$$\left(\frac{\partial^2}{\partial \bar{\eta}^2} - \frac{\partial^2}{\partial \bar{\zeta}^2}\right)_P \phi = 0, \quad \left(\frac{\partial^2}{\partial \bar{\tau}^2} - \frac{\partial^2}{\partial \bar{\epsilon}^2}\right)_L \phi = 0. \quad (10)$$

We now introduce the light-cone coordinates, valid throughout all space-time:

$$V = t + z, \quad U = t - z \quad (11)$$

and their analogs in the above four coordinate systems:

$$(F) \nu = \eta + \zeta, \quad \mu = \eta - \zeta, \quad (12)$$

$$(P) \bar{\nu} = -\bar{\eta} - \bar{\zeta}, \quad \bar{\mu} = -\bar{\eta} + \bar{\zeta}, \quad (13)$$

$$(R) \chi = \tau + \epsilon, \quad \kappa = \tau - \epsilon, \quad (14)$$

$$(L) \bar{\chi} = -\bar{\tau} - \bar{\epsilon}, \quad \bar{\kappa} = -\bar{\tau} + \bar{\epsilon}. \quad (15)$$

These are related in the following way

$$(F) V = a^{-1} e^{a\nu}, \quad U = a^{-1} e^{a\mu}, \quad (16)$$

$$(P) V = -a^{-1} e^{-\bar{\nu}}, \quad U = -a^{-1} e^{-a\bar{\mu}}, \quad (17)$$

$$(R) V = a^{-1} e^{a\chi}, \quad U = -a^{-1} e^{-a\kappa}, \quad (18)$$

$$(L) V = -a^{-1} e^{-a\bar{\chi}}, \quad U = a^{-1} e^{a\bar{\kappa}}. \quad (19)$$

Field expansion, Bogoliubov transformations, and entanglement.—Using the light-cone coordinates, the field may be expanded in plane waves as $\hat{\Phi}(V, U) = \int_0^\infty \frac{dk}{(4\pi k)^{1/2}} [\hat{a}_k^1 e^{-ikV} + \hat{a}_k^{1\dagger} e^{ikV} + \hat{a}_k^2 e^{-ikU} + \hat{a}_k^{2\dagger} e^{ikU}]$. Since all \hat{a}^1 's commute with all \hat{a}^2 's (corresponding to left and right moving modes, respectively), we make a common simplification, and treat only the left moving sector of the field, $\hat{\Phi}(V) = \int_0^\infty \frac{dk}{(4\pi k)^{1/2}} [\hat{a}_k^1 e^{-ikV} + \hat{a}_k^{1\dagger} e^{ikV}]$, with the understanding that analogous results hold for the right moving sector $\hat{\Phi}(U)$ as well.

We can expand $\hat{\Phi}(V)$ in the following sets of functions, in their respective quadrants. The Rindler modes

$$g_\omega^R(\chi) = (4\pi\omega)^{-1/2} e^{-i\omega\chi}, \quad (20)$$

$$g_\omega^L(\bar{\chi}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\chi}} \quad (21)$$

and their analogs in the future and past

$$g_\omega^F(\nu) = (4\pi\omega)^{-1/2} e^{-i\omega\nu} \quad (22)$$

$$g_\omega^P(\bar{\nu}) = (4\pi\omega)^{-1/2} e^{-i\omega\bar{\nu}}. \quad (23)$$

We note that $g_\omega^F(\nu)$ is the same solution as $g_\omega^R(\chi)$, extended from R into F —a fact pointed out by Gerlach [4,9] in the context of massive fields. This can be seen by expanding these functions in plane waves

$$\theta(V) g_\omega^R(\chi) = \int_0^\infty \frac{dk}{(4\pi k)^{1/2}} (\alpha_{\omega k}^R e^{-ikV} + \beta_{\omega k}^R e^{ikV}), \quad (24)$$

$$\theta(V) g_\omega^F(\nu) = \int_0^\infty \frac{dk}{(4\pi k)^{1/2}} (\alpha_{\omega k}^F e^{-ikV} + \beta_{\omega k}^F e^{ikV}). \quad (25)$$

