Stable Charged Cosmic Strings

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We study the quantum stabilization of a cosmic string by a heavy fermion doublet in a reduced version of the standard model. We show that charged strings, obtained by populating fermionic bound state levels, become stable if the electroweak bosons are coupled to a fermion that is less than twice as heavy as the top quark. This result suggests that extraordinarily large fermion masses or unrealistic couplings are not required to bind a cosmic string in the standard model. Numerically we find the most favorable string profile to be a simple trough in the Higgs vacuum expectation value of radius $\approx 10^{-18}$ m. The vacuum remains stable in our model, because neutral strings are not energetically favored.

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Introduction.—Various field theories suggest the existence of stringlike configurations, which are the particle physics analogues of vortices or magnetic flux tubes in condensed matter physics. They are called cosmic (or Z) strings to distinguish them from the fundamental variables in string theory and to indicate that they can stretch over cosmic length scales. They can have significant cosmological effects [1] and thus may be relevant to the early universe. Stable strings within the standard model of particle physics would be particularly interesting because they could be observable today.

In the standard model, string configurations [2–4] are not topologically stable and thus can only be stabilized dynamically. Here we focus on the role heavy fermions can play in this stabilization. Since fermions can lower their energy by binding to the string, their binding energy can overcome the classical energy required to form the string background. However, once we include the contribution to the energy from bound fermions, we must also include the contribution from the distortion of the entire fermion spectrum, i.e., the vacuum polarization energy, since both contributions enter at order $\hbar$.

A string configuration with a vortex structure introduces nontrivial behavior at spatial infinity. This property invalidates the straightforward application of standard methods to compute the vacuum polarization energy. Only recently developed techniques [5,6] have made it possible to extend these techniques to string configurations. Earlier Naculich [7] showed that in the limit of weak coupling, fermion fluctuations destabilize the string. The quantum properties of Z strings have been connected to nonperturbative anomalies [8]. A first attempt at a full calculation of the quantum corrections to the Z-string energy was carried out in Ref. [9]. Those authors were only able to compare the energies of two string configurations, rather than comparing a single string configuration to the vacuum; these limitations arise from the nontrivial behavior at spatial infinity. The fermionic vacuum polarization energy of the Abelian Nielson-Oleson vortex has been estimated in Ref. [10] with regularization limited to the subtraction of the divergences in the heat-kernel expansion. Quantum energies of bosonic fluctuations in string backgrounds were calculated in Ref. [11]. Previously, we have pursued the idea of stabilizing cosmic strings by populating fermionic bound states in a $2+1$ dimensional model [12]. Many such bound states emerge and some configurations even induce an exact zero mode [7]. Nonetheless, stable configurations were only obtained for extreme values of the model parameters. In $3+1$ dimensions, stability is more likely because quantization of the momentum parallel to the symmetry axis yields an additional multiplicity of bound states.

Model and ansatz.—We consider a model of the electroweak interactions in $D = 3 + 1$ dimensions with some technical simplifications, which we will justify *a posteriori*. We set the Weinberg angle to zero, so that electromagnetism is decoupled from the theory. We also neglect QCD interactions, though we include the $N_C = 3$ color degeneracy. Finally, we consider a single heavy doublet that is degenerate in mass, neglecting flavor mixing and mass splitting within the doublet. The classical Higgs and gauge fields are described by the Lagrangian

$$\mathcal{L}_{\phi,W} = -\frac{1}{2}\mathrm{tr}(G^{\mu\nu}G_{\mu\nu}) + \frac{1}{2}\mathrm{tr}(D^\mu\Phi)^\dagger D_\mu\Phi - \frac{\lambda}{2}\mathrm{tr}(\Phi^\dagger\Phi - v^2)^2. \tag{1}$$

We represent the Higgs doublet $\phi = (\phi_+, \phi_0)$ as a matrix,

$$\Phi = \begin{pmatrix} \phi_0^* & \phi_+ \\ -\phi_+^* & \phi_0 \end{pmatrix}.$$

