Covariant Closed String Coherent States

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We give the first construction of covariant coherent closed string states, which may be identified with fundamental cosmic strings. We outline the requirements for a string state to describe a cosmic string, and provide an explicit and simple map that relates three different descriptions: classical strings, light cone gauge quantum states, and covariant vertex operators. The resulting coherent state vertex operators have a classical interpretation and are in one-to-one correspondence with arbitrary classical closed string loops.

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The construction of covariant closed string coherent states with an arbitrary distribution of harmonics has been sought after for many years. The naive construction based on oscillator coherent states does not lead to physical states [1,2]. Ways out include a nonstandard gauge fixing [2] and a nonstandard light cone gauge quantization [3]. See also [4,5]. With the realization that cosmic superstrings may lead to observational signatures for string theory the necessity of understanding macroscopic string states with a classical interpretation has become of paramount importance. Cosmic superstrings are expected to be produced in the early Universe at the end of (e.g., D3-$\overline{\mathrm{D3}}$) brane inflation, in, e.g., models with warped throats ($\mathbb{KLMT}$) or large compact dimensions (see, e.g., [6,7], and references therein).

Almost all predictions to date concerning cosmic superstrings are either classical and neglect effects of gravitational backreaction (which can be important even for order of magnitude estimates [8]), involve cosmic strings in their vacuum state (with no harmonics excited) [9], or involve mass eigenstates (with only first harmonics excited) [10–14] which are not expected to reproduce the classical evolution [2,12]. These computations need to be extended to more realistic cosmic superstrings and in what follows we discuss the first construction of a closed string covariant coherent state [15] with arbitrarily excited harmonics, a large fundamental cosmic string loop. Further details and the corresponding open string construction are presented in a companion paper [16].

Classically, a cosmic string with position $X^\mu_{\rm cl}$ depending on world sheet coordinates z, $\bar{z}$ (see [17]) evolves according to the equations of motion and constraints [18], $\partial\bar{\partial}X^\mu_{\rm cl}=0$, $(\partial X_{\rm cl})^2=(\bar{\partial}X_{\rm cl})^2=0$. Explicit solutions are easily obtained in light cone gauge where one takes $X^+_{\rm cl}=2p^+\tau$, and for the transverse directions one finds

$$X^i_{\rm cl}(z,\bar{z})-x^i=-ik^i\ln|z|^2+i\sum_{n\neq0}\frac{1}{n}(\xi^i_n z^{-n}+\bar{\xi}^i_n\bar{z}^{-n}). \quad (1)$$

In string theory cosmic strings are described by vertex operators. These are composed of the fields present in the theory, $X(z,\bar{z})$ and $g_{\alpha\beta}(z,\bar{z})$. Because of conformal invariance the explicit dependence on $g_{\alpha\beta}$ drops out [19,20], states in the underlying Hilbert space transform like one-particle states under Poincaré transformations [21], and therefore normal ordered closed string vertices are of the form $V(z,\bar{z})=\sum_\alpha\mathcal{P}_\alpha[\partial^\# X]e^{ik_L^{(\alpha)}X(z)}\bar{\mathcal{P}}_\alpha[\bar{\partial}^\# X]e^{ik_R^{(\alpha)}X(\bar{z})}$, with $\mathcal{P}_\alpha$, $\bar{\mathcal{P}}_\alpha$ (to be determined) polynomials and $k_L^{(\alpha)}$, $k_R^{(\alpha)}$ left- and right-moving momenta associated to the momentum eigenstate α. We wish to derive the explicit form of $V(z,\bar{z})$ and to do so we search for vertex operators which (a) transform correctly under all symmetries of string theory, (b) ideally possess spacetime covariance, (c) are macroscopic and massive, (d) possess classical expectation values, e.g., $\langle X^\mu\rangle=X^\mu_{\rm cl}$, $\langle J^{\mu\nu}\rangle=J^{\mu\nu}_{\rm cl}$, provided these are compatible with (a), and (e) have small uncertainty in momentum and position (relative to the center of mass). Requirement (a) is dictated by string theory, while (b) is preferred for compatibility with standard string technology (e.g., [20,22]) for string amplitude computations. Requirements (c)–(e) would be our targets for a quantum state most closely approximating a large classical string.

