Quantum Reading of a Classical Digital Memory

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We consider a basic model of digital memory where each cell is composed of a reflecting medium with two possible reflectivities. By fixing the mean number of photons irradiated over each memory cell, we show that a nonclassical source of light can retrieve more information than any classical source. This improvement is shown in the regime of few photons and high reflectivities, where the gain of information can be surprising. As a result, the use of quantum light can have nontrivial applications in the technology of digital memories, such as optical disks and barcodes.

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In recent years, nonclassical states of radiation have been exploited to achieve marvellous results in quantum information and computation [1]. In the language of quantum optics, the bosonic states of the electromagnetic field are called ''classical'' when they can be expressed as probabilistic mixtures of coherent states. Classical states describe practically all the radiation sources which are used in today's technological applications. By contrast, a bosonic state is called ''nonclassical'' when its decomposition in coherent states is nonpositive [2,3]. One of the key properties which makes a state nonclassical is quantum entanglement. In the bosonic framework, this is usually present under the form of Einstein-Podolsky-Rosen (EPR) correlations, meaning that the position and momentum quadrature operators of two bosonic modes are so correlated as to beat the standard quantum limit [1]. This is a well-known feature of the two-mode squeezed vacuum (TMSV) state [1], one of the most important states routinely produced in today's quantum optics labs.

In this Letter, we show how the use of nonclassical light possessing EPR correlations can *widely* improve the readout of information from digital memories. To our knowledge, this is the first study which proves and quantifies the advantages of using nonclassical light for this fundamental task, being absolutely nontrivial to identify the physical conditions that can effectively disclose these advantages (as an example, see the recent no-go theorems of Ref. [4] applied to quantum illumination [5]). Our model of digital memory is simple but can potentially be extended to realistic optical disks, like CDs and DVDs, or other kinds of memories such as barcodes. In fact, we consider a memory where each cell is composed of a reflecting medium with two possible reflectivities, r_0 and r_1, used to store a bit of information. This memory is irradiated by a source of light which is able to resolve every single cell. The light focused on, and reflected from, a single cell is then measured by a detector, whose outcome provides the value of the bit stored in that cell. Besides the ''signal'' modes irradiating the target cell, we also consider the possible presence of ancillary ''idler'' modes which are directly sent to the detector. The general aim of these modes is to improve the performance of the output measurement by exploiting possible correlations with the signals. Adopting this model and fixing the mean number of photons irradiated over each memory cell, we show that a *nonclassical* source of light with EPR correlations between signals and idlers can retrieve more information than any classical source of light. In particular, this is proven for high reflectivities (typical of optical disks) and few photons irradiated. In this regime the difference of information can be surprising, up to 1 bit per cell (corresponding to the extreme situation where only quantum light can retrieve information). As we will discuss in the conclusion, the chance of reading information using few photons can have remarkable consequences in the technology of digital memories, e.g., in terms of data-transfer rates and storage capacities.

Let us consider a digital memory where each cell can have two possible reflectivities, r_0 or r_1, encoding the two values of a logical bit u [see Fig. 1]. Close to the memory,

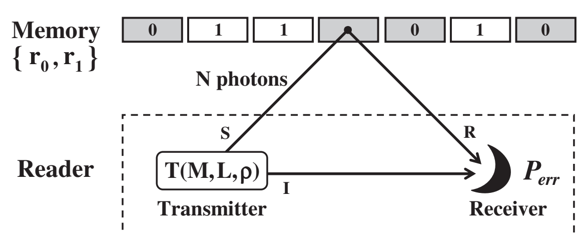

FIG. 1. Basic model of memory: Digital information is stored in a memory whose cells have different reflectivities: $r = r_0$ encoding bit value $u = 0$, and $r = r_1$ encoding bit value $u = 1$. Readout of the memory: In general, a digital reader consists of transmitter and receiver. The transmitter $T(M, L, \rho)$ is a bipartite bosonic system, composed by a signal system S (with M modes) and an idler system I (with L modes), which is given in some global state ρ. The signal S emitted by this source has bandwidth M and energy N (mean number of photons). The signal is directly shined over the cell, and its reflection R is detected together with the idler I at the output receiver, where a suitable measurement retrieves the value of the bit up to an error probability P_{err}.

