New Solution of the Cosmological Constant Problems

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We extend the usual gravitational action principle by promoting the bare cosmological constant (CC) to a field which can take many possible values. Variation gives a new integral constraint equation for the classical value of the effective CC that dominates the wave function of the Universe. The expected value of the effective CC, is calculated from measurable quantities to be $O(t_U^{-2})$ as observed, where t_U is the present age of the Universe in Planck units. This also leads to a falsifiable prediction for the observed spatial curvature parameter of $\Omega_{k0} = -0.0055$. Our proposal requires no fine-tunings or extra dark-energy fields but suggests a new view of time evolution.

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The cosmological constant (CC) was introduced by Einstein in 1917 to ensure that general relativity (GR) admitted a static cosmological solution. Introducing a CC, λ, required the addition of a term $-\lambda g_{\mu\nu}$ to the original field equations:

$$G^{\mu\nu} = \kappa\langle T^{\mu\nu}\rangle \rightarrow G^{\mu\nu} = \kappa\langle T^{\mu\nu}\rangle - \lambda g^{\mu\nu},$$

where $G^{\mu\nu} = R^{\mu\nu} - Rg^{\mu\nu}/2$, $R_{\mu\nu}$ is the Ricci curvature of $g_{\mu\nu}$, and $\langle T^{\mu\nu}\rangle$ is the expected energy-momentum tensor of matter; $\kappa = 8\pi G$, $c = \hbar = 1$. It was later appreciated that there were fundamental, reasons for its presence. Quantum fluctuations result in a vacuum energy, ρ_{vac}, that contributes to the $\langle T^{\mu\nu}\rangle$

$$\langle T^{\mu\nu}\rangle = T_m^{\mu\nu} - \rho_{\mathrm{vac}} g^{\mu\nu},$$

where $T_m^{\mu\nu}$ vanishes *in vacuo* and hence

$$G^{\mu\nu} = \kappa T_m^{\mu\nu} - \Lambda g^{\mu\nu}, \qquad \Lambda = \lambda + \kappa\rho_{\mathrm{vac}}.$$

The vacuum energy contributes $\kappa\rho_{\mathrm{vac}}$ to the effective CC, Λ. Even if the "bare" CC, λ, is assumed to vanish, the effective CC will generally be nonzero. For $\Lambda = 0$, the λ, and $\kappa\rho_{\mathrm{vac}}$ terms must exactly cancel. With no *a priori* link between the values of λ and $\kappa\rho_{\mathrm{vac}}$ this seems improbable. With no cancellation, we expect $|\Lambda| \gtrsim O(\kappa\rho_{\mathrm{vac}})$.

At late cosmic times ρ_{vac} does not evolve. Given the standard model of particle physics, and reasonable (e.g., supersymmetric) extensions of it, a late-time ρ_{vac} of at least $M_{\mathrm{EW}}^4 \sim (246\ \mathrm{GeV})^4$ appears to be unavoidable. Hence, it seems natural that $\rho_{\mathrm{vac}}^{\mathrm{eff}} = \kappa^{-1}\Lambda \gtrsim M_{\mathrm{EW}}^4$. This *cannot* be the case because measurements of the expansion rate give $\rho_{\mathrm{vac}}^{\mathrm{eff}} \approx (2.4 \times 10^{-12}\ \mathrm{GeV})^4$ [1], at least 10^{56} times smaller than the expected quantum contribution. This is the *cosmological constant problem*. Equivalently, assuming the estimate of ρ_{vac} from quantum fluctuations is accurate we ask why $\lambda \approx -\kappa\rho_{\mathrm{vac}}$ to at least 56 decimal places? Furthermore, the time $t_\Lambda = \Lambda^{-1/2} \approx 9.7$ Gyr is curiously close to the present age of the Universe, $t_U \approx 13.7$ Gyr. First Barrow and Tipler [2], then Efstathiou [3] and Weinberg [4], derived anthropic upper limits on $|\Lambda|$ by requiring that inhomogeneities grow by gravitational instability long enough for galaxies to form. For $\Lambda > 0$ this requires $t_\Lambda \gtrsim 0.7$ Gyr. But, there is still no reason why the fixed time, t_Λ, should correlate with an observer-dependent time like t_U. This is the *coincidence problem*.

