

New Approach to Gutenberg-Richter Scaling

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We introduce a new model for an earthquake fault system that is composed of noninteracting simple lattice models with different levels of damage denoted by q . The undamaged lattice models ($q = 0$) have Gutenberg-Richter scaling with a cumulative exponent $\beta = 1/2$, whereas the damaged models do not have well defined scaling. However, if we consider the “fault system” consisting of all models, damaged and undamaged, we get excellent scaling with the exponent depending on the relative frequency with which faults with a particular amount of damage occur in the fault system. This paradigm combines the idea that Gutenberg-Richter scaling is associated with an underlying critical point with the notion that the structure of a fault system also affects the statistical distribution of earthquakes. In addition, it provides a framework in which the variation, from one tectonic region to another, of the scaling exponent, or b value, can be understood.

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It has been known for over half a century that the frequency-magnitude distribution of seismic events occurring on a fault system obeys the Gutenberg-Richter (GR) scaling relation [1,2] which states that magnitudes are distributed according to

$$N_{\mu} \sim 10^{-b\mu}, \quad (1)$$

where μ is the local or Richter magnitude [3]. The Richter magnitude has been superseded by the more physically significant moment magnitude, μ_w , [4] which is related to the average slip, $\overline{\Delta u}$, over the rupture surface by

$$\mu_w = \frac{2}{3} \log_{10} M - 10.7, \quad \text{where } M = GA \overline{\Delta u}, \quad (2)$$

A is the area of and G is the shear modulus of the rupture surface, and M is the seismic moment. The exponential distribution in the magnitude then becomes a power law distribution in the moment, transforming Eq. (1) into

$$N_M \sim M^{-\beta}, \quad \text{with } \beta \equiv \frac{2b}{3}. \quad (3)$$

Here, N_M is the total number of events with moment greater than or equal to M [1,2]. The b value in Eq. (1) is distributed in the range $0.75 \leq b \leq 1.2$ [5,6] which corresponds to the range $0.5 \leq \beta \leq 0.8$.

The GR distribution in Eqs. (1) and (3) has been the subject of considerable investigation by seismologists as well as physicists and geophysicists who have associated the scaling with an underlying critical point. This apparent critical phenomena has led to the creation of several simple models that exhibit a GR-like distribution. The first,

proposed by Burridge and Knopoff [7], consists of blocks connected by linear springs resting on a rough surface and is evolved according to Newton’s Laws. Using the Mohr-Coulomb friction law and ignoring inertial effects, Rundle and Jackson [8] introduced a cellular automaton version of the Burridge-Knopoff model which was reintroduced in lattice form by Olami *et al.* (OFC) [9]. In addition to the association with critical points, several authors [10,11] proposed that GR scaling is a form of self organized criticality. Klein *et al.* [12] proposed that the scaling involves fluctuations about a spinodal critical point. All of these models share at least one shortcoming: they are best understood as describing individual faults rather than fault systems while it is the latter which are observed to scale according to Eqs. (1) and (3). Seismic events on single faults may or may not follow the GR distribution, depending on the fault, as we will show later.

In this Letter, we propose a different approach that models a *fault system* as a collection of noninteracting OFC models or “faults” with varying degrees of damage. The frequency of events on *individual* faults does not, in general, scale as a power law in the event’s moment; however, when a collection of model faults with varying degrees of damage are assembled into a “fault system”, the aggregate distribution of moments is well described by the GR scaling relation. The OFC model consists of a regular lattice of linear size L with open boundary conditions and a stress failure threshold, σ^F , and a residual stress, σ^R , at each vertex (site). Initially, the sites are assigned a random stress, $\sigma^R \leq \sigma_j < \sigma^F$, where the index j labels the position of the site on the lattice. The model is

evolved via a plate update in which stress is simultaneously added to each site until the stress on the site nearest its failure threshold reaches σ^F . This loading method is the so-called zero velocity limit [9]. When a site fails its stress is reduced to σ^R and $(1 - \alpha)(\sigma_j - \sigma^R)/ar^2$ is distributed to each of its ar^2 neighbors where a is a dimensionless number depending on the shape of the interaction, r is the range of the interaction, and $0 \leq \alpha < 1$ is the dissipation parameter. Failure of a site may cause neighboring sites to fail, resulting in an earthquake or avalanche. We continue to search the lattice until $\sigma_j < \sigma^F \forall j$, at which point the lattice is reloaded as described above.

We add a small amount of noise to the model by resetting each site to $\sigma^R \pm \eta$, where η is drawn from a flat distribution with zero mean. For this paper, we take $\sigma^F = 2$, $\sigma^R = 1 \pm 0.025$, $L = 512$, and stress from a failed site is redistributed within a circle of radius $r = 20$. We choose $r = 20$ to mimic the long-range stress transfer in fault systems [13,14]. We simulate the system for 4×10^6 plate updates to remove any transient effects before recording the size (number of failed sites) of each of the next 10^7 events.

