Rotonlike Fulde-Ferrell Collective Excitations of an Imbalanced Fermi Gas in a Two-Dimensional Optical Lattice

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We address the question of whether superfluidity can survive in the case of fermion pairing between different species with mismatched Fermi surfaces using as an example a population-imbalanced mixture of ^{6}Li atomic Fermi gas loaded in a two-dimensional optical lattice at nonzero temperatures. The collective mode is calculated from the Bethe-Salpeter equations in the general random phase approximation assuming a Fulde-Ferrell order parameter. The numerical solution shows that, in addition to low-energy (Goldstone) mode, two rotonlike minima exist, and therefore, the superfluidity can survive in this imbalanced system.

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Introduction.—Although the Fulde and Ferrell (FF) [1] and the Larkin and Ovchinnikov (LO) [2] phases were introduced quite a long time ago, they are still of very high interest because the question of whether the superconductivity or superfluidity can survive in polarized systems remains unanswered. In the FFLO phase, Cooper pairing occurs between a fermion (or up and down quarks) with momentum $\mathbf{k}+\mathbf{q}$ and spin $\uparrow$ and a fermion with momentum $-\mathbf{k}+\mathbf{q}$, and spin $\downarrow$. As a result, the pair-momentum is $2\mathbf{q}$ and the order parameter becomes spatially dependent. The mean-field treatment of the FFLO phase in a variety of systems, such as superconductors with Zeeman splitting and heavy-fermion superconductors [3], atomic Fermi gases with population imbalance loaded in optical lattices [4–6] and harmonic traps [7], and dense quark matter [8] shows that the FFLO state competes with a number of other states, such as the Sarma ($\mathbf{q}=0$) states [9], but in some regions of momentum space the FFLO phase provides the minimum of the mean-field expression of the Helmholtz free energy. Since it is known that with decreasing dimensionality, the pair fluctuation becomes increasingly important, and therefore, there is no *a priori* justification for applying the mean-field calculations in one-dimensional (1D) systems. Thus, in what follows we consider an imbalanced mixture of a ^{6}Li atomic Fermi gas of two hyperfine states $|\uparrow\rangle$ and $|\downarrow\rangle$ with contact interaction loaded into a 2D square optical lattice. The total number of atoms is $M=M_\uparrow+M_\downarrow$, and they are distributed along N sites. The FFLO state is expected to occur on the BCS side of a Feshbach resonance, where the effective attractive interaction between fermion atoms leads to BCS type pairing. We also assume that the lattice potential is sufficiently deep such that the tight-binding approximation is valid and the system is well described by the single-band attractive Hubbard model (on the BCS side the Hubbard parameter U is negative, but in what follows U denotes its absolute value). The tight-binding form of the electron energy is $\xi_{\uparrow,\downarrow}(\mathbf{k})=2J(1-\sum_\nu \cos k_\nu a)-\mu_{\uparrow,\downarrow}$, where $\mu_{\uparrow,\downarrow}$ is the corresponding chemical potential, J is the tunneling strength of the atoms between nearest-neighbor sites, and the lattice constant $a=\lambda/2$ (λ is the laser wavelength and in our numerical calculations we use $\lambda=1032$ nm). The order parameter is assumed to vary as a single plane wave $\Delta_\mathbf{q}=\Delta\exp(2\imath\mathbf{q}.\mathbf{r})$. Unlike the population-balanced systems, for which the spectrum of the collective excitations has been obtained by linearizing the Anderson-Rickayzen equations [10], by the Kadanoff and Baym approach [11], and by the Bethe-Salpeter (BS) formalism [12], to the best of our knowledge the FFLO collective modes have been studied in (i) a 1D population-unbalanced trapped system [7] by using the linear response of the equilibrium system by supplementing the Bogoliubov–de Gennes (BdG) equations with a self-consistent random phase approximation, (ii) a 1D superconductor [13] by transforming slow deformations of the order parameter into small corrections to the BdG Hamiltonian, and (iii) a cold-atom rotated system [14] by locating the poles of the many-body scattering function. We present here a theory which is the first calculation to find the spectrum of the collective excitations in the presence of FF phase which goes beyond the mean-field gap, number, and pair-momentum equations by solving the BS equations for the spectrum of the collective excitations in the general random phase approximation. Since at a finite temperature the FF states compete with the Sarma states, before calculating the collective modes we have obtained the phase diagram. In Fig. 1, we show the phase separation between the FF, Sarma, and normal states for a total filling factor $f=f_\uparrow+f_\downarrow=0.5$ ($f_{\uparrow,\downarrow}=M_{\uparrow,\downarrow}/N$) and an interaction strength $U/J=2.64$, which is similar to the phase diagram in the case of 3D optical lattice [5]. The polarization $P=(f_\uparrow-f_\downarrow)/(f_\uparrow+f_\downarrow)$ that we shall use in collective-mode calculations is $P=0.1$. In this case, the FF states lower the system free energy compared to the corresponding Sarma states at low temperatures. As