We propose a simple extension of the usual action principle in which the bare CC, λ, will be promoted from a parameter to a "field." The variation leads to a new field equation which determines the value of λ, and hence the effective CC, in terms of other properties of the observed Universe. Crucially, one finds that the observed classical history naturally has $t_\Lambda \sim t_U$. Fuller details are presented elsewhere [5]. When it is applied to GR, λ (and hence Λ except when ρ_{vac} evolves due to, say, a phase transition) is a true constant and is *not* seen to evolve. Hence, the resulting history is indistinguishable from GR with the value of Λ put in by hand. Nonetheless, for given theory of gravity such as GR, our model produces a firm prediction for Λ in terms of other measurable quantities and is testable by future observations. It should be stressed that our proposal is equally applicable to theories of gravity other than GR and to theories with more than 4 space-time dimensions. As in 4-d GR, t_Λ is still expected to be $O(t_U)$.

If our model is correct, assuming an (approximately) homogeneous and isotropic GR cosmology, the measured value of Λ requires a specific value for the dimensionless spatial curvature, Ω_{k0}, of the observable Universe. The predicted Ω_{k0} is consistent with current observational limits and large enough to be detected in the near future. Our model also specifies the probability, $f(\Lambda)d\Lambda$ observing a CC in the range $[\Lambda, \Lambda + d\Lambda]$. Crucially, $f(\Lambda)$ is independent of the prior weighting given to different values of Λ in the wave function of the Universe. We find that the observed value of Λ is indeed typical, as is a coincidence between t_Λ and t_U. Our proposal provides a realistic and falsifiable model of the Universe that avoids the CC and coincidence problems.

Define the total action of the Universe on a manifold $\mathcal{M}$ with boundary $\partial\mathcal{M}$ and effective CC Λ, matter fields Ψ^a, and metric $g_{\mu\nu}$, to be $I_{\rm tot}[g_{\mu\nu}, \Psi^a, \Lambda; \mathcal{M}]$. Usually, λ is a fixed parameter and the wave (partition) function of the Universe, $Z[\lambda; \mathcal{M}] \equiv Z_\Lambda[\mathcal{M}]$, is given by

$$Z_\Lambda[\mathcal{M}] = \sum e^{iI_{\rm tot}}[\times \text{gauge fixing terms}],$$

where $\{Q^A\}$ are some fixed boundary quantities (generalized "charges") on $\partial\mathcal{M}$, and the sum is over all histories (i.e., configurations of the metric and matter, $g_{\mu\nu}$, Ψ^a) consistent with these fixed charges. The dominant contribution to $Z_\Lambda[\mathcal{M}]$ is from the histories for which $I_{\rm tot}$ is stationary for $g_{\mu\nu}$ and Ψ^a variations that preserve the $\{Q^A\}$. In these dominant histories, the matter and metric fields obey their classical field equations.

When the surface terms in the gravitational action are chosen to make $I_{\rm tot}$ first order in derivatives of the metric, for a non-null $\partial\mathcal{M}$ with induced 3-metric $\gamma_{\mu\nu}$, a small general metric variation gives

$$2\kappa\delta I_{\rm tot} = \int_{\partial\mathcal{M}} |\gamma|^{(1/2)} d^3x N^{\mu\nu}\delta\gamma_{\mu\nu} + \int_{\mathcal{M}} |g|^{(1/2)} d^4x E^{\mu\nu}\delta g_{\mu\nu}.$$