For nearest neighbor stress transfer ($r = 1$), there is no apparent GR scaling [15–17], however, if we take $r \gg 1$, the system becomes near mean-field [15] and there appears to be GR or cluster scaling associated with a spinodal critical point [12,15]. In this limit, the number of events, n_s , of size $s \in [s, s + ds]$ scales as

$$n_s = n_0 \frac{\exp[-\Delta h s]}{s^\tau}, \quad (4)$$

with $\tau = 3/2$. Here, n_0 is a measure of the seismic activity of the fault and Δh is a measure of the distance from the spinodal [15,18]. The spinodal is a critical-like point that becomes pronounced in systems with long-range interactions or stress transfer. It is characterized by a set of exponents that are different in value than those associated with standard critical points. For example, spinodals have $\tau = 3/2$ rather than $4/3$ as found at standard mean-field critical points. To make contact with observations on real faults, we note that for $r \gg 1$, the size, s in Eq. (4), is proportional to the moment, M in Eq. (2) [15]. To obtain the cumulative distribution, N_M , we simply integrate $N_M \propto N_s = \int_s^\infty d\xi n_\xi$.

We now modify the model by randomly eliminating a fraction of sites, q [19]. These eliminated or dead sites do not hold stress; hence, when stress is transferred to a dead site from a failing neighbor, it is dissipated. In real faults, this can be caused by different levels of fracture or gouge, which are known to vary over a fault system [20]. We do this for many lattices with $0 \leq q < 1$. As can be seen from Fig. 1, the larger the value of q the less the distribution resembles a simple power law. This is consistent with the idea that the OFC model in the zero velocity limit is only critical with a zero dissipation constant [21]. Nevertheless,

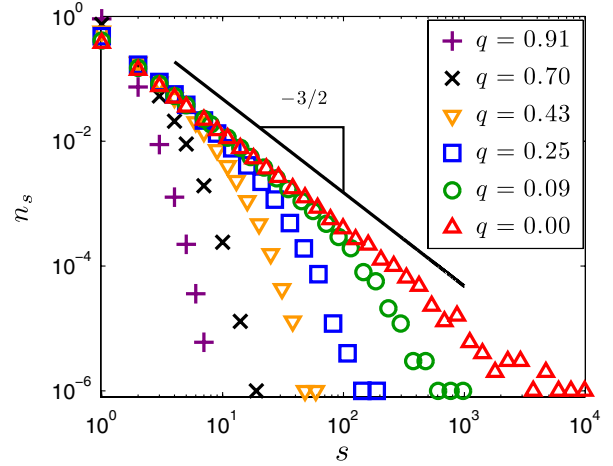


FIG. 1 (color online). The frequency-size distribution for a model fault for various values of q . For $q = 0$ (i.e., no damage) we observe scaling up to the linear system size. As q is increased, the distribution is no longer described by a power law, rather, large events are exponentially suppressed according to Eq. (4). The black line has the slope predicted for the scaling on undamaged faults.

the data are still well described by Eq. (4), but with q -dependent Δh and n_0 . To see the data more clearly, in Fig. 1 (as well as Figs. 2 and 4) we plot the data at evenly distributed points in log-space. Additionally, each of the distributions in these figures contain the same number of events.

To determine the dependence of Δh and n_0 on q , we consider the theoretical description of the model as derived by Klein *et al.* [12,15], from which one can argue for the dependence

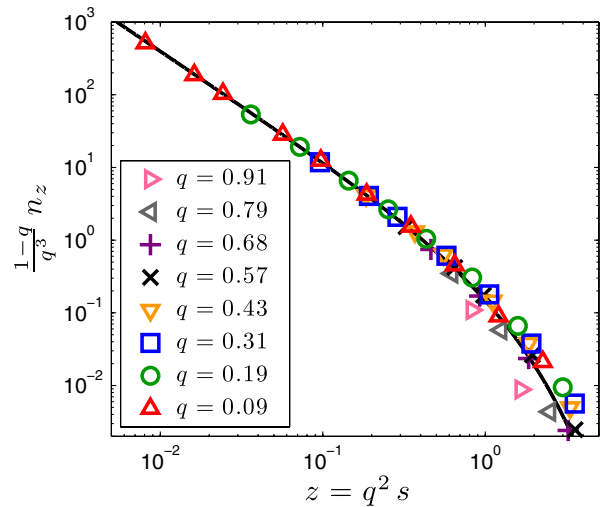


FIG. 2 (color online). The frequency-size distribution plotted in terms of the scaling variables. We explicitly account for the q dependence in n_s by plotting $(1 - q)/q^3 n_z$ versus $z = q^2 s$. Having removed the q dependence in these distributions, the data collapse to a single curve (black line) for all values of q .