Now note that $g_\omega^R(\chi)$ and $g_\omega^F(\nu)$ are identical as functions of V , since $\chi(V) = \nu(V)$, and so they must be built of exactly the same plane waves. The same relationship holds for $g_\omega^L(\bar{\chi})$ and $g_\omega^P(\bar{\nu})$. In other words, the Bogoliubov coefficients satisfy

$$\alpha_{\omega k}^F = \alpha_{\omega k}^R, \quad \beta_{\omega k}^F = \beta_{\omega k}^R, \quad (26)$$

$$\alpha_{\omega k}^P = \alpha_{\omega k}^L, \quad \beta_{\omega k}^P = \beta_{\omega k}^L. \quad (27)$$

Thus, the well-known relations between Bogoliubov coefficients in R and L are duplicated in F and P . In particular, solving (24) and (25) (and the analogous P and L relations) leads to Bogoliubov coefficients which satisfy the relations, $\beta_{\omega k}^P = -e^{-\pi\omega/a} \alpha_{\omega k}^{F*}$ and $\beta_{\omega k}^F = -e^{-\pi\omega/a} \alpha_{\omega k}^{P*}$.

From this point forward, the demonstration of future-past entanglement of the Minkowski vacuum is *exactly the same* as the standard demonstration of right-left entanglement, with a change of labels $R \rightarrow F$ and $L \rightarrow P$.

We review the basic argument, and refer the reader to Crispino, Higuchi, and Matsas [4] for detail.

Using the above Bogoliubov relations, the following pure-positive frequency modes can be defined

$$G_\omega(V) = \theta(V)g_\omega^F(\nu) + \theta(-V)e^{-\pi\omega/a}g_\omega^{P*}(\bar{\nu}), \quad (28)$$

$$\bar{G}_\omega(V) = \theta(-V)g_\omega^P(\bar{\nu}) + \theta(V)e^{-\pi\omega/a}g_\omega^{F*}(\nu). \quad (29)$$

When the field is expanded in terms of these functions, the annihilation operators for the G and \bar{G} quanta can readily be seen to be $(\hat{a}_\omega^F - e^{-\pi\omega/a}\hat{a}_\omega^{P\dagger})$ and $(\hat{a}_\omega^P - e^{-\pi\omega/a}\hat{a}_\omega^{F\dagger})$ (where the \hat{a} 's here refer to the g quanta). Since both G and \bar{G} are pure-positive frequency functions of Minkowski time, their vacuum coincides with the Minkowski vacuum $|0_M\rangle$, and we obtain the relations

$$(\hat{a}_\omega^F - e^{-\pi\omega/a}\hat{a}_\omega^{P\dagger})|0_M\rangle = 0, \quad (30)$$

$$(\hat{a}_\omega^P - e^{-\pi\omega/a}\hat{a}_\omega^{F\dagger})|0_M\rangle = 0, \quad (31)$$

which in turn imply the following:

$$(\hat{a}_\omega^{F\dagger}\hat{a}_\omega^F - \hat{a}_\omega^{P\dagger}\hat{a}_\omega^P)|0_M\rangle = 0. \quad (32)$$

Define the vacuum $|0_T\rangle$ to satisfy $\hat{a}_\omega^F|0_T\rangle = \hat{a}_\omega^P|0_T\rangle = 0$. Using the approximation that there are a discrete set of modes labeled by ω_i , the relations (30)–(32) imply that the Minkowski vacuum restricted to F - P may then be expressible in the following form:

$$|0_M\rangle = \prod_i C_i \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i/a}}{n_i!} (\hat{a}_{\omega_i}^{F\dagger} \hat{a}_{\omega_i}^{P\dagger})^{n_i} |0_T\rangle, \quad (33)$$

which is clearly entangled. Also mirroring the Rindler case, the state of the future (or the past) alone is a ‘‘thermal’’ state of the g_ω -modes, where ω is a frequency with respect to the conformal time coordinate η .

$$\hat{\rho}_F = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i/a} |n_i^F\rangle \langle n_i^F| \right], \quad (34)$$

where $C_i = \sqrt{1 - e^{-2\pi\omega_i/a}}$.