The gauge coupling constant enters via the covariant derivative $D^\mu = \partial^\mu - igW^\mu$, and the $SU(2)$ field strength tensor is $G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu]$. We then have the fermion Lagrangian

$$\mathcal{L}_\Psi = i\bar{\Psi}(P_L D\!\!\!/ + P_R \partial\!\!\!/)\Psi - f\bar{\Psi}(\Phi P_R + \Phi^\dagger P_L)\Psi, \quad (2)$$

where the Yukawa coupling f controls the strength of the Higgs-fermion interaction, which generates the fermion mass. Our model is thus characterized by the fermion mass $m_f = f v$, the gauge boson mass $m_W = gv/\sqrt{2}$, the Higgs boson mass $m_H = 2v\sqrt{\lambda}$, and the Higgs vacuum expectation value (VEV) v. When we introduce the fermionic quantum corrections, we impose on-shell renormalization conditions, in which we hold fixed m_H, v, and the residue of the pole in each particle's propagator. These choices exhaust the available counterterms, so we have to adjust the gauge coupling g to match the physical gauge boson mass. Since we neglect boson loops, this renormalization scheme also leaves the fermion mass unchanged.

We construct the string as a classical background field that is translationally invariant in the z direction. We work in Weyl gauge $W^0 = 0$ and also introduce a parameter ξ_1 that allows us to include a gauge field with winding number n. We set n to unity in the actual calculations. The gauge and Higgs fields are then

$$\vec{W} = ns\frac{f_G(\rho)}{g\rho}\hat{\varphi}\begin{pmatrix} s & ic\mathrm{e}^{-in\varphi} \\ -ic\mathrm{e}^{in\varphi} & -s \end{pmatrix} \quad \text{and}$$
$$\Phi = v f_H(\rho)\begin{pmatrix} s\mathrm{e}^{-in\varphi} & -ic \\ -ic & s\mathrm{e}^{in\varphi} \end{pmatrix}, \quad (3)$$

where $s = \sin(\xi_1)$ and $c = \cos(\xi_1)$, and (ρ, φ) are polar coordinates in the plane perpendicular to the string axis. This ansatz yields the classical energy per unit length

$$E_{\mathrm{cl}} = 2\pi\int_0^\infty d\rho\rho\bigg[\frac{2}{g^2}\bigg(\frac{f_G'}{\rho}\bigg)^2 + v^2(f_H')^2 + \frac{v^2}{\rho^2}f_H^2(1-f_G)^2 + \lambda v^4(1-f_H^2)^2\bigg], \quad (4)$$

where primes denote derivatives with respect to ρ. Variational width parameters w_H and w_G enter through the respective profile functions for each field,

$$f_H(\rho) = 1 - \mathrm{e}^{-\rho/w_H}, \qquad f_G(\rho) = 1 - \mathrm{e}^{-\rho^2/w_G^2}. \quad (5)$$

Energy considerations.—We compute the total binding energy per unit length as a sum of three terms:

$$E_{\mathrm{tot}} = E_{\mathrm{cl}} + N_C(E_{\mathrm{vac}} + E_b). \quad (6)$$

The classical energy per unit length depends on the model parameters and the variational parameters w_H, w_G, and ξ_1. The two contributions in Eq. (6) proportional to N_C summarize the fermionic effects. We measure all dimensionful quantities in comparison to appropriate powers of m_f, so that E_{vac} and E_b only depend on the ansatz parameters w_H, w_G and ξ_1. (There is a weak, logarithmic dependence on the model parameters introduced via the on-shell renormalization conditions; it is small for the values of the coupling constants we consider.) QCD effects only enter via the degeneracy factor N_C. Since the considered energy scales are well above the QCD scale, these interactions can be neglected due to asymptotic freedom.