Put $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g^{(\mathcal{M})}_{\mu\nu}$, $\bar{g}_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g^{(\partial\mathcal{M})}_{\mu\nu}$, where the $\delta g^{(\mathcal{M})}_{\mu\nu}$ vanish on $\partial\mathcal{M}$ but $\delta g^{(\partial\mathcal{M})}_{\mu\nu}$ do not. The vanishing of $\delta I_{\rm tot}$ in $\mathcal{M}$ implies that $E^{\mu\nu}[g^{(0)}_{\mu\nu}] = E^{\mu\nu}[\bar{g}_{\mu\nu}] = 0$. The classical field equations for the metric are $E^{\mu\nu} = 0$. The variation $\delta I_{\rm tot} = 0$ then requires that $\gamma_{\mu\nu}$ be fixed on $\partial\mathcal{M}$. However, if some part, $\partial\mathcal{M}_u$, of $\partial\mathcal{M}$ lies in the causal future of another part, $\partial\mathcal{M}_I$, the choice of fixed $\gamma_{\mu\nu}$ is constrained by $E^{\mu\nu} = 0$. In this example, we define $\{Q^A\}$ to be the smallest data set on $\partial\mathcal{M}$ that can be freely specified which, when combined with $E^{\mu\nu} = 0$, fixes $\gamma_{\mu\nu}$ up to a gauge choice on $\partial\mathcal{M}$. This definition is then extended to the matter sector (for which the classical field equations are $\Phi_a = 0$). This is just a restatement of the usual variational principle allowing for a causally interconnected $\partial\mathcal{M}$. Since $E^{\mu\nu} = 0$ depends on Λ, fixed $\{Q^A\}$ and $E^{\mu\nu} = \Phi_a = 0$ only fixes $\gamma_{\mu\nu}$ and boundary matter fields for given Λ, and we have $\delta\gamma_{\mu\nu}|_{\partial\mathcal{M}} = \mathcal{H}_{\mu\nu}\delta\Lambda$ and $\delta\Psi^a|_{\partial\mathcal{M}} = \mathcal{P}^a\delta\Lambda$, which define $\mathcal{H}_{\mu\nu}$ and $\mathcal{P}^a$.

Our proposal for solving the CC problems is simply to promote the bare cosmological constant, λ, from a fixed parameter to a field (albeit one that is constant in space and time). A similar promotion of λ occurs in studies of unimodular gravity. Equally, it can arise in a fundamental theory, e.g., string theory, where there are many vacua with different minima of the vacuum energy.

The wave function of the Universe, $Z[\mathcal{M}]$, now includes a sum over all possible values of λ in addition to the usual sum over configurations of $g_{\mu\nu}$ and Ψ^a. The effective CC, Λ, is equal to $\lambda + \text{const}$ and so a sum over all possible values of λ is equivalent to a sum over all Λ and so

$$Z[\mathcal{M}] = \sum_\lambda \mu[\lambda] Z[\lambda; \mathcal{M}] = \sum_\Lambda \mu[\lambda] Z_\Lambda[\mathcal{M}],$$

where $\mu[\lambda]$ is some unknown prior weighting on the different values of λ. Provided $\mu[\lambda]$ is not strongly peaked at a particular λ value, we find that (at least classically) our model is independent of the choice of μ. The classical histories that dominate the wave function are those for which, with fixed $\{Q^A\}$, $\delta I_{\rm tot} = 0$ for variations in the summed-over fields. For variations of $g_{\mu\nu}$ and Ψ^a, this gives $E^{\mu\nu} = \Phi_a = 0$ as before. Since λ is summed over, a stationary $I_{\rm tot}$ also now requires $\delta I_{\rm tot}/\delta\lambda = \delta I_{\rm tot}/\delta\Lambda = 0$.

We define $I_{\rm class}(\Lambda; \mathcal{M})$ to be $I_{\rm tot}$ evaluated at the classical solution for $g_{\mu\nu}$ and Ψ^a and fixed $\{Q^A\}$; $\delta I_{\rm tot}/\delta\Lambda = 0$ is then equivalent to

$$dI_{\rm class}(\Lambda; \mathcal{M})/d\Lambda = 0. \tag{1}$$

Equation (1) yields a field equation for determining the classical value of the effective CC. An observer sees a classical history with effective CC, Λ, which satisfies Eq. (1). Since λ is a true space-time constant, the effective CC will not be seen to evolve in this classical history.

The solutions of Eq. (1) depend on the definition of $\mathcal{M}$, fixed $\{Q^A\}$ and surface terms in $I_{\rm tot}$; these choices should be well-motivated and consistent with the symmetries of nature. We demand that all observables including Λ should be influenced only by parts of the Universe causally connected to the observer. As Eq. (1) involves integrals over $\mathcal{M}$ and $\partial\mathcal{M}$, the only coordinate independent choice consistent with this demand is that $\mathcal{M}$ is the observer's causal past. If our model's predictions are accurate, this requirement could indicate that a notion of causal order is a fundamental rather than emergent property of quantum space-time. The wave function, $Z[\mathcal{M}]$, is then a sum over all possible configurations in the causal past, and $\partial\mathcal{M}$ is composed of the observer's past-light cone, $\partial\mathcal{M}_u$, and initial spacelike singularity $\partial\mathcal{M}_I$, (where say $\tau = 0$). As we move towards $\partial\mathcal{M}_I$, the CC has less and less influence on the evolution of the Universe. This motivates specifying the $\{Q^A\}$ so that the initial state on $\partial\mathcal{M}_I$ is fixed independently of λ. On $\partial\mathcal{M}_u$, the fields then depend on Λ through the classical field equations in a calculable fashion. The canonical surface term choice is the minimal term that renders the total action first order in metric and matter derivatives. There are no obvious and well-motivated alternatives.