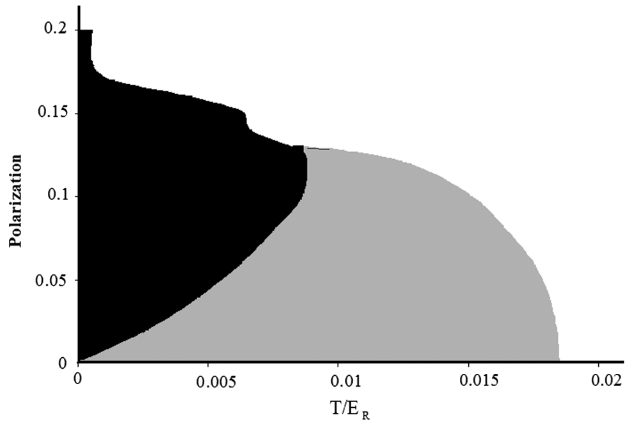

FIG. 1. The phase diagram of the imbalanced Fermi gas in a 2D lattice (FF = black, Sarma = gray, normal = white). The total filling factor is $f = 0.5$. The interaction strength is $U/J = 2.64$, where $J/E_R = 0.0783$. For ^{6}Li atomic Fermi gas and $\lambda = 1030$ nm, $E_R = 1.293 \times 10^{-11}$ eV.

the temperature increases the Sarma states provide the minimum of the free energy. If the temperature is increased even further the normal polarized Fermi gas becomes energetically favorable. The results from our numerical solutions of the BS equation show that (i) there exists a low-energy (Goldstone) mode in the FF state, corresponding to the fluctuations of the order parameter phase, but since the FF state breaks both gauge and translational symmetry there are two different sound velocities in the long wavelength limit, and (ii) the Goldstone mode has rotonlike minima.

Spectrum of the collective excitations.—We concern ourselves with a population-imbalanced 2D Fermi gas with attractive interactions described by the Hubbard model. In this case the Fourier transform of the single-particle Green's function is a 2×2 matrix $\hat{G} = \begin{pmatrix} G^{\uparrow\uparrow} & G^{\uparrow\downarrow} \\ G^{\downarrow\uparrow} & G^{\downarrow\downarrow} \end{pmatrix}$. In the mean-field approximation the corresponding matrix elements are as follows:

$$G^{\uparrow,\uparrow}_{\mathbf{q}}(\mathbf{k}, \imath\omega_m) = \frac{(u^{\mathbf{q}}_{\mathbf{k}})^2}{\imath\omega_m - \omega_+(\mathbf{k},\mathbf{q})} + \frac{(v^{\mathbf{q}}_{\mathbf{k}})^2}{\imath\omega_m + \omega_-(\mathbf{k},\mathbf{q})},$$

$$G^{\downarrow,\downarrow}_{\mathbf{q}}(\mathbf{k}, \imath\omega_m) = \frac{(v^{\mathbf{q}}_{\mathbf{k}})^2}{\imath\omega_m - \omega_+(\mathbf{k},\mathbf{q})} + \frac{(u^{\mathbf{q}}_{\mathbf{k}})^2}{\imath\omega_m + \omega_-(\mathbf{k},\mathbf{q})},$$

$$G^{\uparrow,\downarrow}_{\mathbf{q}}(\mathbf{k}, \imath\omega_m) = G^{\downarrow,\uparrow}_{\mathbf{q}}(\mathbf{k}, \imath\omega_m) = u^{\mathbf{q}}_{\mathbf{k}} v^{\mathbf{q}}_{\mathbf{k}}\left[\frac{1}{\imath\omega_m - \omega_+(\mathbf{k},\mathbf{q})} - \frac{1}{\imath\omega_m + \omega_-(\mathbf{k},\mathbf{q})}\right].$$