The fermionic effects are computed from the single particle Dirac Hamiltonian in the two-dimensional subspace orthogonal to the symmetry axis of the string. (We refrain from displaying this Hamiltonian, which we extract using Eq. (3). For actual computations a specific gauge must be adopted, complicating its presentation [6,13].) The profiles f_G and f_H act as potentials in this Hamiltonian. The vacuum polarization energy per unit length in the string background E_{vac} is the computationally most expensive part of the calculation. It is computed from the scattering solutions using the spectral method [5,14,15], adapted to handle the long-ranged string potential [6,13]. Finally, the single particle Hamiltonian has many bound state solutions; for $\xi_1 = \frac{\pi}{2}$ there exists an exact zero mode. By explicitly populating these bound states, we add charge to the string. Let $\epsilon_i \leq m_f$ be an eigenvalue of the two-dimensional Dirac Hamiltonian. Then a state has energy $[\epsilon_i^2 + p^2]^{1/2}$, where p is its conserved momentum along the symmetry axis. To count the populated states, we introduce a chemical potential μ such that $\min\{|\epsilon_i|\} \leq \mu \leq m_f$. States with $[\epsilon_i^2 + p^2]^{1/2} < \mu$ are filled while states with $[\epsilon_i^2 + p^2]^{1/2} > \mu$ remain empty, which gives a Fermi momentum $P_i(\mu) = [\mu^2 - \epsilon_i^2]^{1/2}$ for each bound state. According to the Pauli exclusion principle we can occupy each state only once. This yields the charge density per unit length of the string

$$Q(\mu) = \frac{1}{\pi}\sum_{\epsilon_i \leq \mu} P_i(\mu), \quad (7)$$

where the sum runs over all bound states available for a given chemical potential.

Equation (7) can be inverted to give $\mu = \mu(Q)$. In numerical computations we prescribe the left-hand side of Eq. (7) and increase μ from $\min\{|\epsilon_i|\}$ until the right-hand side matches. From this value $\mu = \mu(Q)$, the binding energy per unit length

$$E_b(Q) = \frac{1}{\pi}\sum_{\epsilon_i \leq \mu}\int_0^{P_i(\mu)} dp\Big[\sqrt{\epsilon_i^2 + p^2} - m_f\Big] \quad (8)$$

can be computed as a function of the prescribed charge. Filling the available states up to a common chemical potential minimizes E_b: if the towers of states built upon two different ϵ_i had different upper limits, the energy would be lowered by moving a state from the tower with the larger limit to that with the lower one, without changing the charge.

Our central task is to find Higgs-gauge field configurations that yield $E_{\mathrm{tot}} < 0$ for a prescribed value of the charge density, Q. In doing so, we must take care that any binding we observe is not an artifact of the Landau pole, which eventually sends E_{vac} to minus infinity as w_H and/or w_G tend to zero. It arises because in our approximation (neglecting contributions from bosonic loops) the model is not asymptotically free. Once we identify a configuration and parameter set with interesting numerical results we use a method similar to that of Ref. [16] to ensure that the Landau pole contribution is negligible.

Results.—The similarity to the standard model suggests the model parameters $g = 0.72$, $v = 177$ GeV, $m_H = 140$ GeV, and $f = 0.99$. The Yukawa coupling estimate is obtained from the top-quark mass $m_t = 175$ GeV. To consider a fourth generation with a heavy fermion doublet that couples to the standard model bosons, we will vary the Yukawa coupling but keep all other model parameters fixed.

For the configurations we consider, the classical energy, Eq. (4), is dominated by the Higgs potential contribution, which scales as $\lambda w_H^2/(f^4 N_C)$ compared to the fermionic contributions. As $\xi_1 \to 0$, the gauge field contribution goes to zero, so this choice is favored classically. We will see that $\xi_1 \approx 0$ remains favored when $E_{\rm vac}$ and E_b are included, so that the stable charged string obtained in our model is simply a trough in the Higgs VEV, without significant gauge field contributions.

We give all numerical results in units of m_f or $1/m_f$ as appropriate. In Fig. 1 we display the fermion contributions for various sets of ansatz parameters. These lines terminate at an end point where all available bound states (for all longitudinal momenta) are populated and the charge cannot be increased any further. The fermion contributions favor a wide string for large charges, while they cause the string to shrink for small charges. For very small charges, corresponding to small widths, the calculation is unreliable because of the Landau pole. (The problem arises for widths much less than unity and coupling constants of order five or larger. In our numerical search for stable configurations we only consider $w_H \geq 2$ and $w_G \geq 2$.)