we have a digital reader, made up of transmitter and receiver, whose goal is to retrieve the value of the bit stored in a target cell. In general, we call the "transmitter" a bipartite bosonic system, composed by a signal system S with M modes and an idler system I with L modes, and globally given in some state ρ. This source can be completely specified by the notation $T(M, L, \rho)$. By definition, we say that the transmitter T is classical (nonclassical) when the corresponding state ρ is classical (nonclassical), i.e., $T_c = T(M, L, \rho_c)$ and $T_{\rm nc} = T(M, L, \rho_{\rm nc})$. The signal S emitted by the transmitter is associated with two basic parameters: the number of modes M, that we call the "bandwidth" of the signal, and the mean number of photons N, that we call the "energy" of the signal [6]. The signal S is shined directly on the target cell, and its reflection R is detected together with the idler I at the output receiver. Here a suitable measurement yields the value of the bit up to an error probability $P_{\rm err}$. Repeating the process for each cell of the memory, the reader retrieves an average of $1 - H(P_{\rm err})$ bits per cell, where $H(\cdot)$ is the binary Shannon entropy.

The basic mechanism in our model of digital readout is quantum channel discrimination. In fact, encoding a logical bit $u \in \{0, 1\}$ in a pair of reflectivities $\{r_0, r_1\}$ is equivalent to encoding u in a pair of attenuator channels $\{\mathcal{E}(r_0), \mathcal{E}(r_1)\}$, with linear losses $\{r_0, r_1\}$ acting on the signal modes. The readout of the bit consists in the statistical discrimination between r_0 and r_1, which is formally equivalent to the channel discrimination between $\mathcal{E}(r_0)$ and $\mathcal{E}(r_1)$. The error probability affecting the discrimination $\mathcal{E}(r_0) \neq \mathcal{E}(r_1)$ depends on both transmitter and receiver. For a *fixed* transmitter $T(M, L, \rho)$, the pair $\{\mathcal{E}(r_0), \mathcal{E}(r_1)\}$ generates two possible output states at the receiver, $\sigma_0(T)$ and $\sigma_1(T)$. These are expressed by $\sigma_u(T) = [\mathcal{E}(r_u)^{\otimes M} \otimes \mathcal{I}^{\otimes L}](\rho)$, where $\mathcal{E}(r_u)$ acts on the signals and the identity $\mathcal{I}$ on the idlers. By optimizing over the output measurements, the minimum error probability which is achievable by the transmitter T in the channel discrimination $\mathcal{E}(r_0) \neq \mathcal{E}(r_1)$ is equal to $P_{\rm err}(T) = (1 - D)/2$, where D is the trace distance between $\sigma_0(T)$ and $\sigma_1(T)$. Now the crucial point is the minimization of $P_{\rm err}(T)$ over the transmitters T. Clearly, this optimization must be constrained by fixing basic parameters of the signal. Here we consider the most general situation where only the signal energy N is fixed. Under this energy constraint the optimal transmitter T which minimizes $P_{\rm err}(T)$ is unknown. For this reason, it is nontrivial to ask the following question: does a nonclassical transmitter which outperforms any classical one exist? In other words, given two reflectivities $\{r_0, r_1\}$, i.e., two attenuator channels $\{\mathcal{E}(r_0), \mathcal{E}(r_1)\}$, and a fixed value N of the signal energy, can we find any $T_{\rm nc}$ such that $P_{\rm err}(T_{\rm nc}) < P_{\rm err}(T_c)$ for every T_c? In the following we reply to this basic question, characterizing the regimes where the answer is positive. The first step in our derivation is providing a bound which is valid for every classical transmitter (see Ref. [7] for the proof).