There is now a simple argument for why $t_\Lambda \sim t_U$ is natural in our model. With $I_{\rm tot}$ at most first order in metric and matter derivatives, Eq. (1) is

$$\int_{\mathcal{M}} |g|^{(1/2)} d^4x = \frac{1}{2}\int_{\partial\mathcal{M}} |\gamma|^{(1/2)}[N^{\mu\nu}\mathcal{H}_{\mu\nu} + \Sigma_a \mathcal{P}^a] d^3x. \tag{2}$$

The left-hand side is just the 4-volume, $V_{\mathcal{M}}$, of $\mathcal{M}$. The right-hand side is a "holographic" term defined on the boundary (of area $A_{\partial\mathcal{M}}$, say). Cosmologically $N^{\mu\nu}\mathcal{H}_{\mu\nu} + \Sigma_a \mathcal{P}^a \sim O(\mathrm{tr}N/\Lambda) \sim O(H/\Lambda)$, where H is the Hubble constant [with $H(t_U) \equiv H_0$ today]. Hence, the right-hand side of Eq. (2) is $O(\Lambda^{-1} H_0 A_{\partial\mathcal{M}})$. So, we expect solutions of Eq. (2) to have $\Lambda \sim O(H_0) A_{\partial\mathcal{M}}/V_{\mathcal{M}}$. Typically, $H_0 \sim A_{\partial\mathcal{M}}/V_{\mathcal{M}}$ and H_0^{-1} is determined by $t_\Lambda = \Lambda^{-1/2}$ and the age of the Universe t_U. Equation (2) links the values of t_Λ and t_U and, in the absence of fine-tunings, we naturally expect $t_\Lambda \sim O(t_U)$ and hence $\Lambda \sim O(1) t_U^{-2}$ ($\sim 10^{-122}$ in units where $G \equiv 1$). If there are extra dimensions with volume V_D, then $A_{\partial\mathcal{M}}$ and $V_{\mathcal{M}}$ would both be multiplied by V_D leaving $A_{\partial\mathcal{M}}/V_{\mathcal{M}}$ and the expectation $\Lambda \sim t_U^{-2}$ is unaltered [6]. If Eq. (2) admits a classical solution, then the classical value of the effective CC will have the observed magnitude, $O(t_U^{-2}) \sim 10^{-122}$, without fine-tuning.

We now apply our model to our Universe where gravity is described by GR to a good approximation. The observed CC is given by the requirement that the total action $I_{\rm cl}$ be stationary with respect to small changes in λ, i.e., Eq. (1) We expand this equation by first evaluating $I_{\rm cl}$ as a implicit function of λ. $I_{\rm cl}$ is the total action $I_{\rm tot}$ modulo the matter and metric field equations, with

$$I_{\rm tot} = I_{\rm EH} + I_{\rm CC} + I_{\rm GHY}^{(u)} + I_m + \ldots,$$

where the ... represent the λ-independent surface terms on $\partial M_{\rm I}$. $I_{\rm EH}$ is the usual Einstein-Hilbert action, i.e., the integral of $(2\kappa)^{-1}\sqrt{-g}R$ over $\mathcal{M}$; $I_{\rm CC}$ and I_m are the cosmological constant and matter actions, respectively, and $I_{\rm GHY}^{(u)}$ is the standard Gibbons-Hawking-York surface term on $\partial\mathcal{M}_u$. We remove the quantum vacuum energy from I_m and absorb it into the effective CC, $\Lambda = \lambda + \kappa\rho_{\rm vac}$. $I_{\rm CC}$ and I_m are then the integrals of $-\kappa^{-1}\sqrt{-g}\Lambda$ and $\sqrt{-g}\mathcal{L}_m$ over $\mathcal{M}$ respectively. $\mathcal{L}_m$ is the effective matter Lagrangian density defined to vanish *in vacuo*; $T_m^{\mu\nu}$ is the associated energy-momentum tensor. Einstein's equations give $(2\kappa)^{-1}R = 2\kappa^{-1}\Lambda - T_m/2$, which we substitute into $I_{\rm EH}$. $I_{\rm GHY}^{(u)}$ can be transformed so that $I_{\rm tot}$ and $I_{\rm cl}$ can be written as a volume integral on $\mathcal{M}$ (see Ref. [5] for details).