When we add more configurations, we observe a linear relation between the charge and the *minimal* fermion contribution to the energy, even though for any given configuration, the fermion energy depends quadratically on the charge, cf. Eqs. (7) and (8). This linear dependence arises from a delicate balance between the vacuum polarization (which determines the y intercept for a given configuration) and the binding energies (which determine the Q dependence). Figure 1 also suggests that the width of the Higgs profile, w_H, is the dominating scale (which is corroborated in Fig. 3, where $E_b + E_{\rm vac}$ is seen to be nearly independent of ξ_1, and thus of w_G.) Both the number of two-dimensional bound states and the magnitude of their binding energies $\epsilon_i - m_f$ vary roughly linearly with w_H. As a result, the minimal fermion contribution scales quadratically with w_H, as the classical energy does. To decide if the string is stable we have to compare the leading scaling with w_H^2 in $E_{\rm cl}$ and $E_{\rm vac} + E_b$. For large widths, the string is stable if the resulting coefficient of the scaling with w_H^2 is negative. For the physically motivated parameters mentioned above, the classical energy dominates and there is no binding for any charge. However, as mentioned above, the relative contribution from $E_{\rm cl}$ decreases like $1/f^4$. So even a moderate increase of the fermion mass could lead to binding. We remark that extrapolating the straight line in Fig. 1 predicts that the vacuum energy should vanish for very narrow strings, as we would expect. This estimate overcomes the Landau pole obstacles that arise in a direct computation.

To search for a stable string of fixed charge Q, we have computed the vacuum polarization energy and the bound state energies from the two-dimensional Hamiltonian for several hundred configurations characterized by w_H, w_G, and ξ_1. We then prescribe the charge Q and, for those configurations that can accommodate it according to Eq. (7), we compute the binding energy as in Eq. (8). Once we have computed the fermionic contribution to $E_{\rm tot}$, the classical energy is a simple spatial integral, which requires a negligible amount of additional computation. As a result, in this procedure it is most efficient to vary the Yukawa coupling. For a given charge, we then have a large set of configurations that are labeled by given (discrete) values of the variational parameters. We scan this set for

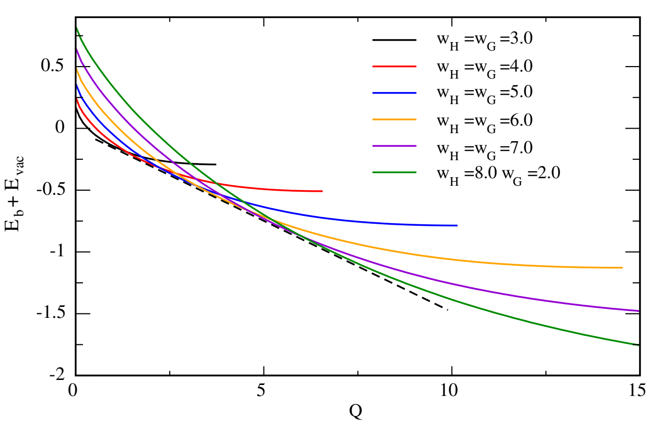

FIG. 1 (color online). Total bound state and vacuum energy per unit length as a function of charge density per unit length, in units of the fermion mass, for $\xi_1 = 0.4\pi$. The dashed line indicates the minimal fermionic contribution to the energy.

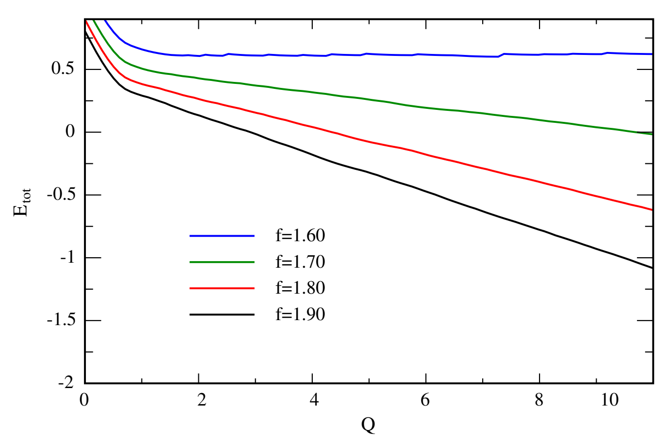

FIG. 2 (color online). Total energy per unit length of optimal string configurations as a function of charge per unit length, in units of the fermion mass.