Theorem 1 (classical discrimination bound).— Let us consider the discrimination of two reflectivities $\{r_0, r_1\}$ using a classical transmitter T_c which signals N photons. The corresponding error probability satisfies

$$P_{\rm err}(T_c) \geq \mathcal{C}(N, r_0, r_1) := \frac{1 - \sqrt{1 - e^{-N(\sqrt{r_1} - \sqrt{r_0})^2}}}{2}. \quad (1)$$

According to this theorem, all the classical transmitters T_c irradiating N photons on a memory with reflectivities $\{r_0, r_1\}$ cannot beat the classical discrimination bound $\mathcal{C}(N, r_0, r_1)$; i.e., they cannot retrieve more than $1 - H(\mathcal{C})$ bits per cell. Clearly, the next step is constructing a nonclassical transmitter which can violate this bound. A possible design is the "EPR transmitter," composed by M signals and M idlers, that are entangled pairwise via two-mode squeezing. This transmitter has the form $T_{\rm epr} = T(M, M, |\xi\rangle\langle\xi|^{\otimes M})$, where $|\xi\rangle\langle\xi|$ is a TMSV state entangling signal mode $s \in S$ with idler mode $i \in I$. In the number-ket representation $|\xi\rangle = (\cosh\xi)^{-1} \times \sum_{n=0}^{\infty} (\tanh\xi)^n |n\rangle_s |n\rangle_i$, where the squeezing parameter ξ quantifies the signal-idler entanglement. An arbitrary EPR transmitter, composed by M copies of $|\xi\rangle\langle\xi|$, irradiates a signal with bandwidth M and energy $N = M\sinh^2\xi$. As a result, this transmitter can be completely characterized by the basic parameters of the emitted signal; i.e., we can set $T_{\rm epr} = T_{M,N}$. Then, let us consider the discrimination of two reflectivities $\{r_0, r_1\}$ using an EPR transmitter $T_{M,N}$ which signals N photons. The corresponding error probability is upper bounded by the quantum Chernoff bound [8]

$$P_{\rm err}(T_{M,N}) \leq \mathcal{Q}(M, N, r_0, r_1) := \frac{1}{2}\left[\inf_{t \in (0,1)} {\rm Tr}\left(\theta_0^t \theta_1^{1-t}\right)\right]^M, \quad (2)$$

where $\theta_u := [\mathcal{E}(r_u) \otimes \mathcal{I}](|\xi\rangle\langle\xi|)$. In other words, at least $1 - H(\mathcal{Q})$ bits per cell can be retrieved from the memory. Exploiting Eqs. (1) and (2), our main question simplifies to finding $\bar{M}$ such that $\mathcal{Q}(\bar{M}, N, r_0, r_1) < \mathcal{C}(N, r_0, r_1)$. In fact, this implies $P_{err}(T_{\bar{M},N}) < \mathcal{C}(N, r_0, r_1)$, i.e., the existence of an EPR transmitter $T_{\bar{M},N}$ able to outperform any classical transmitter T_c. This is the result of the following theorem (see Ref. [7] for the proof).

Theorem 2 (threshold energy).—For every pair of reflectivities $\{r_0, r_1\}$ with $r_0 \neq r_1$, and signal energy

$$N > N_{\rm th}(r_0, r_1) := \frac{2\ln 2}{2 - r_0 - r_1 - 2\sqrt{(1 - r_0)(1 - r_1)}}, \quad (3)$$

there is an $\bar{M}$ such that $P_{\rm err}(T_{\bar{M},N}) < \mathcal{C}(N, r_0, r_1)$.

Thus we get the central result of the Letter: for every memory and above a threshold energy, there is an EPR transmitter which outperforms any classical transmitter. Remarkably, the threshold energy $N_{\rm th}$ turns out to be low

$(<10^2)$ for most of the memories $\{r_0, r_1\}$ outside the region $r_0 \approx r_1$. This means that we can have an enhancement in the regime of few photons ($N < 10^2$). Furthermore, for low energy N, the critical bandwidth $\bar{M}$ can be low, too. In other words, in the regime of few photons, narrow band EPR transmitters are generally sufficient to overcome every classical transmitter. To confirm and quantify this analysis, we introduce the "minimum information gain" $G(M, N, r_0, r_1) := 1 - H(\mathcal{Q}) - [1 - H(\mathcal{C})]$. For given memory $\{r_0, r_1\}$ and signal energy N, this quantity lower bounds the number of bits per cell which are gained by an EPR transmitter $T_{M,N}$ over any classical transmitter T_c [9]. Numerical investigations [see Fig. 2] show that narrow band EPR transmitters are able to give $G > 0$ in the regime of few photons and high reflectivities, corresponding to having r_0 or r_1 sufficiently close to 1 (as typical of optical disks). In this regime, part of the memories display remarkable gains ($G > 0.5$).