Let us elaborate on (d). Recall that $L_0^\perp-\bar{L}_0^\perp$ generates rigid spacelike world sheet translations [23], $\sigma\to\sigma+\epsilon$, so that $\langle V|[L_0^\perp-\bar{L}_0^\perp,X^i]|V\rangle=\langle V|\partial_\sigma X^i|V\rangle$, with $L_0^\perp$, $\bar{L}_0^\perp$ the transverse Virasoro generators (defined below). As pointed out in [2], we see that states invariant under shifts, $e^{-i\epsilon(L_0^\perp-\bar{L}_0^\perp)}|V\rangle=|V\rangle$, obey $\partial_\sigma\langle X^i\rangle=0$, implying that $\langle X^\mu\rangle=X^\mu_{\rm cl}$ in (d) cannot be realized. This is, nevertheless, a good condition for classicality when $(L_0^\perp-\bar{L}_0^\perp)|V\rangle\neq0$ and $\langle X^i\rangle$ is evaluated in light cone gauge and we will see that this is only possible when the underlying spacetime manifold is compactified in a lightlike direction, $X^-\sim X^-+2\pi R^-$.

For light cone or covariant gauge states that do not live in a null-compactified background [which satisfy $(L_0^\perp-\bar{L}_0^\perp)|V\rangle=0$ or $(L_0-\bar{L}_0)|V\rangle=0$, respectively], the fact that $\langle X\rangle\neq X_{\rm cl}$ is a gauge problem and says nothing about the classicality of the corresponding quantum states. That this is a gauge problem was shown in [2], by adding a gauge fixing term, $\lambda(c-\oint d\!\!{}^-z z\partial_z X^1)$ with c a c number and

λ a Lagrange multiplier, to the Polyakov action. This fixes the remaining world sheet translation invariance and enforces the constraint $(c - \alpha_1^1)|V\rangle = 0$. The Virasoro constraint $L_0 - \bar{L}_0 = 0$ determines α_{-1}^1 in terms of the remaining (in light cone gauge, transverse) oscillators. This enables one to construct eigenstates of the annihilation operators (out of the *remaining* oscillators), for which $\langle X^\mu \rangle = X_{\rm cl}^\mu$, which need not be manifestly level matched, thus proving that the argument that led to $\langle X^\mu \rangle \neq X_{\rm cl}^\mu$ is a gauge problem; see also [3]. This procedure is somewhat messy. Instead, working in light cone or covariant gauge we shall replace the classicality condition $\langle X^\mu \rangle = X_{\rm cl}^\mu$ in (d) with

$$\langle X^i(\sigma', \tau) X^j(\sigma, \tau)\rangle = \int_0^{2\pi} đs X_{\rm cl}^i(\sigma' - s, \tau) X_{\rm cl}^j(\sigma - s, \tau), \tag{2}$$

modulo zero mode contributions, with $X_{\rm cl}^i$ defined in (1), X^i given by a similar expression with operators α_n^i, $\tilde{\alpha}_n^i$, $\hat{x}^i$, $\hat{p}^i$ replacing ξ_n^i, $\bar{\xi}_n^i$, x^i, k^i and $đs \equiv ds/(2\pi)$. Rather than fixing the invariance under σ translations on the quantum side (as done in [2]) we average over σ translations on the classical side.

We first construct states which satisfy the requirements (a)–(e). If we proceed by analogy to the harmonic oscillator coherent states $e^{\lambda a^\dagger}|0\rangle$ (with $a|0\rangle = 0$ and $[a, a^\dagger] = 1$) which have classical expectation values, $\partial_t^2 \langle x(t)\rangle = -\omega^2 \langle x(t)\rangle$, and consider the naive closed string state $V \sim e^{\lambda_n \alpha_{-n}} e^{\bar{\lambda}_n \tilde{\alpha}_{-n}} e^{ikX(z,\bar{z})}$ we find that the Virasoro constraints are not satisfied [1]. One possibility is to work in light cone gauge where the Virasoro constraints are automatically satisfied. Rather than drop spacetime covariance our approach will be to make use of the spectrum generating Del Guidice, Di Vecchia, and Fubini (DDF) operators [24,25] which can be used to generate covariant [18,26] physical states.