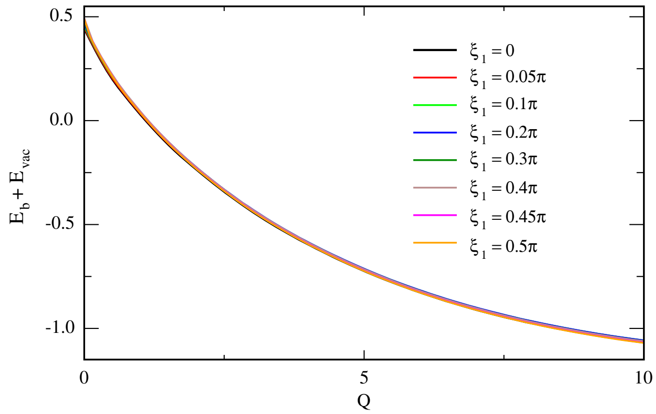

FIG. 3 (color online). Fermionic contribution to the string binding energy per unit length as a function of charge density per unit length, in units of the fermion mass, for a variety of values of ξ_1 and $w_H = 6.0$ and $w_G = 6.0$.

the minimal total energy. If the variational parameters covered the full configuration space, this treatment would be equivalent to the self-consistent construction of the minimal energy configuration. With our restriction to the variational space, however, we only find an upper limit to the exact minimum; if our treatment detects a bound configuration, the existence of a stable charged cosmic string is established.

In Fig. 2, we show the full energy per unit length $E_{\rm tot}$ as a function of the charge density per unit length for a variety of Yukawa couplings f. The sharp increase at small Q is an artifact of the restriction of the ansatz parameters to avoid the Landau pole. Increasing the Yukawa coupling from its top-quark value decreases the relative contribution from $E_{\rm cl}$ to $E_{\rm tot}$. We see that at $f \approx 1.6$ the large width pieces from $E_{\rm cl}$ and $E_{\rm vac} + E_b$ approximately cancel. Increasing the Yukawa coupling only slightly more, e.g., to $f \gtrsim 1.7$, yields a negative total energy per unit length at large charge densities, which indicates that the string is lighter than the corresponding density of free fermions. This limit corresponds to a fermion mass of about 300 GeV with a typical width for the stable charged string of about 10^{-18} m ($w_H \approx 4/m_f$).

Surprisingly, we find that the fermion contribution to the energy is nearly independent of the ansatz parameter ξ_1, as shown in Fig. 3. Even though the bound state spectrum varies strongly with ξ_1, and $E_{\rm vac} + E_b$ depends only weakly on ξ_1 [6], there are subtle cancellations within the bound state spectrum itself that yield such a tiny gauge field dependence of the fermion energy. As $g \ll f$, the gauge field terms increase $E_{\rm cl}$ for $\xi_1 \neq 0$. Hence $E_{\rm tot}$ is minimized for $\xi_1 \approx 0$ in the cases we have studied.

Discussion.—We have seen that a heavy fermion doublet can stabilize a nontrivial string background in a simplified version of the electroweak standard model for a nonzero fixed charge density. Light fermions would contribute only weakly to the binding of the string, since their Yukawa couplings are small. As a result, we can add them to our model, e.g., to accommodate anomaly cancellation, without significantly changing the result. The resulting configuration is essentially of pure Higgs structure. Any additional (variational) degree of freedom can only lower the total energy. Hence embedding this configuration in the full standard model, with the $U(1)$ gauge field included, also yields a bound object. We see binding set in at $m_f \approx 300$ GeV, which is still within the range of energy scales at which the standard model should provide an effective description of the relevant physics, and also within the range to be probed at the LHC. For such fermion masses, recent calculations have also suggested the potential stability of multifermion bound states in a Higgs background [17,18].

The fermion bound states carry nonzero angular momenta, implying that the bound state wave functions depend on the azimuthal angle. This might induce a more complicated spatial structure of the string configuration than the one adopted in Eq. (3). In particular, the cylindrical analog of spherical "hedgehog" configurations, representing a Higgs field with unit winding within a $U(1)$ subgroup of the full $SU(2)$ isospin group, could be an interesting extension of our work. Such alterations can only lower the total energy, however.

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