The symbol ω_m denotes $\omega_m = (2\pi/\beta)(m + 1/2)$; $m = 0, \pm1, \pm2, \ldots, \beta = (k_B T)^{-1}$, k_B is the Boltzmann constant, T is the temperature. As can be seen, the one-particle excitations in a mean-field approximation are coherent combinations of electronlike $\omega_+(\mathbf{k},\mathbf{q}) = E_{\mathbf{q}}(\mathbf{k}) + \eta_{\mathbf{q}}(\mathbf{k})$ and holelike $\omega_-(\mathbf{k},\mathbf{q}) = E_{\mathbf{q}}(\mathbf{k}) - \eta_{\mathbf{q}}(\mathbf{k})$ excitations. The coherent factors $u_{\mathbf{q}}(\mathbf{k})$ and $v_{\mathbf{q}}(\mathbf{k})$ give the probability amplitudes of these states in the actual mixture. Here, $E_{\mathbf{q}}(\mathbf{k}) = \sqrt{\chi^2_{\mathbf{q}}(\mathbf{k}) + \Delta^2}$, $u^{\mathbf{k}}_{\mathbf{k}} = \sqrt{\frac{1}{2}[1 + \frac{\chi_{\mathbf{q}}(\mathbf{k})}{E_{\mathbf{q}}(\mathbf{k})}]}$, $v^{\mathbf{k}}_{\mathbf{k}} = \sqrt{\frac{1}{2}[1 - \frac{\chi_{\mathbf{q}}(\mathbf{k})}{E_{\mathbf{q}}(\mathbf{k})}]}$, and we have used the following notations: $\eta_{\mathbf{q}}(\mathbf{k}) = \frac{1}{2}[\xi_\uparrow(\mathbf{k}+\mathbf{q}) - \xi_\downarrow(\mathbf{q}-\mathbf{k})]$, $\chi_{\mathbf{q}}(\mathbf{k}) = \frac{1}{2}[\xi_\uparrow(\mathbf{q}+\mathbf{k}) + \xi_\downarrow(\mathbf{q}-\mathbf{k})]$.

The thermodynamic potential Ω at temperature T in a mean-field approximation can be evaluated as a summation of quasiparticles with energy $\omega_\pm(\mathbf{k},\mathbf{q})$ [4]. Having obtained the thermodynamic potential in the mean-field approximation, we set $\mathbf{q}$ to be in the x direction and minimize the Helmholtz free energy $F(\Delta, q_x, f_\uparrow, f_\downarrow) = \Omega + \mu_\uparrow f_\uparrow + \mu_\downarrow f_\downarrow$ with respect to $\mu_\uparrow$, $\mu_\downarrow$, Δ, and q_x. As a result, we obtain the number and gap equations, as well as the equation for $\mathbf{q} = (q_x, 0)$:

$$f_\uparrow = \frac{1}{N}\sum_{\mathbf{k}}[u^2_{\mathbf{q}}(\mathbf{k}) f(\omega_+(\mathbf{k},\mathbf{q})) + v^2_{\mathbf{q}}(\mathbf{k}) f(-\omega_-(\mathbf{k},\mathbf{q}))],$$

$$f_\downarrow = \frac{1}{N}\sum_{\mathbf{k}}[u^2_{\mathbf{q}}(\mathbf{k}) f(\omega_-(\mathbf{k},\mathbf{q})) + v^2_{\mathbf{q}}(\mathbf{k}) f(-\omega_+(\mathbf{k},\mathbf{q}))],$$