Thus the enhancement provided by quantum light can be dramatic in the regime of few photons and high reflectivities. To investigate more closely this regime, we consider the case of ideal memories, defined by $r_0 < r_1 = 1$. As an analytical result, we have the following [7].

Theorem 3 (ideal memory).—For every $r_0 < r_1 = 1$ and $N \geq N_{\mathrm{th}} := 1/2$, there is a minimum bandwidth $\bar{M}$ such that $P_{\mathrm{err}}(T_{M,N}) < \mathcal{C}(N, r_0, r_1)$ for every $M > \bar{M}$.

Thus, for ideal memories and signals above $N_{\mathrm{th}} = 1/2$ photon, there are infinitely many EPR transmitters able to outperform every classical transmitter. For these memories, the threshold energy is so low that the regime of few photons can be fully explored. The gain G increases with the bandwidth, so that optimal performances are reached by broadband EPR transmitters ($M \to \infty$). However, narrow band EPR transmitters are sufficient to give remarkable advantages, even for $M = 1$ (i.e., using a single TMSV state). This is shown in Fig. 3, where G is plotted in terms of r_0 and N, considering the two extreme cases $M = 1$ and $M \to \infty$. According to Fig. 3, the value of G can approach 1 for ideal memories and few photons even if we consider narrow band EPR transmitters.

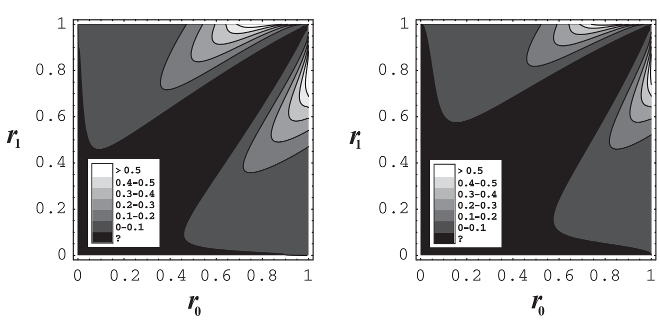

FIG. 2. Left panel: Minimum information gain G over the memory plane $\{r_0, r_1\}$. For a few-photon signal ($N = 30$), we compare a narrow band EPR transmitter ($M = 30$) with all the classical transmitters. Inside the black region ($r_0 \approx r_1$) our investigation is inconclusive. Outside the black region, we have $G > 0$. Right panel: G plotted over the plane $\{r_0, r_1\}$ in the presence of decoherence ($\varepsilon = \bar{n} = 10^{-5}$). For a few-photon signal ($N = 30$), we compare a narrow band EPR transmitter ($M = 30$) with all the classical transmitters $T(M, L, \rho_c)$ having $M \leq M^* = 5 \times 10^6$.