$$n_s = \frac{N}{1-q} \frac{\exp[-q^2 s]}{s^\tau}, \quad (5)$$

where $\tau = 3/2$, as above, and N is a q -independent constant related to the average seismic activity on an undamaged ($q = 0$) fault. We check this q dependence in two ways. First, for various values of q , we fit n_s , using a least-squares, nonlinear, weighted fit, for Δh and n_0 to Eq. (5). Additionally, we introduce a scaling variable $z \equiv q^2 s$ such that plots of $n_s(1-q)/q^3$ should collapse to a single curve for all values of q . The data collapse in Fig. 2 suggests we have the correct scaling form for n_s . For values of q approaching unity, we do not expect the scaling form in Eq. (5) to be valid given the singularity at $q = 1$. As expected, the numerical data generated for $q = 0.91$ do not fit the scaling form.

If this mechanism is responsible for the frequency-magnitude distribution on *single faults*, then we expect the data obtained from *real* faults to vary from fault to fault but, this variation should be captured by a single parameter in an otherwise universal distribution, as in Eq. (5). Using scaling arguments, one can rewrite Eq. (4) in terms of the slip area, A , introduced in Eq. (2), as

$$N_A \propto \frac{\exp[-\Delta h A^{2/3}]}{A}. \quad (6)$$

In Fig. 3, we plot the cumulative number of events with area A from the Advanced National Seismic System catalog [22] on three faults from the southern California system. As can be seen from the figure, the data are consistent with Eq. (6).

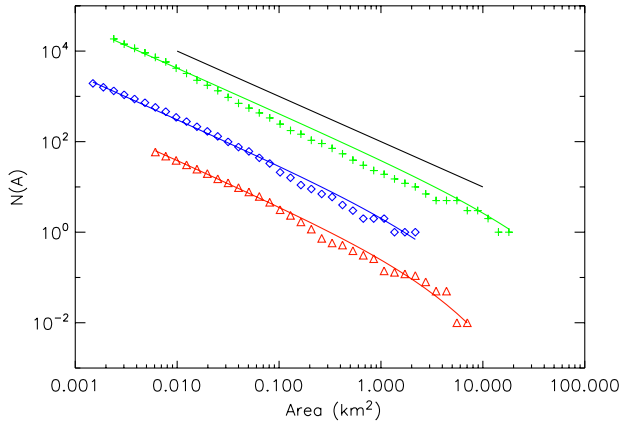


FIG. 3 (color online). The number of events with magnitude greater than or equal to the minimum magnitude of completeness [24] versus the area of each event [25] occurring from 1980–2008 within a 20 km swatch on either side of the San Jacinto fault (green crosses), Ft. Tejon segment of the San Andreas Fault (blue diamonds), and the Creeping section of the San Andreas Fault (red triangles). To see the data clearly, the various faults have been offset by factors of ten. The fits (solid colored lines) are to Eq. (6) and the solid black line has unit slope. The data are from the Advanced National Seismic System catalog [22].

To obtain the frequency-size distribution for a *fault system*, we must integrate Eq. (5) over all values of q , that is,

$$\tilde{n}_s = \int_0^1 dq n_s \sim \int_0^1 dq \frac{D_q}{1-q} \frac{\exp[-q^2 s]}{s^\tau} \sim \frac{1}{s^{\tilde{\tau}}}, \quad (7)$$

where the weighting factor, D_q , is the fraction of faults with damage $q \in [q, q + dq]$. For equal weighting ($D_q = 1$) we find $\tilde{n}_s \sim 1/s^2$ which implies a cumulative exponent $\beta = \tilde{\tau} - 1 = 1$. In Fig. 4, we plot (purple crosses) the number of events of a given size versus s . As noted above, β is observed to vary between 0.5 and 0.8 in real fault systems [5,6]; however, in real fault systems we do not know D_q . If we associate the damage with microcracks, which are known to have a fractal spatial distribution [20], it is reasonable to assume a scale-free or power law distribution for $D_q \sim q^{-x}$, where the exponent varies depending on the fault system. Therefore, the frequency-size distribution in Eq. (7) can be written in terms of the incomplete gamma function. If we perform an asymptotic expansion in the limit $s \gg 1$, we find

$$\tilde{n}_s \sim \frac{1}{s^{2-x/2}} \left(1 + \frac{\Gamma[(2-x)/2]}{\Gamma[(1-x)/2]} \frac{1}{s^{1/2}} + \mathcal{O}(s^{-1}) \right), \quad (8)$$