For simplicity we focus on a homogeneous and isotropic cosmology with metric

$$ds^2 = a^2(\tau)[-d\tau^2 + (1 + kx^2/4)^{-2} dx^i dx^i],$$

where k determines the spatial curvature. The observer is at $(\tau, x) = (\tau_0, 0)$ and $\partial\mathcal{M}_I$ is the surface $\tau = 0$ where $a = 0$. We take $T_m^{\mu\nu} = (\rho_m + P_m)U^\mu U^\nu + P_m g^{\mu\nu}$; $U^\mu = -a^{-1}\nabla^\mu\tau$. With $H = a_{,\tau}/a^2$, Einstein's equations give $H^2 = \kappa\rho_m/3 + \Lambda/3 - k/a^2$ and $a^{-1}\rho_{m,\tau} = -3H(\rho_m + P_m)$. We find that to linear order in $O(kx^2)$, $I_{\rm cl}$ is [5]:

$$I_{\rm cl} = \frac{4\pi}{3}\int_0^{\tau_0} a^4(\tau)(\tau_0 - \tau)^3[\kappa^{-1}\Gamma - P_{\rm eff}(a)]d\tau,$$

where $P_{eff} = P_m - \mathcal{L}_m$ and $\Gamma = (k/a^2)[2/3 + \tau/(\tau_0 - \tau)]$. Contributions to $P_{\rm eff}$ can come from radiation, dark matter and baryonic matter (labeled "rad", "dm" and 'b', respectively). For radiation and dark matter, $P_{\rm rad} = \rho_{\rm rad}/3$, $\mathcal{L}_{\rm rad}/\rho_{\rm rad} \approx 0$ and $P_{\rm dm}/\rho_{\rm dm}$, $\mathcal{L}_{\rm dm}/\rho_{\rm dm} \approx 0$. For baryonic matter, $P_b/\rho_b \approx 0$, $\mathcal{L}_b = -\zeta_b\rho_b$, where for some $\zeta_b \sim O(1)$ is calculable in principle from QCD. The chiral bag model for baryon structure gives the estimate $\zeta_b \approx 1/2$ [5]. Since $\rho_b \gg \rho_{\rm rad}$, the dominant contribution to $P_{\rm eff}$ comes from baryonic matter and $P_{\rm eff} \approx \zeta_b\rho_b$. The terms in $I_{\rm cl}$ only depend on λ through the scale factor $a(\tau)$. We define $\delta \ln a/\delta\lambda = \mathcal{A}(\tau)$. $\Gamma \propto a^{-2}$ and $P_{\rm eff} \approx \zeta_b\rho_b \propto 1/a^3$, so $\delta(a^4\Gamma)/\delta\lambda = 2\Gamma\mathcal{A}(\tau)$ and $\delta(a^4 P_{\rm eff})/\delta\lambda \approx \zeta_b\rho_b\mathcal{A}(\tau)$; $\mathcal{A}(\tau)$ follows from perturbing Einstein's equations with respect to Λ and using $\delta\ln a/\delta\Lambda = 0$ initially. We find [5]

$$\mathcal{A}(\tau) = \frac{a(\tau)H(\tau)}{6}\int_0^\tau \frac{d\tau^*}{H^2(\tau^*)}.$$

Varying $I_{\rm cl}$ with respect to λ, we find that Eq. (2) for the CC is equivalent to

$$k = \frac{\kappa\int_0^{\tau_0}(\tau_0 - \tau)^3 a^4\zeta_b\rho_b\mathcal{A}(\tau)d\tau}{\int_0^{\tau_0} a^2(\tau)(\tau_0 - \tau)^2(4(\tau_0 - \tau) + 6\tau)\mathcal{A}(\tau)d\tau}. \quad (3)$$

Note that this k is the average spatial curvature in the causal past rather than necessarily the average spatial curvature of the whole space-time; hence $k > 0$ does *not* require the Universe to have a closed topology.