$$1 = \frac{U}{N}\sum_{\mathbf{k}}\frac{1 - f(\omega_-(\mathbf{k},\mathbf{q})) - f(\omega_+(\mathbf{k},\mathbf{q}))}{2E_{\mathbf{q}}(\mathbf{k})},$$

$$0 = \frac{1}{N}\sum_{\mathbf{k}}\left\{\frac{\partial\eta_{\mathbf{q}}(\mathbf{k})}{\partial q_x}[f(\omega_+(\mathbf{k},\mathbf{q})) - f(\omega_-(\mathbf{k},\mathbf{q}))] + \frac{\partial\chi_{\mathbf{q}}(\mathbf{k})}{\partial q_x} \times\left[1 - \frac{\chi_{\mathbf{q}}(\mathbf{k})}{E_{\mathbf{q}}(\mathbf{k})}[1 - f(\omega_+(\mathbf{k},\mathbf{q})) - f(\omega_-(\mathbf{k},\mathbf{q}))]\right]\right\},$$

where $f(x) = [\exp(\beta x) + 1]^{-1}$ is the Fermi distribution function.

The BS equations for the collective mode $\omega = \omega_{\mathbf{q}}(\mathbf{Q})$ and the corresponding BS amplitude $\hat{\Psi}_{\mathbf{q}}(\mathbf{k},\mathbf{Q})$ can be derived in a similar manner as in population-balanced systems [12]. We have used the Hubbard-Stratonovich transformation which allows us to apply a functional derivative technique. As a result, the following BS equations have been derived:

$$\hat{\Psi}_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) = -\frac{U}{2N}\hat{D}\sum_{\mathbf{p}}\hat{\Psi}_{\mathbf{q}}(\mathbf{p},\mathbf{Q}) + \frac{U}{2N}\hat{M}\sum_{\mathbf{p}}\hat{\Psi}_{\mathbf{q}}(\mathbf{p},\mathbf{Q}).$$

The BS amplitude is a 4×1 matrix $\hat{\Psi}_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) = (\Psi^{\downarrow,\uparrow}_{\mathbf{q}}(\mathbf{k},\mathbf{Q})\ \Psi^{\uparrow,\downarrow}_{\mathbf{q}}(\mathbf{k},\mathbf{Q})\ \Psi^{\uparrow,\uparrow}_{\mathbf{q}}(\mathbf{k},\mathbf{Q})\ \Psi^{\downarrow,\downarrow}_{\mathbf{q}}(\mathbf{k},\mathbf{Q}))^T$ (T means transpose of a matrix), and $U\hat{D}$ and $U\hat{M}$ represent the direct and exchange interactions, respectively:

$$\hat{D} = \begin{pmatrix} K_{\mathbf{q}}^{(\downarrow,\downarrow,\uparrow,\uparrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p), & K_{\mathbf{q}}^{(\downarrow,\uparrow,\downarrow,\uparrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p) & 0 & 0 \\ K_{\mathbf{q}}^{(\uparrow,\downarrow,\uparrow,\downarrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p), & K_{\mathbf{q}}^{(\uparrow,\uparrow,\downarrow,\downarrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p) & 0 & 0 \\ K_{\mathbf{q}}^{(\uparrow,\downarrow,\uparrow,\uparrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p), & K_{\mathbf{q}}^{(\uparrow,\uparrow,\downarrow,\uparrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p) & 0 & 0 \\ K_{\mathbf{q}}^{(\downarrow,\downarrow,\uparrow,\downarrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p), & K_{\mathbf{q}}^{(\downarrow,\uparrow,\downarrow,\downarrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p) & 0 & 0 \end{pmatrix}, \qquad \hat{M} = \begin{pmatrix} 0 & 0 & K_{\mathbf{q}}^{(\downarrow,\downarrow,\downarrow,\uparrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p), & K_{\mathbf{q}}^{(\downarrow,\uparrow,\uparrow,\uparrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p) \\ 0 & 0 & K_{\mathbf{q}}^{(\uparrow,\downarrow,\downarrow,\downarrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p), & K_{\mathbf{q}}^{(\uparrow,\uparrow,\uparrow,\downarrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p) \\ 0 & 0 & K_{\mathbf{q}}^{(\uparrow,\downarrow,\downarrow,\uparrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p), & K_{\mathbf{q}}^{(\uparrow,\uparrow,\uparrow,\uparrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p) \\ 0 & 0 & K_{\mathbf{q}}^{(\downarrow,\downarrow,\downarrow,\downarrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p), & K_{\mathbf{q}}^{(\downarrow,\uparrow,\uparrow,\downarrow)}(\mathbf{k},\mathbf{Q},\imath\omega_p) \end{pmatrix}.$$

