## Influence of Energetic Ions on Tearing Modes

Huishan Cai,[\\*](#page-3-0) Shaojie Wang, Yinfeng Xu, Jintao Cao, and Ding Li

<span id="page-0-1"></span>Department of Modern Physics, School of Physical Sciences, University of Science and Technology of China,

Hefei 230026, People's Republic of China

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In contrast with the stability effects of trapped energetic ions on tearing modes, the effects of circulating energetic ions (CEI) on tearing modes depend on the toroidal circulating direction, and are closely related to the momentum of energetic ions. CEI provide an additional source or sink of momentum to affect tearing modes. For co-CEI, tearing modes can be stabilized if the momentum of energetic ions is large enough. On the other hand, the growth of tearing modes can be enhanced by counter-CEI. Further, a possibility to suppress the island growth of neoclassical tearing modes by co-CEI is pointed out.

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The tearing mode, one of the most dangerous magnetohydrodynamics (MHD) instabilities in tokamak discharges, can change the topology of the magnetic field, lead to the formation of magnetic islands, increase local radial transport, and degrade plasma confinement. If the island becomes large, tearing modes can cause a macroscopic relaxation event. The neoclassical tearing mode (NTM) is driven by bootstrap current and can limit the  $\beta = 8\pi P/B^2$  value (where P is pressure, B is the magnetic field) in tokamak discharges [\[1\]](#page-3-1). Hence, understanding the physics of tearing modes (including NTMs) in tokamak plasmas is one of the critical physics problems to achieving steady-state and high confinement plasmas [\[2\]](#page-3-2).

Recently, some theories and experiments have shown that energetic ions can interact with tearing modes effectively [\[3–](#page-3-3)[8](#page-3-4)]. Many works have been devoted to the influences of energetic ions on ideal MHD instabilities [\[9](#page-3-5)–[13\]](#page-3-6), such as the internal kink mode and the Alfvén eignmodes. For example, it has been shown that energetic ions can stabilize the internal kink mode [[10](#page-3-7)]. However, the study of the interaction between energetic ions and resistive stability of  $m > 1$  modes (such as tearing modes) has only begun. The redistribution and loss of energetic ions due to NTMs and the stabilization of energetic ions on NTMs were found in some experiments [[3](#page-3-3)[–5\]](#page-3-8). In 1990, Hegna et. al. [[7\]](#page-3-9) showed that energetic ions have stabilizing effects on nonlinear tearing modes by the interaction with the inner region. In the presence of the nonlinear magnetic island, the island current generated due to the magnetic drift of energetic ions depends on the density gradient of energetic ions outside the rational surface and affects the stability of nonlinear tearing modes. Recently, the effects of energetic ions on linear tearing modes have been simulated by Takahashi et. al. [[8\]](#page-3-4). It has been found that the kinetic effects of energetic ions can play a crucial role in the stability of  $m/n = 2/1$  tearing modes, and energetic ions affect the stability of tearing modes through the interaction with the ideal outer region. Since the drift orbit radius of energetic ions is much larger than the island width of linear tearing modes, the effects of energetic ions on tearing modes in the nonideal region can be assumed unimportant. Hence, we will focus on the interaction between energetic ions and the outer region in this article.