The DDF operators, A_n^i, $\bar{A}_n^i$, satisfy an oscillator algebra, $[A_n^i, A_m^j] = n\delta^{ij}\delta_{n+m,0}$, in direct analogy to $[\alpha_n^i, \alpha_m^j] = n\delta^{ij}\delta_{n+m,0}$. Explicitly,

$$A_n^i = \oint đz \partial X^i e^{{\rm inq}X(z)}, \qquad \bar{A}_n^i = \oint đ\bar{z}\, \bar{\partial}\, X^i e^{{\rm inq}X(\bar{z})}. \tag{3}$$

Indices i are transverse to the null vector q^μ, $q^2 \equiv 0$. Vertex operators, $V(z, \bar{z})$, have the correct symmetries provided [27] they are annihilated by $L_{n>0}$, $\bar{L}_{n>0}$, $(L_0 - 1)$, and $(\bar{L}_0 - 1)$ Virasoro generators. The DDF operators are gauge invariant, $[L_n, A_m^i] = 0$, and so given a physical vacuum, $e^{ipX(z,\bar{z})}$, for which $Qe^{ipX(z,\bar{z})} \cong 0 \cong A_{n>0}^i e^{ipX(z,\bar{z})}$, vertex operators of the form $\xi_{i\ldots}\bar{\xi}_{j\ldots} A_{-n}^i \ldots \bar{A}_{-\bar{n}}^j \ldots e^{ipX(z,\bar{z})}$, are physical and covariant provided $\xi_{\ldots i\ldots} q^i = \bar{\xi}_{\ldots i \ldots} q^i = 0$ and,

$$pq = 1, \quad p^2 = 2, \quad \text{and} \quad q^2 = 0. \tag{4}$$

Such vertex operators are transverse to null states (see, e.g., [18]) and represent a complete set [26] of covariant vertex operators.

The equivalent light cone gauge states are obtained by [26] the mapping $A_{-n}^i \to \alpha_{-n}^i$ and $e^{ipX(z,\bar{z})} \to |p^+, p^i\rangle$, with $|p^+, p^i\rangle$ an eigenstate of $\hat{p}^+$, $\hat{p}^i$ and annihilated by the lowering operators $\alpha_{n>0}^i$, $\tilde{\alpha}_{n>0}^i$. Here the constraints $(\partial X)^2 = (\bar{\partial} X)^2 = 0$ imply the operator equations

$$p^+ \alpha_0^- = L_0^\perp - 1, \qquad p^+ \tilde{\alpha}_0^- = \bar{L}_0^\perp - 1, \tag{5}$$

with $L_0^\perp$, $\bar{L}_0^\perp$ the transverse Virasoro generators, $L_0^\perp = \frac{1}{2}\hat{p}^i\hat{p}^i + N^\perp$, $\bar{L}_0^\perp = \frac{1}{2}\hat{p}^i\hat{p}^i + \bar{N}^\perp$, and $N^\perp = \sum_{n>0}\alpha_{-n}^i\alpha_n^i$, $\bar{N}^\perp = \sum_{n>0}\tilde{\alpha}_{-n}^i\tilde{\alpha}_n^i$. Recall that $(L_0^\perp - \bar{L}_0^\perp)$ generates spacelike world sheet shifts. From (5) it follows that $(\alpha_0^- - \tilde{\alpha}_0^-)|V\rangle = \frac{1}{p^+}(L_0^\perp - \bar{L}_0^\perp)|V\rangle$, and so as α_0^- and $\tilde{\alpha}_0^-$ are the left- and right-moving momentum operators, $\hat{p}_L^-$ and $\hat{p}_R^-$, respectively, the light cone gauge state is only invariant under shifts, $(L_0^\perp - \bar{L}_0^\perp)|V\rangle = 0$, when the corresponding eigenvalues, $k_{L,R}^-$, are equal.