Here, $\omega_p = (2\pi/\beta)p$; $p = 0, \pm 1, \pm 2, \ldots$ is a Bose frequency, and we have introduced the two-particle propagator $K_{\mathbf{q}}^{(i,j,k,l)}(\mathbf{k},\mathbf{Q},\imath\omega_p) = \sum_{\omega_m} G_{\mathbf{q}}^{i,j}(\mathbf{k}+\mathbf{Q};\imath\omega_p+\imath\omega_m)G_{\mathbf{q}}^{k,l}(\mathbf{k};\imath\omega_m)$, where $i, j, k, l = \{\uparrow,\downarrow\}$. The condition for existing a nontrivial solution of the Bethe-Salpeter equations leads to the following secular determinant:

$$Z = \begin{vmatrix} U^{-1} + (I_{\gamma,\gamma} - L_{\tilde{\gamma},\tilde{\gamma}}) & (J_{\gamma,l} - K_{m,\tilde{\gamma}}) & (I_{\gamma,\tilde{\gamma}} + L_{\gamma,\tilde{\gamma}}) & (J_{\gamma,m} + K_{l,\tilde{\gamma}}) \\ (J_{\gamma,l} - K_{m,\tilde{\gamma}}) & U^{-1} + (I_{l,l} - L_{m,m}) & (J_{l,\tilde{\gamma}} + K_{m,\gamma}) & (I_{l,m} + L_{l,m}) \\ (I_{\gamma,\tilde{\gamma}} + L_{\gamma,\tilde{\gamma}}) & (J_{l,\tilde{\gamma}} + K_{m,\gamma}) & -U^{-1} + (I_{\tilde{\gamma},\tilde{\gamma}} - L_{\gamma,\gamma}) & (J_{\tilde{\gamma},m} - K_{\gamma,l}) \\ (J_{\gamma,m} + K_{l,\tilde{\gamma}}) & (I_{l,m} + L_{l,m}) & (J_{\tilde{\gamma},m} - K_{\gamma,l}) & U^{-1} + (I_{m,m} - L_{l,l}) \end{vmatrix}, \tag{1}$$

where the following symbols are used:

$$I_{a,b} = \frac{1}{2N}\sum_{\mathbf{k}} a_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} b_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} \left[\frac{1 - f(\omega_-(\mathbf{k},\mathbf{q})) - f(\omega_+(\mathbf{k}+\mathbf{Q},\mathbf{q}))}{\omega + \Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) - \varepsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q})]} - \frac{1 - f(\omega_+(\mathbf{k},\mathbf{q})) - f(\omega_-(\mathbf{k}+\mathbf{Q},\mathbf{q}))}{\omega + \Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) + \varepsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q})]}\right],$$

$$J_{a,b} = \frac{1}{2N}\sum_{\mathbf{k}} a_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} b_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} \left[\frac{1 - f(\omega_-(\mathbf{k},\mathbf{q})) - f(\omega_+(\mathbf{k}+\mathbf{Q},\mathbf{q}))}{\omega + \Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) - \varepsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q})]} + \frac{1 - f(\omega_+(\mathbf{k},\mathbf{q})) - f(\omega_-(\mathbf{k}+\mathbf{Q},\mathbf{q}))}{\omega + \Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) + \varepsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q})]}\right],$$

$$K_{a,b} = \frac{1}{2N}\sum_{\mathbf{k}} a_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} b_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} \left[\frac{f(\omega_-(\mathbf{k},\mathbf{q})) - f(\omega_-(\mathbf{k}+\mathbf{Q},\mathbf{q}))}{\omega + \Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) + \epsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q})]} + \frac{f(\omega_+(\mathbf{k},\mathbf{q})) - f(\omega_+(\mathbf{k}+\mathbf{Q},\mathbf{q}))}{\omega + \Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) - \epsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q})]}\right],$$