where $\Gamma(z)$ is the gamma function. To leading order, we find $\tilde{n}_s \sim s^{-\tilde{\tau}}$ with $\tilde{\tau} = 2 - x/2$, and so if we wish to obtain a b value of unity, we simply solve

$$b = \frac{3(2-x)}{4}, \quad (9)$$

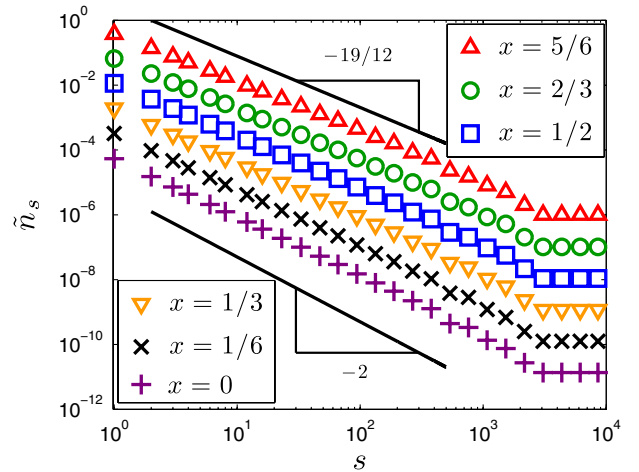


FIG. 4 (color online). The cumulative frequency-size distribution for a model fault system for various damage distributions, $D_q \sim q^{-x}$. By varying the exponent x , the GR distribution scales with different exponents $\tilde{\tau}$. The solid black lines are the predictions of Eq. (8). The flattening of the distribution at large s is a finite size effect. We have offset the various distributions by scaling the data by factors of 10.

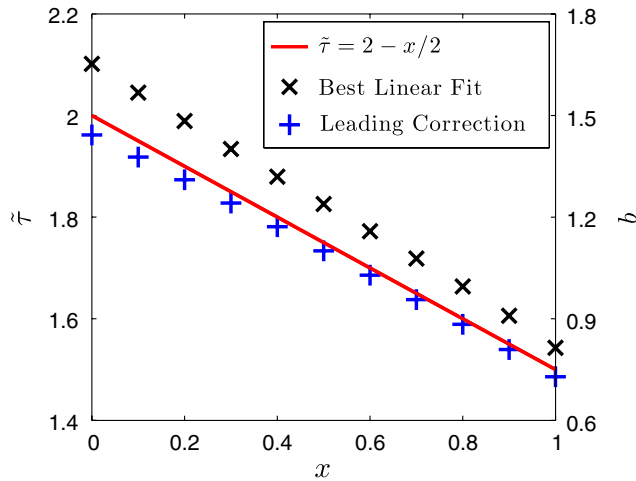


FIG. 5 (color online). The scaling exponents, $\tilde{\tau}$ and b , versus the damage distribution exponent, x . The black exes represent the value of $\tilde{\tau}$ determined by fitting the data to a power law, the blue crosses come from fits keeping the leading asymptotic correction to the scaling form, and the red line is the predicted value of $\tilde{\tau}$.

with $b = 1$, so $x = 2/3$. Furthermore, it is clear from Eq. (9) that a range of b values can be obtained by varying the exponent in D_q [23] as shown in Fig. 4. In Fig. 5, we plot the scaling exponents $\tilde{\tau}$ and b versus the exponent, x , of the damage distribution, D_q . The solid red line is the relationship predicted by Eq. (8) while the black exes represents the value of $\tilde{\tau}$ determined by a linear fit of the data to the form $N/s^{\tilde{\tau}}$ for N and $\tilde{\tau}$. To get better agreement between this fit value and our predicted value, we perform a two parameter nonlinear fit to the form $N/s^{\tilde{\tau}}(1 + \gamma/s^{1/2})$ where $\gamma = \Gamma[(2-x)/2]/\Gamma[(1-x)/2]$ is not a fit parameter. The value of $\tilde{\tau}$ as determined from this fit is shown in blue crosses.

In summary, we have introduced a new paradigm for understanding GR scaling which links scaling associated with a (spinodal) critical point on a single fault with GR scaling on a fault system. The GR scaling on a fault system is affected by both the degree of damage on each fault (q), and the distribution of damage over the fault system (D_q). This paradigm offers an explanation for the observation of GR scaling on fault systems comprised of individual faults: none of which produce the same event magnitude statistics as the aggregate system. It also provides a framework in which one can understand the observed variation of the GR exponent, or b value, from one region to another. In addition, since the various faults that comprise the fault system are noninteracting by construction, our model provides a mechanism in which scaling can arise on fault system length scales that are ten to one-hundred times larger than any length scale associated with the critical phenomena occurring on a single fault.

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