The map between the DDF operators and the light cone oscillators suggests that we can define a gauge invariant "position operator" [28],

$$\mathsf{X}^i(z,\bar{z}) - \hat{\mathsf{x}}^i = -\hat{p}^i \ln|z|^2 + i\sum_{n\neq 0}\frac{1}{n}(A_n^i z^{-n} + \bar{A}_n^i \bar{z}^{-n}), \tag{6}$$

with $= A_0^i = \alpha_0^i$, $\hat{\mathsf{x}}^i = q_\mu J^{i\mu}$ and the angular momentum $J^{\mu\nu} = \oint đz X^{[\mu}\partial X^{\nu]} - \oint đ\bar{z} X^{[\mu}\bar{\partial} X^{\nu]}$, integrals being along a spacelike curve, $|z|^2 = 1$, and $a^{[\mu\nu]} = \frac{1}{2}(a^{\mu\nu} - a^{\mu\nu})$. Writing $\mathsf{X}^i(z,\bar{z}) = \mathsf{X}^i(z) + \mathsf{X}^i(\bar{z})$ and $\hat{\mathsf{x}}^i = \hat{\mathsf{x}}_L^i + \hat{\mathsf{x}}_R^i$, this satisfies $[L_n, \mathsf{X}^i(z)] = 0$ for all n, $[\mathsf{X}^i(z), \partial_\tau \mathsf{X}^j(z')] = \delta^{ij}\delta(\sigma - \sigma')$ and similarly for the antiholomorphic piece. Furthermore, $[\hat{\mathsf{x}}^i, \hat{p}^j] = i\delta^{ij}$. Equation (6) is not essential for what follows but is useful because (reasonably behaved) functionals $F[A]$, satisfy

$$\langle F[\mathsf{X}^i(z,\bar{z}) - \hat{\mathsf{x}}^i]\rangle_{\rm cov} = \langle F[X^i(z,\bar{z}) - x^i]\rangle_{\rm lc}, \tag{7}$$

which follows from the isomorphism of light cone (in terms of the α_n^i, $\tilde{\alpha}_n^i$) and covariant states (in terms of the A_n^i, $\bar{A}_n^i$), the isomorphism of the light cone gauge and gauge invariant position operators, the isomorphism of the corresponding oscillator algebras and finally the fact that the light cone and covariant states are equivalent.

Now, a candidate vertex operator to describe bosonic cosmic string loops is the following:

$$V(\lambda, \bar{\lambda}) = C\exp\left\{\sum_{n=1}^{\infty}\frac{1}{n}\lambda_n A_{-n}\right\} \times \exp\left\{\sum_{m=1}^{\infty}\frac{1}{m}\bar{\lambda}_m \bar{A}_{-m}\right\} e^{ipX(z,\bar{z})}, \tag{8}$$

with $(\lambda, \bar{\lambda}) = \{\lambda_n^i, \bar{\lambda}_n^i\}$ and $C = e^{-\sum_{n=1}^{\infty}\times((1/2n)|\lambda_n|^2 + (1/2n)|\bar{\lambda}_n|^2)}$ a normalization constant. The polarization tensors λ_n^i, $\bar{\lambda}_n^i$ are such that $\bar{\lambda}_n q = \lambda_n q = 0$.

The string theory requirements [see (a),(b) above] are satisfied because any combination of DDF operators on the vacuum yields covariant vertex operators which satisfy the Virasoro constraints. The cosmic string requirements (c)–(e) above are also satisfied: $V(\lambda, \bar{\lambda})$ is an eigenstate of the annihilation operator, $A^i_{n>0}V \cong \lambda^i_n V$, and hence both $\langle \mathsf{X}^i(\sigma, \tau) - \hat{\mathsf{x}}^i \rangle_{\rm cov}$ and $\langle X^i(\sigma, \tau) - x^i \rangle_{\rm lc}$ on account of (7) are identical to (1) with λ^i_n, $\bar{\lambda}^i_n$ replacing ξ^i_n, $\bar{\xi}^i_n$. From the standard coherent state properties it follows that choosing the $|\lambda_n|$, $|\bar{\lambda}_n|$ appropriately (large) ensures that the cosmic string requirements are satisfied.