$$L_{a,b} = \frac{1}{2N}\sum_{\mathbf{k}} a_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} b_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} \left[\frac{f(\omega_-(\mathbf{k},\mathbf{q})) - f(\omega_-(\mathbf{k}+\mathbf{Q},\mathbf{q}))}{\omega + \Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) + \epsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q})]} - \frac{f(\omega_+(\mathbf{k},\mathbf{q})) - f(\omega_+(\mathbf{k}+\mathbf{Q},\mathbf{q}))}{\omega + \Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) - \epsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q})]}\right].$$

Here, $\varepsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) = E_{\mathbf{q}}(\mathbf{k}+\mathbf{Q}) + E_{\mathbf{q}}(\mathbf{k})$, $\epsilon_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) = E_{\mathbf{q}}(\mathbf{k}+\mathbf{Q}) - E_{\mathbf{q}}(\mathbf{k})$, $\Omega_{\mathbf{q}}(\mathbf{k},\mathbf{Q}) = \eta_{\mathbf{q}}(\mathbf{k}) - \eta_{\mathbf{q}}(\mathbf{k}+\mathbf{Q})$, and a and b are one of the following form factors: $\gamma_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} = u_{\mathbf{k}}^{\mathbf{q}} u_{\mathbf{k}+\mathbf{Q}}^{\mathbf{q}} + v_{\mathbf{k}}^{\mathbf{q}} v_{\mathbf{k}+\mathbf{Q}}^{\mathbf{q}}$, $l_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} = u_{\mathbf{k}}^{\mathbf{q}} u_{\mathbf{k}+\mathbf{Q}}^{\mathbf{q}} - v_{\mathbf{k}}^{\mathbf{q}} v_{\mathbf{k}+\mathbf{Q}}^{\mathbf{q}}$, $\tilde{\gamma}_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} = u_{\mathbf{k}}^{\mathbf{q}} v_{\mathbf{k}+\mathbf{Q}}^{\mathbf{q}} - u_{\mathbf{k}+\mathbf{Q}}^{\mathbf{q}} v_{\mathbf{k}}^{\mathbf{q}}$, $m_{\mathbf{k},\mathbf{Q}}^{\mathbf{q}} = u_{\mathbf{k}}^{\mathbf{q}} v_{\mathbf{k}+\mathbf{Q}}^{\mathbf{q}} + u_{\mathbf{k}+\mathbf{Q}}^{\mathbf{q}} v_{\mathbf{k}}^{\mathbf{q}}$. As $\mathbf{Q} \to 0$ in accordance with the well-known Goldstone theorem, there exists a solution $\omega \to 0$. In this case all J, K, and L vanishes, and the secular equation reduces to the gap equation written as $0 = 1 + UI_{\gamma=1,\gamma=1}$.

Numerical results.—We have solved numerically the number, gap, and q equations assuming a two-dimensional lattice. The total filling factor is $f = 0.5$, while the interaction strength and the temperature are chosen to be $U/J = 2.64$ and $T = 2 \times 10^{-6} E_R$, respectively. In Fig. 2 we present the results of our calculations of the chemical potentials $\mu_\uparrow$ and $\mu_\downarrow$, the gap Δ and q_x for different polarization. The FF states lower the system free energy compared to the corresponding Sarma states. For example, in the case when the polarization $P = 0.1$ ($f_\uparrow = 0.275$ and $f_\downarrow = 0.225$), the Sarma gap and chemical potentials are $\Delta/E_R = 0.022$, $\mu_\uparrow/E_R = 0.219$, and $\mu_\downarrow/E_R = 0.176$, respectively. For the same polarization the solution of mean-field equations provides the following results for the FF states: $q_x a = 0.04227\pi$, $\Delta/E_R = 0.01765$, $\mu_\uparrow/E_R = 0.21777$, and $\mu_\downarrow/E_R = 0.18182$, and therefore, the FF free energy is 99.95% of the free energy of the corresponding Sarma state. Similarly to the 3D case [4], the energy gap Δ decreases and q_x increases when the polarization increases.