Detectors.—The above result suggests that an inertial detector switched on at $t = 0$ and sensitive to frequency E with respect to conformal time η should register a thermal response. This can indeed be seen to be the case. For completeness, we move now to a 4- d description.

The Schrödinger equation in the conformal time η along the worldline $\vec{x} = 0$ takes the following form: $i \frac{\partial}{\partial \eta} \Psi = \frac{\partial t}{\partial \eta} H \Psi = atH\Psi$. To use perturbation theory in η , we take the Hamiltonian to be $H = H_0/at + H_I$, where H_I is the standard interaction term for an Unruh-DeWitt detector, $\alpha \hat{m} \hat{\phi}$. The time scaling of the H_0 term means the detector will have a constant η -frequency gap, E . Converting to η , the Schrödinger equation reads,

$$i \frac{\partial}{\partial \eta} \Psi = (H_0 + e^{a\eta} H_I) \Psi. \quad (35)$$

The $e^{a\eta}$ factor in the interaction term means that perturbation theory will eventually break down, as the fixed coupling to the field eventually dominates the changing energy gap of the detector. Here, we assume the constants α and a have been chosen such that perturbation theory remains valid over times that are large compared to any other relevant timescale in the problem, and interpret infinite integrals over η as integrals to ‘‘arbitrarily large η within this approximation.’’

In the Heisenberg picture in η , the detector’s monopole moment operator evolves like $\hat{m}(\eta) = e^{iH_0\eta} \hat{m} e^{-iH_0\eta}$, while the field operator transforms as a scalar under the change $t \rightarrow \eta$, so that $\hat{\phi}(\eta) = \hat{\phi}(t(\eta))$. The detector response can thus be obtained in the standard way

$$F(E) = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' e^{-iE(\eta-\eta')} e^{a(\eta+\eta')} D^+(\eta, \eta'), \quad (36)$$

where $D^+(\eta, \eta') = \langle 0_M | \phi(\eta) \phi(\eta') | 0_M \rangle$.

This differs from the usual form only in the presence of the $e^{a(\eta+\eta')}$ factor in the integrand, and the fact that the limits of integration correspond to a detector switched on at $t = 0$. The usual regularized form of $D^+(x, x')$ $\langle 0_M | \phi(x) \phi(x') | 0_M \rangle$ is given by [1]

$$D^+(x, x') = -\frac{1}{4\pi^2} [(t - t' - i\epsilon)^2 - (\vec{x} - \vec{x}')^2]^{-1}. \quad (37)$$

We now note that the functional form of $e^{a(\eta+\eta')} D^+(\eta, \eta')$ for two points on the inertial trajectory $t = a^{-1} e^{a\eta}$, $\vec{x} = 0$ is identical to that of $D^+(x(\tau), x(\tau'))$ for two points on the accelerated trajectory $t = a^{-1} \sinh(a\tau)$, $x = y = 0$, $z = a^{-1} \cosh(a\tau)$, up to rescaling of ϵ . This can be seen through a coordinate substitution, noting for the inertial trajectory that

$$\frac{1}{(t - t')^2} = \frac{a^2 e^{-a(\eta+\eta')}}{4 \sinh^2(\frac{a}{2}(\eta - \eta'))} \quad (38)$$

and for the accelerated trajectory:

$$\frac{1}{(t - t')^2 - (z - z')^2} = \frac{a^2}{4 \sinh^2(\frac{a}{2}(\tau - \tau'))}. \quad (39)$$

This correspondence thus yields a formally identical response function integral, in the two cases. Through the standard evaluation of the response function integral [1], this leads us to a thermal response at temperature $T = \frac{\hbar a}{2\pi k}$. Note the factor of ‘‘ c ’’ difference compared with the Unruh temperature, due to the dimensionality of a in this case (our scaling parameter). We thus conclude that an inertial, two-state Unruh-DeWitt detector, whose energy gap is continuously scaled as $\frac{1}{at}$ responds to the Minkowski vacuum in a manner identical to an accelerated detector with a fixed proper-energy gap.