Equation (3) is a consistency condition that relates the value of k to $\Omega_{b0} = \kappa\rho_{\rm baryon}(\tau_0)/3H_0^2$, the observation time τ_0 and, through $a(\tau)$ and $\mathcal{A}(\tau)$, to Λ. So it gives $k = k_0(\Lambda; \tau_0)$ and hence $\Lambda = \Lambda_0(k; \tau_0)$. If our model is valid, a measurement of Λ at a given time predicts a specific value of k and hence $\Omega_{k0} = -k/a_0^2H_0^2$. There are no free parameters in this prediction. Equation (3) requires $k > 0$; i.e., the observable Universe has a positive spatial curvature. For our Universe, taking $\Omega_{\Lambda 0} \approx 0.73$, $\Omega_{b0} \approx 0.0423$ and $T_{\rm CMB} = 2.725$ K we predict

$$\Omega_{k0} = -0.0055(\zeta_b/0.5).$$

This is consistent (for all $\zeta_b \in (0, 1]$) with the current 95% CI of $\Omega_{k0} \in (-0.0133, 0.0084)$ [1]. A combination of data from the current Planck satellite CMB survey with measurements of baryon acoustic oscillations (BAO) will be able to test this prediction of Ω_{k0}.

Inflation in the early Universe is usually invoked to explain why the curvature term is so small today. The duration of inflation, given by the number of e folds N, depends on initial conditions since different inflating regions in the same Universe will have different N. Hence, Ω_k is an environmental parameter which is stochastically different in each inflating region. In our model the extent to which the observed value, $\Lambda_{\rm obs}$, is natural is determined by the probability of living in a bubble universe where k is such that $\Lambda_0(k) \sim O(\Lambda_{\rm obs})$. Larger values of Λ require smaller k, and hence larger N. We define $f(\Lambda)d\Lambda$ to be the probability that $\Lambda \in [\Lambda, \Lambda + d\Lambda]$ and $f_N(N)dN$ is the probability that $N \in [N, N + d\Lambda]$. Gibbons and Turok (GT) calculated $f_N(N) = c(N)e^{-3N}$ for single field, inflation using the natural measure on classical solutions in GR [7]; $c(N)$ has a relatively weak N dependence.

Alternatively, a volume weight e^{3N} gives $f_N \approx c(N)$. With $N(k) = \bar{N} - \ln(k/\bar{k})/2$ (and $\bar{N} > 50$–62 for $\bar{k}/a_0^2 H_0^2 < 0.02$ in realistic models), we find (up to a normalization factor):

$$f(\Lambda) = f_N(N[K_0(\Lambda)])|d \ln K_0(\Lambda)/d\Lambda|.$$

If $f_N \propto e^{-3N}$ then $\Lambda_{\rm obs}$ is inside the 80% CI from $f(\Lambda)$.

Including Bayesian selection makes the observed Λ appear even more typical and reduces the dependence on $f_N(N)$. If Λ is too large the formation of galaxies is greatly suppressed [2]. This limits observable values by $\Lambda \lesssim 10^3 \Lambda_{\rm obs}$. Bayesian selection (in the context of a multiverse) is sufficient to explain why Λ is not too large, but whether or not the $\Lambda_{\rm obs}$ is typical is heavily dependent on the unknown relative weighting of different values of the CC in the multiverse (i.e., the prior distribution, here represented by $\mu[\lambda]$). In our theory, the unknown weighting μ is effectively replaced by the calculable prior $f(\Lambda)$. In the allowed Λ range the N changes by <2.5% and so $f(\Lambda)$ depends only weakly on f_N. We follow Ref. [8] and use the number of galaxies for the number of observers. If $f_N \approx$ const in the allowed range, $\Lambda_{\rm obs}$ lies just out the 68% CI, whereas with $f_N \propto \exp(-3N)$ it lies just inside. In either case, $\Lambda_{\rm obs}$ is typical in our model.

The "coincidence" of $t_U/t_\Lambda \sim O(1)$ or $\Omega_\Lambda/\Omega_m \sim O(1)$ is also a typical occurrence in our model. Observations give $R \equiv \ln(t_\Lambda/t_U) \approx 0.35$. We calculate $|R| < 0.35$ has a probability of 9%–15%, depending on f_N. For $|R| < \ln 2$ it is 16%–25%. Bayesian selection with an assumed uniform prior gives $\approx 4\%$ and 8.5%, respectively. Similarly seeing $\Omega_{\Lambda 0} \in [0.2, 0.8]$ has a 14%–22% chance in our model, and 6.8% with just Bayesian selection.