In Fig. 3, we show the Goldstone mode spectrum along the $(\pi, 0)$ direction when $f_\uparrow = 0.275$ and $f_\downarrow = 0.225$, $U/J = 2.64$, and $T/E_R = 2 \times 10^{-6}$. There are two

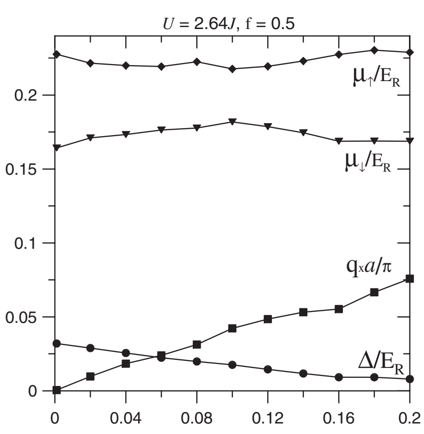

FIG. 2. The energy gap Δ, the magnitude of $\mathbf{q}$, and the chemical potentials $\mu_\uparrow$ and $\mu_\downarrow$ as functions of polarization P (solid lines are guides to the eyes). The total filling factor is $f = 0.5$. The temperature is $T/E_R = 2 \times 10^{-6}$. All other parameters are the same as in Fig. 1.

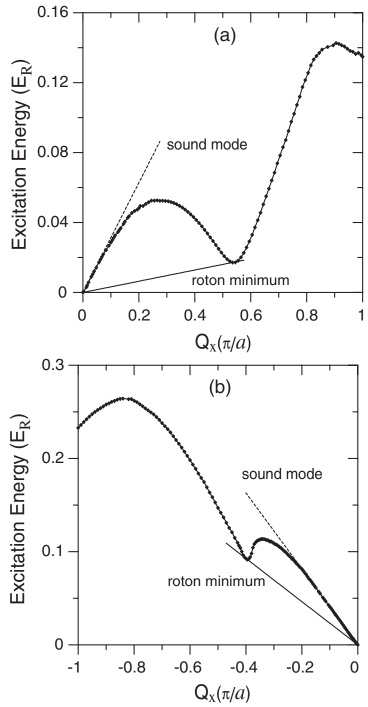

FIG. 3. The calculated excitation energy in 2D optical lattice along the $(Q_x, 0)$ direction for polarization $P = 0.1$ ($f_\uparrow = 0.275$ and $f_\downarrow = 0.225$). The interaction strength and the temperature are the same as in Fig. 1. (a) The slope of the curve at small Q determines the velocity of sound $c = 10$ mm/s. The roton minima corresponding to a speed of $v_r = 1$ mm/s. (b) $c = 13.1$ mm/s and $v_r = 7.5$ mm/s.

distinct sound velocities in the long wavelength limit (13.1 mm/s and 10 mm/s), as shown in Figs. 3(a) and 3(b). The rotonlike structure is clearly seen, and the minimum requirements on the flow velocities to be able to slow down (obtained from the two roton slopes) are 7.5 mm/s and 1 mm/s, respectively. The asymmetry of the sound mode and the roton minima originates from the fact that the population imbalance is achieved when either $\omega_+(\mathbf{k} + \mathbf{Q}, q_x)$ or $\omega_-(\mathbf{k} + \mathbf{Q}, q_x)$ is negative in some regions of momentum space, but the regions are different for positive and negative Q_x. The answer to the question of how this asymmetry is related to $f_\uparrow$, $f_\downarrow$, and U/J requires analytical expressions for the two regions. This ambitious task will be left as a subject of future research.

In summary, we applied the BS approach to the attractive Hubbard model to calculate the collective-mode spectrum of imbalanced Fermi gas in a deep optical lattice. Assuming a plane wave order parameter, we obtained a rotonlike spectrum of the Goldstone mode, which means the superfluidity can survive in polarized systems.