The normal ordered version of (8) assumes a simple form when $\lambda_n\lambda_m = \bar{\lambda}_n\bar{\lambda}_m = 0$ (as appropriate for the Burden solutions [29]), in a frame where $\lambda_n p = \bar{\lambda}_n p = 0$ (otherwise see [16]), $V(\lambda, \bar{\lambda}) = C \exp\{\sum_{n=1}^{\infty} \frac{1}{n} \lambda_n P_n(z) e^{-{\rm inq}X(z)}\} \exp\{\sum_{m=1}^{\infty} \frac{1}{m} \bar{\lambda}_m \bar{P}_m(\bar{z}) e^{-{\rm imq}X(\bar{z})}\} e^{ipX(z,\bar{z})}$, with $P^i_n(z)$, $\bar{P}^i_n(\bar{z})$ related to elementary Schur polynomials, see (12). This expression follows from bringing the DDF operators close to the vacuum and carrying out the corresponding contour integrals [16].

A series expansion of the exponentials shows that we are in fact superimposing momentum eigenstates with (in general) asymmetric left-right momenta, $k^\mu_L - k^\mu_R = wq^\mu$, with winding number $w = N - \bar{N}$ and $q^2 = 0$. Nonzero w and null q^μ implies that the underlying spacetime manifold is null compactified. Any choice of q^μ is permitted provided (4) and $\bar{\lambda}_n q = \lambda_n q = 0$ are satisfied. We choose $q^+ = q^i = 0$ and $q^- = -R^-$ which implies the identification (with X^+ noncompact):

$$X^- \sim X^- + 2\pi R^-.$$

In a frame where $k^i = 0$, the constraints (4) lead to $k^0 = \frac{1}{\sqrt{2}}(\frac{1}{R^-} + \frac{m^2 R^-}{2})$, $k^D = \frac{1}{\sqrt{2}}(\frac{1}{R^-} - \frac{m^2 R^-}{2})$, with $k^\mu = \frac{1}{2}(k_L + k_R)^\mu$ and mass squared $m^2 = N + \bar{N} - 2$. The full vertex, $V(\lambda, \bar{\lambda})$, has an effective mass given by $\langle m^2 \rangle = \langle N \rangle + \langle \bar{N} \rangle - 2$, with $\langle N \rangle = \sum_{n=1}^{\infty} |\lambda_n|^2$ and $\langle \bar{N} \rangle = \sum_{n=1}^{\infty} |\bar{\lambda}_n|^2$. There are similar expressions to k^0, k^D for $\langle k^0 \rangle$, $\langle k^D \rangle$ with $\langle m^2 \rangle$ replacing m^2. Note that (i) the mass spectrum is as in the noncompact case, $m^2 = N + \bar{N} - 2$, but with N not necessarily equal to $\bar{N}$, (ii) the string only fluctuates in directions transverse to the null direction (as $\lambda_n q = 0$) implying that the various geometrical features of the string (such as cusps) are not affected by the compactification, (iii) $\langle X^\mu(\sigma + 2\pi, \tau) \rangle = \langle X^\mu(\sigma, \tau) \rangle$ for $\mu = (\pm, i)$ when $\langle N \rangle = \langle \bar{N} \rangle$, implying that classically compact and noncompact X^- are indistinguishable. At the quantum level, scattering amplitudes are expected to be affected by the lightlike compactification in general and will, in particular, contain additional terms when compared to the corresponding amplitude computations involving the projected states (defined below). Quantizing on a null compact background is known as discrete light cone quantization (DLCQ) [30], and is a crucial component in the M (matrix) theory to string theory correspondence [31].