At any given location and time, the wave function is dominated by a classical history in which Λ takes a single constant value. This means that, classically, no evolution of Λ can be observed. Yet the history that dominates, and its associated Λ value, is different at different observation times [9]. We see a history with CC, Λ_1. A observer in our past would see a different history with CC $\Lambda_2 > \Lambda_1$. Yet, for measurements of Λ_1 and Λ_2 to be compared, information would have to be sent from one history to another. At the level of classical physics there is no mechanism for this. Observers will only see a history consistent with the constant Λ given by Eq. (2). Crucially, this includes registering all previous measurements of Λ as being consistent with $\Lambda = \Lambda_1$. Put simply, we do not see the past as an observer in the past would have seen it. This behavior implies a new view of time in which the whole history changes slowly. It arises as a consequence of taking $\mathcal{M}$ to be the observer's causal past which in turn was necessary for causality when λ was promoted from an external parameter to a field.

As this behavior is an integral part of our model, it is tested indirectly through the $\Omega_{k0} = -0.0055(\zeta_b/0.5)$ prediction. Classically, this movement from one history to another has no directly detectable consequences. From a quantum perspective, the wave function is dominated by a superposition of histories with a small spread in Λ of $\Delta\Lambda \sim (\delta^2 I_{\rm tot}/\delta\Lambda^2)^{-1/2}$. This superposition could give rise to new effects if a system were sensitive to shifts of $O(\Delta\Lambda)$. However, with $\Omega_{\Lambda 0} \sim O(1)$, $\Delta\Lambda/\Lambda \sim \Lambda^{1/2}/M_{\rm pl} \sim 10^{-60} \ll 1$ today, this effect looks undetectably small.

In summary: we have introduced a new approach to solving the CC and coincidence problems. The bare CC, λ, or equivalently the minimum of the vacuum energy, is allowed to take many possible values in the wave function, Z, of the Universe. The value of the effective CC in the classical history that dominates Z is given by a new equation, Eq. (1). This proposal is agnostic about the theory of gravity and the number of space-time dimensions. We have applied it to a universe in which gravity is described by GR. The observed classical history will be completely consistent with a nonevolving CC. In an homogeneous and isotropic universe with realistic matter content we find that the observed value of the effective CC is typical, as is a coincidence between $1/\sqrt{\Lambda}$ and the present age of the Universe, t_U. Unlike explanations of the CC problem that rely only on Bayesian selection in a multiverse, our model in independent of the unknown prior weighting of different Λ values, and makes a numerical prediction for the observed spatial curvature parameter, $\Omega_{k0} = -0.0055(\zeta_b/0.5)$, that is consistent with current observations but can be tested in the near future. We describe a new solution of the CC problems, consistent with observations and free of fine-tunings, new forms of dark energy, or modifications to GR. It implies a new view of time and is subject to high-precision test.

[1] E. Komatsu *et al.*, Astrophys. J. Suppl. Ser. **192**, 18 (2011).
[2] J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle* (Oxford UP, Oxford, 1986), chap. 6.9.
[3] G. Efstathiou, Mon. Not. R. Astron. Soc. **274**, L73 (1995).
[4] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).
[5] D. J. Shaw and J. D. Barrow, Phys. Rev. D **83**, 043518 (2011).
[6] This is unlike the ever-present Λ model of R. Sorkin, Int. J. Theor. Phys. **36**, 2759 (1997); S. Dodelson, M. Ahmed, P. B. Greene, and R. Sorkin, Phys. Rev. D **69**, 103523 (2004); see J. D. Barrow, Phys. Rev. D **75**, 067301 (2007).
[7] G. W. Gibbons and N. Turok, Phys. Rev. D **77**, 063516 (2008).
[8] M. Tegmark *et al.*, Phys. Rev. D **73**, 023505 (2006).
[9] Differently placed observers are most likely to observe when $t \sim t_{\rm ms}$, where $t_{\rm ms} \sim O(10^{10}\ {\rm yr})$ is the main-sequence lifetime. In our model they will all most likely observe $\Lambda \sim O(t_{\rm ms}^{-2})$ because t_Λ is close to $t_{\rm ms}$. The quantum aspect of our theory produces a very precise anthropic effect because the observed value of t_Λ is numerically determined by the time at which it is observed, not merely bounded above by the requirement that stars form.