Presence of decoherence.—Note that the previous analysis does not consider the presence of thermal noise. Actually this is a good approximation in the optical range, where the number of thermal background photons is around 10^{-26} at about 1 μm and 300 K. However, to complete the analysis, we now show that the quantum effect exists even in the presence of stray photons hitting the upper side of the memory and decoherence within the reader. The scattering is modeled as white thermal noise with $\bar{n}$ photons per mode entering each memory cell. Numerically we consider $\bar{n} = 10^{-5}$ corresponding to nontrivial diffusion. This scenario may occur when the light, transmitted through the cells, is not readily absorbed by the drive (e.g., using a bucket detector just above the memory) but travels for a while diffusing photons which hit neighboring cells. Assuming the presence of one photon per mode traveling the "optimistic distance" of 1 m and undergoing Rayleigh scattering, we get roughly $\bar{n} \simeq 10^{-5}$ [10]. The internal decoherence is modeled as a thermal channel $\mathcal{N}(\varepsilon)$ adding Gaussian noise of variance ε to each signal and reflected mode, and 2ε to the each idler mode (numerically we consider the nontrivial value $\varepsilon = \bar{n} = 10^{-5}$). Now, distinguishing between two reflectivities $\{r_0, r_1\}$ corresponds to discriminating between two Gaussian channels $\mathcal{S}_u \otimes \mathcal{N}(2\varepsilon)$ for $u \in \{0, 1\}$. Here $\mathcal{S}_u := \mathcal{N}(\varepsilon) \circ \mathcal{E}(r_u, \bar{n}) \circ \mathcal{N}(\varepsilon)$ acts on each signal mode, and contains the attenuator channel $\mathcal{E}(r_u, \bar{n})$ with conditional loss r_u and thermal noise $\bar{n}$. To solve this scenario we use Theorem 1 with the proviso of generalizing the classical discrimination bound. In general, we have $\mathcal{C} = (1 - \sqrt{1 - F^M})/2$, where F is the fidelity between $\mathcal{S}_0(|\sqrt{n_S}\rangle\langle\sqrt{n_S}|)$ and $\mathcal{S}_1(|\sqrt{n_S}\rangle \times \langle\sqrt{n_S}|)$, the two outputs of a single-mode coherent state $|\sqrt{n_S}\rangle$ with $n_S := N/M$ mean photons. Here the expression for $\mathcal{C}$ depends also on the bandwidth M of the classical

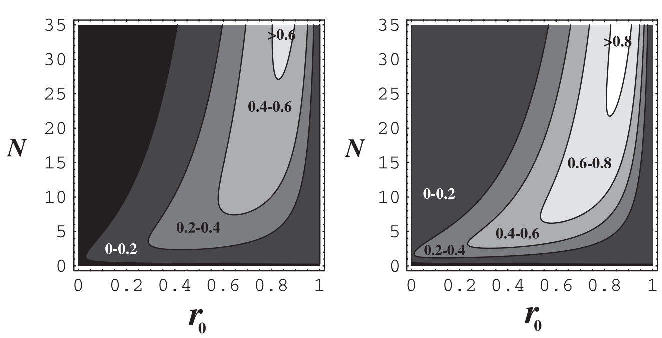

FIG. 3. Minimum information gain G versus r_0 and N. Left picture refers to $M = 1$, right picture to $M \to \infty$. (For arbitrary M the scenario is intermediate.) Outside the inconclusive black region we have $G > 0$. For $M \to \infty$ the black region is completely collapsed below $N_{\mathrm{th}} = 1/2$.

transmitter $T_c = T(M, L, \rho_c)$. Since $\mathcal{C}$ decreases to zero for $M \rightarrow \infty$, our quantum-classical comparison is now restricted to classical transmitters $T(M, L, \rho_c)$ with M less than a maximal value $M^* < \infty$. Remarkably we find that, in the regime of few photons and high reflectivities, narrow band EPR transmitters are able to outperform all the classical transmitters up to an extremely large bandwidth M^*. This is confirmed by the numerical results of Fig. 2, proving the robustness of the quantum effect $G > 0$ in the presence of decoherence. Note that we can neglect classical transmitters with extremely large bandwidths (i.e., with $M > M^*$) since they are not meaningful for the model. In fact, in a practical setting, the signal is an optical pulse with carrier frequency ν high enough to completely resolve the target cell. This pulse has frequency bandwidth $w \ll \nu$ and duration $\tau \simeq w^{-1}$. Assuming an output detector with response time $\delta t \lesssim \tau$ and "reading time" $t > \tau$, the number of modes which are excited is roughly $M = wt$. In other words, the bandwidth of the signal M is the product of its frequency bandwidth w and the reading time of the detector t. Now, the limit $M \rightarrow \infty$ corresponds to $\delta t \rightarrow 0$ (infinite detector resolution) or $t \rightarrow \infty$ (infinite reading time). As a result, transmitters with too large an M can be discarded.