Note that in light cone gauge, $\langle X^i \rangle_{\rm lc} = X^i_{\rm cl}$, for arbitrary classical solutions. In covariant gauge, however, $\langle X^i \rangle_{\rm cov} \neq X^i_{\rm cl}$, as the covariant vertex (8) is still invariant under shifts.

Although the above states (8) satisfy the requirements (a)–(e), the necessity of a null-compactified spacetime manifold is perhaps too constraining, because this breaks four-dimensional Lorentz invariance. When the spacetime background is not compactified in a lightlike direction, only states with $k^-_L = k^-_R$ can propagate consistently. We therefore next discuss the construction of cosmic strings in noncompact spacetimes.

Define a projection operator, $G_w = \int_0^{2\pi} d\!\!\!{}^-\! s\, e^{is(\hat{W} - w)}$, with $\hat{W} = \hat{p}^+_L \hat{p}^-_L - \hat{p}^+_R \hat{p}^-_R$, and $\hat{p}^\mu_L = \oint d\!\!\!{}^-\! z \partial X^\mu$, $\hat{p}^\mu_R = -\oint d\!\!\!{}^-\! \bar{z}\, \bar{\partial} X^\mu$. $\hat{W}$ is the null winding number operator. This satisfies $G_n G_m = \delta_{n,m} G_n$ and when applied to arbitrary vertices projects out all states in the underlying Hilbert space except for those with null winding number w. When there are no transverse compact directions, $\hat{W} = -p(\hat{p}_L - \hat{p}_R)$, with p^μ defined in (4). Covariant vertex operators in noncompact spacetimes are therefore given by $V_0(\lambda, \bar{\lambda}) \cong G_0 V(\lambda, \bar{\lambda})$. With $V(\lambda, \bar{\lambda})$ as given in (8), we commute G_0 through the DDF operators and find that

$$V_0(\lambda, \bar{\lambda}) = \mathcal{C}_{\lambda\bar{\lambda}} \int_0^{2\pi} d\!\!\!{}^-\! s \exp\left\{\sum_{n=1}^{\infty} \frac{1}{n} \zeta_n(s) A_{-n}\right\} \times \exp\left\{\sum_{m=1}^{\infty} \frac{1}{m} \bar{\zeta}_m(s) \bar{A}_{-m}\right\} e^{ipX(z,\bar{z})}, \qquad (9)$$

with $\zeta^i_n(s) \equiv \lambda^i_n e^{\rm ins}$, $\bar{\zeta}^i_n(s) \equiv \bar{\lambda}^i_n e^{-\rm ins}$, and the normalization constant $\mathcal{C}_{\lambda\bar{\lambda}} = [\int_0^{2\pi} d\!\!\!{}^-\! s \exp(\sum_{n=1}^{\infty} \frac{1}{n} |\lambda_n|^2 e^{\rm ins} + \frac{1}{n} |\bar{\lambda}_n|^2 e^{-\rm ins})]^{-1/2}$. This leads us to suggest that the resulting vertex operators $V_0(\lambda, \bar{\lambda})$ represent arbitrary classical loops in noncompact spacetime. One can also show that this is a coherent state, the definition of which is given in [15] (with unit operator $\mathbb{1} = G_0$). The normal ordered version of $V_0(\lambda, \bar{\lambda})$ can be derived from the normal ordered expression $V(\lambda, \bar{\lambda})$ [as given below (8)] by computing the operator product $G_0 V(\lambda, \bar{\lambda})$. One finds

$$V_0(\lambda, \bar{\lambda}) = \mathcal{C}_{\lambda\bar{\lambda}} \int_0^{2\pi} d\!\!\!{}^-\! s \exp\left\{\sum_{n=1}^{\infty} \frac{1}{n} \zeta_n(s) P^i_n(z) e^{-{\rm inq}X(z)}\right\} \times \exp\left\{\sum_{m=1}^{\infty} \frac{1}{m} \bar{\zeta}_m(s) \bar{P}^i_m(\bar{z}) e^{-{\rm imq}X(\bar{z})}\right\} e^{ipX(z,\bar{z})}.$$