Our results have previously been hinted at in the literature by Bunch, Christensen, and Fulling [1,10], who studied a field quantized in F alone, and who noted a

“thermal correspondence” in the particle content of two different (but pure in F) vacua. More recently, Martinetti and Rovelli [11] have predicted that a time-dependent temperature, $T = \frac{1}{2\pi t}$, should be seen by an observer “born at $t = 0$,” but no detector interpretation was given. Recently, the response of an inertial Unruh-Dewitt detector (without frequency scaling) switched on abruptly at $t = 0$ was computed by Louko and Satz [12]—the response is not thermal. However, if a detector is designed to compensate for the changing field temperature with a changing energy gap, then $\frac{E}{T}$ becomes constant in time, and one might expect to see a constant thermal response—exactly what we have shown above. We believe this is the proper interpretation of the result of Martinetti and Rovelli, in terms of detectors. Further, our result reveals the source of thermalization via F - P entanglement.

Feasibility of experimental detection.—The Unruh effect is notoriously difficult to observe, since the temperature is so tiny for accessible values of the acceleration, a , namely $T_U = \frac{\hbar a}{2\pi c k}$. To observe a temperature of 1° K requires an acceleration on the order of $10^{20} \frac{m}{s^2}$.

However, we have seen that scaling the energy gap of an inertial detector allows interaction with precisely the same field modes in the same thermal state. In our case, a is simply a scaling constant with units of $\frac{1}{\text{sec}}$. The factor of c^{-1} disappears from the temperature, $T = \frac{\hbar a}{2\pi k}$. To encounter a temperature of 1° K requires a on the scale of 100 GHz.

We also require that the energy gap of the detector is scaled over a long enough period to allow thermalization. Experimentally, we imagine a detector which is scaled between times η_1 and η_2 , and we demand many oscillations at the constant η frequency E of the detector, within the interaction time period. This requirement reads $\eta_2 - \eta_1 \gg E^{-1}$. If expressed in ordinary t times and frequencies (in which \tilde{E}_1 is the initial t -frequency gap of the detector at t_1 , etc.), it then reads $\frac{t_2}{t_1} \gg e^{(1/t_1 \tilde{E}_1)} = e^{(1/t_2 \tilde{E}_2)}$.

Now, if t_1 is taken to be the characteristic time scale $1/\tilde{E}_1$, then thermalization requires $t_2 \gg 2.7t_1$.

In other words, simple analysis suggests that the effect could be visible on frequency scales in the vicinity of 100 GHz, scaled over a single order of magnitude.

Conclusions.—In contrast to earlier work which attempted to define “entanglement in time” as a new and different quantity [13], the definition of entanglement we have used is the standard one—the nonseparability of a pure state (in this case the vacuum). The implications of timelike entanglement have not been explored in depth. The thermal effect we describe here is only the first such consequence. We speculate, however, that most consequences of ordinary entanglement have a directly analogous interpretation in timelike entanglement. For example, the no-signaling theorem [14] may be interpreted to forbid the use of timelike entanglement as a means of communication with the past. Nevertheless, projecting onto states in F should collapse the state in P .

It was noted above that the entangled modes we have described in F - P are the same mode solutions as the entangled Rindler modes in R - L . In other words, F - P entanglement is not merely *analogous* to R - L entanglement—it is *precisely the same entanglement*, viewed in a different region of space-time. Recently, theoretical work has focused on manipulating or extracting vacuum entanglement by interacting with the R - L entangled modes [15–20]—this illustrates “exotic effects” which are in principle allowed by relativistic quantum field theory. We speculate that due to the dimensional improvement and the lack of need for acceleration, some of these effects may in fact become experimentally accessible, when converted to equivalent interactions in F - P .

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