Suboptimal receiver.—The former results are valid assuming optimal output detection. Here we show an explicit receiver design which is (i) easy to construct and (ii) able to approximate the optimal results. This suboptimal receiver consists of a continuous variable Bell measurement (i.e., a balanced beam splitter followed by two homodyne detectors) whose output is classically processed by a suitable χ^2 test with significance level φ (see Ref. [7] for details). In this case the information gain G can be optimized jointly over the signal bandwidth M (i.e., the number of input TMSV states) and the significance level of the output test φ. As shown in Fig. 4, the advantages of quantum reading are fully preserved.

Error correction.—In our basic model of memory we store 1 bit of information per cell. In an alternative model, information is stored in block of cells by using error correcting codes, so that the readout of data is practically flawless. In this configuration, we show that the error correction overhead which is needed by EPR transmitters can be made very small. By contrast, classical transmitters are useless since they may require more than 100 cells for retrieving a single bit of information in the regime of few photons (see Ref. [7] for details).

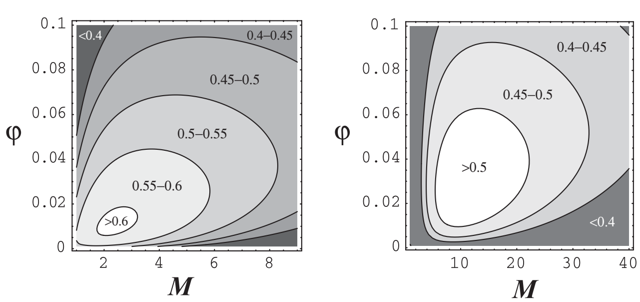

FIG. 4. Left panel: G optimized over M and φ. G can be higher than 0.6 bit per cell. Results are shown in the absence of decoherence ($\varepsilon = \bar{n} = 0$) considering $r_0 = 0.85$, $r_1 = 1$ and $N = 35$. Right panel: G optimized over M and φ. Results are shown in the presence of decoherence ($\bar{n} = \varepsilon = 10^{-5}$) considering $r_0 = 0.85$, $r_1 = 0.95$, $N = 100$, and $M^* = 10^6$.

Conclusion.—Quantum reading is able to work in the regime of few photons. What does it imply? Using fewer photons means that we can reduce the reading time of the cell, thus accessing higher data-transfer rates. This is a theoretical prediction that can be checked with a pilot experiment [7]. Alternatively, we can fix the total reading time of the memory while increasing its storage capacity [7]. The chance of using few photons leads to another interesting application: the safe readout of photodegradable memories, such as dye-based optical disks or photosensitive organic microfilms (e.g., containing confidential information). Here faint quantum light can retrieve the data safely, whereas classical light could only be destructive. More fundamentally, our results apply to the binary discrimination of attenuator channels.

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[2] For any K-mode bosonic state ρ, we can write the $\mathcal{P}$ representation $\rho = \int d^{2K}\boldsymbol{\alpha}\mathcal{P}(\boldsymbol{\alpha})|\boldsymbol{\alpha}\rangle\langle\boldsymbol{\alpha}|$, where $\mathcal{P}(\boldsymbol{\alpha})$ is normalized to 1 and $|\boldsymbol{\alpha}\rangle\langle\boldsymbol{\alpha}|$ is a multimode coherent state. Then, ρ is called "classical" ("nonclassical") if $\mathcal{P}(\boldsymbol{\alpha})$ is positive (nonpositive) [3]. For ρ classical, $\mathcal{P}(\boldsymbol{\alpha})$ is a probability distribution (implying separability).

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[6] In our Letter, N is always a *mean* quantity (averaged over the state). It represents the mean number of photons in the whole M-mode signal system. Clearly, the ratio N/M gives the mean number of photons per signal mode.

[7] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.106.090504.

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[9] $G > 0$ is a sufficient condition for the superiority of quantum reading. $G = 1$ corresponds to the singular case where only quantum light can retrieve information.

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