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[1] P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).
[2] A. I. Larkin and Y. N. Ovchinnikov, Zh. Eksp. Teor. Fiz., Pis'ma Red. **47**, 1136 (1964) [Sov. Phys. JETP **20**, 762 (1965)].
[3] P. Pieri, D. Neilson, and G. C. Strinati, Phys. Rev. B **75**, 113301 (2007); T. Hakioğlu and M. Şahin, Phys. Rev. Lett. **98**, 166405 (2007); T. Zhou and C. S. Ting, Phys. Rev. B **80**, 224515 (2009); Xian-Jun Zuo and Chang-De Gong, Europhys. Lett. **86**, 47004 (2009); H. Shimahara, Phys. Rev. B **80**, 214512 (2009); A. Romano *et al.*, Phys. Rev. B **81**, 064513 (2010); R. Ikeda, Phys. Rev. B **81**, 060510(R) (2010); M. M. Maśka *et al.*, Phys. Rev. B **82**, 054509 (2010).
[4] T. Koponen *et al.*, New J. Phys. **8**, 179 (2006).
[5] T. Koponen *et al.*, Phys. Rev. Lett. **99**, 120403 (2007).
[6] Tung-Lam Dao, A. Georges, and M. Capone, Phys. Rev. B **76**, 104517 (2007); Q. Chen *et al.*, Phys. Rev. B **75**, 014521 (2007); Xia-Ji Liu, H. Hu, and P. D. Drummond, Phys. Rev. A **76**, 043605 (2007); M. Rizzi *et al.*, Phys. Rev. B **77**, 245105 (2008); Xia-Ji Liu, Hui Hu, and P. D. Drummond, Phys. Rev. A **78**, 023601 (2008); M. Reza Bakhtiari, M. J. Leskinen, and P. Törma, Phys. Rev. Lett. **101**, 120404 (2008); A. Lazarides and B. Van Schaeybroeck, Phys. Rev. A **77**, 041602 (2008); T Paananen, J. Phys. B **42**, 165304 (2009); X. Cui and Y. Wang, Phys. Rev. B **79**, 180509(R) (2009); A. Mishra and H. Mishra, Eur. Phys. J. D **53**, 75 (2009); B. Wang, Han-Dong Chen, and S. Das Sarma, Phys. Rev. A **79**, 051604 (R) (2009); Y. Yanase, Phys. Rev. B **80**, 220510(R) (2009); A. Ptok, M. Máska, and M. Mierzejewski, J. Phys. Condens. Matter **21**, 295601 (2009); Yan Chen *et al.*, Phys. Rev. B **79**, 054512 (2009); Yen Lee Loh and N. Trivedi, Phys. Rev. Lett. **104**, 165302 (2010); A. Korolyuk, F. Massel, and P. Törma, Phys. Rev. Lett. **104**, 236402 (2010); F. Heidrich-Meisner *et al.*, Phys. Rev. A **81**, 023629 (2010); S. K. Baur, J. Shumway, and E. J. Mueller, Phys. Rev. A **81**, 033628 (2010); M. J. Wolak *et al.*, Phys. Rev. A **82**, 013614 (2010); L. Radzihovsky and D. Sheehy, Rep. Prog. Phys. **73**, 076501 (2010).
[7] J. M. Edge and N. R. Cooper, Phys. Rev. Lett. **103**, 065301 (2009); Phys. Rev. A **81**, 063606 (2010).
[8] A. Sedrakian and D. H. Rischke, Phys. Rev. D **80**, 074022 (2009).
[9] G. Sarma, J. Phys. Chem. Solids **24**, 1029 (1963).
[10] L. Belkhir and M. Randeria, Phys. Rev. B **49**, 6829 (1994).
[11] R. Cotê and A. Griffin, Phys. Rev. B **48**, 10404 (1993).
[12] Z. G. Koinov, Physica (Amsterdam) **470C**, 144 (2010); Phys. Status Solidi B **247**, 140 (2010); Ann. Phys. (Berlin) **522**, 693 (2010); arXiv:1010.1200.
[13] K. V. Samokhin, Phys. Rev. B **81**, 224507 (2010).
[14] Y.-P. Shim, R. A. Duine, and A. H. MacDonald, Phys. Rev. A **74**, 053602 (2006).