Having projected out the null winding states world sheet translation invariance is restored and, according to the above discussion, the condition for classicality $\langle X \rangle = X_{\rm cl}$ in (d) is replaced by (2). Given that we know the classical solution in light cone gauge, see (1), we establish (2) for the projected states in light cone gauge by making use of (7). Denoting states with null winding w by $V_w(\lambda, \bar{\lambda}) \cong G_w V(\lambda, \bar{\lambda})$ one can show that (2) is satisfied by making use of Eqs. (1) and (6), with $A^i_n |V_0\rangle = \lambda^i_n |V_n\rangle$, $\bar{A}^i_n |V_0\rangle = \bar{\lambda}^i_n |V_{-n}\rangle$ $(n > 0)$, and $\langle V_n | V_m \rangle = \delta_{n,m}$, which follow from the DDF operator commutation relations. We learn that (9) has a classical interpretation given by (1) with $(\xi, \bar{\xi}) = (\lambda, \bar{\lambda})$ and $k^i = p^i$. Furthermore, it is not too hard to show [16] that also the angular momentum $J^{\mu\nu}$ defined below (6), of the states (9) matches the corresponding classical expression. $J^{\mu\nu}$ is gauge invariant and so we expect to find

$$\langle J^{\mu\nu}\rangle_{\rm cov} = \langle J^{\mu\nu}\rangle_{\rm lc} = J^{\mu\nu}_{\rm cl}, \tag{10}$$

which is indeed the case, with the result $\sum_{n>0}\frac{2}{n}{\rm Im}(\lambda_n^{*i}\lambda_n^j + \bar\lambda_n^{*i}\bar\lambda_n^j)$ for the transverse directions and a slightly more complicated expression for the longitudinal components, see [16].

To summarize, we have constructed closed string coherent state vertex operators (in covariant and light cone gauge) and have shown how to map these to arbitrary classical solutions. These new vertex operators may be used to study the cosmic F-string evolution, taking gravitational backreaction into account which is almost always neglected in the classical computations. It can also be used to check whether gravitational radiation is indeed the primary decay channel, and if so what the frequency spectrum is. An example currently under investigation is the graviton emission amplitude from a coherent state, which (due to the optical theorem) can be extracted from the imaginary part of the forward scattering u-channel amplitude, ${\rm Im}\langle V_0^\dagger V_g^\dagger V_g V_0\rangle$. Here V_g is the graviton vertex operator (or any other massless or massive vertex), and V_0 a coherent state (9).

Appendix.—Elementary Schur polynomials are defined [32] by the generating series, $\sum_{m=0}^\infty S_m(a_1,\ldots,a_m)z^m = \exp\sum_{n=1}^\infty a_n z^n$; equivalently,

$$S_m(a_1,\ldots,a_m) = -i\oint_0 \text{đ}w\, w^{-m-1}\exp\sum_{s=1}^m a_s w^s, \tag{11}$$

with $\text{đ}w \equiv dw/(2\pi)$, $S_0 = 1$, and $S_{m<0} = 0$. When $a_s = -\frac{1}{s!}{\rm inq}\partial^s X(z)$, with q^μ defined in (4) we write $S_m(nq;z) \equiv S_m(a_1,\ldots,a_m)$. The following Taylor series is useful, $e^{-{\rm inq}X(z)} = \sum_{a=0}^\infty z^a S_a(nq;0)e^{-{\rm inq}X(0)}$. The polynomials $P_n(z)$, $\bar P_n(\bar z)$ that appear in normal ordered covariant vertex operators are then defined by

$$P_n^i(z) = \sum_{\ell=1}^\infty \frac{i}{(\ell-1)!}\partial^\ell X^i(z) S_{n-\ell}(nq;z), \tag{12}$$

with a similar expression for $\bar P_n^i(\bar z)$, obtained from (12) by the replacement $\partial^s X(z) \to \bar\partial^s X(\bar z)$.

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