To understand the underlying physics, a heuristic interpretation is shown. In the outer region of tearing modes,  $\mathbf{B} \cdot \nabla J_{\parallel} = 0$  (where  $J_{\parallel}$  is the parallel current) in the low- $\beta$ plasmas, so  $J_{\parallel} = J_{\parallel}(\Omega)$  is the flux function, where  $\Omega =$  $Q(\psi) + \delta \psi$ ,  $dQ/d\psi = 1 - q/q_s$  ( $\psi$  and  $\delta \psi$  are the equilibrium and perturbed poloidal magnetic flux, respectively, q is the safety factor,  $q_s = m/n$  is the value of q on the resonant surface), satisfying  $\mathbf{B} \cdot \nabla \Omega = 0$ . Then the perturbed current can be obtained as  $\delta J_{\parallel} =$  $(dJ_{\parallel 0}/dQ)\delta\psi$  in the outer region. However, due to the magnetic drift of energetic ions,  $J_{\parallel}$  is not a flux function, since  $\mathbf{B} \cdot \nabla J_{\parallel} = -\nabla_{\perp} \cdot \mathbf{J}_{\perp,h}$ , where  $\mathbf{J}_{\perp,h}$  is the diamagnetic current due to the kinetic parts of energetic ions. Separating the current into  $J_{\parallel} = J_{\parallel,1} + \delta J_{\parallel,2}$ , where  $J_{\parallel,1} =$  $J_{\parallel,1}(\Omega)$  is still a flux function, and  $\mathbf{B}_0 \cdot \nabla \delta J_{\parallel,2} = -\nabla_\perp \cdot$  $\delta J_{\perp,h}$ , then the perturbed current  $\delta J_{\parallel} = (dJ_{\parallel 0}/dQ)\delta \psi$  +  $\delta J_{\parallel,2}$ . Consequently,  $\delta J_{\parallel,2}$  must be a function of  $\delta \psi$ , and the equation in the outer region is changed. Thus, it can be considered that energetic ions affect the outer region of tearing modes by providing an additional source or sink for the exchange of momentum with waves.

Now, the details of the calculation are shown. In the outer region, the linearized equations are

$$
-\nabla \delta p_c - \nabla \cdot \delta \mathbf{p_h} + \delta \mathbf{J} \times \mathbf{B}_0/c + \mathbf{J} \times \delta \mathbf{B}/c = 0, (1)
$$

$$
\delta \mathbf{p_h} = \delta p_{\perp, h} \mathbf{I} + (\delta p_{\parallel, h} - \delta p_{\perp, h}) \mathbf{bb},\tag{2}
$$

<span id="page-0-0"></span>where  $\delta p_c$  is perturbation of the core plasma pressure, which is assumed to be isotropic.  $\delta \mathbf{p}_h$  is the perturbation form of Chew-Goldberger-Low [[14](#page-3-10)] pressure tensor of energetic ions. I is the unit tensor,  $\mathbf{b} = \mathbf{B}_0 / B_0$  denotes the direction of equilibrium magnetic field. The magnetic field is written as  $\mathbf{B} = I \nabla \zeta + \nabla \zeta \times \nabla (\psi + \delta \psi)$ , where the toroidal geometry is assumed to be axisymmetric.

 $(\psi, \theta, \zeta)$  are chosen as flux coordinates.  $\theta$  and  $\zeta$  are the poloidal and toroidal angle, respectively.

For simplicity, the single helicity is considered. Then the form of perturbation  $\left[\delta\psi, \delta\phi\right] = \left[\delta\hat{\psi}, \delta\hat{\phi}\right](r) \exp(im\theta$ in $\zeta$ ) can be assumed. Operating  $\nabla \cdot (\mathbf{B}_0/B_0^2 \times \cdots)$  on Eq.  $(1)$  $(1)$ , one finds

<span id="page-1-7"></span>
$$
\begin{split}\n&\left(\frac{\partial}{\partial \theta} + q \frac{\partial}{\partial \zeta}\right) \frac{\delta J_{\parallel}}{B_0} + \left[\frac{\partial}{\partial \psi} \left(\frac{J_{\parallel}}{B_0}\right) \frac{\partial}{\partial \theta} (R \delta A_{\parallel})\right] \\
&- \frac{\partial}{\partial \theta} \left(\frac{J_{\parallel}}{B_0}\right) \frac{\partial}{\partial \psi} (R \delta A_{\parallel})\right] \\
&+ \frac{2Ic}{B_0^2} \left(\kappa_1 \frac{\partial \delta p_c}{\partial \theta} - \kappa_2 \frac{\partial \delta p_c}{\partial \psi}\right) \\
&+ \frac{Ic}{B_0^2} \left[\kappa_1 \frac{\partial}{\partial \theta} (\delta p_{\parallel,h} + \delta p_{\perp,h})\right] \\
&- \kappa_2 \frac{\partial}{\partial \psi} (\delta p_{\parallel,h} + \delta p_{\perp,h})\right] = 0,\n\end{split} \tag{3}
$$

where  $\kappa_1 = -(r \cos \theta/R)(\partial \psi/\partial r)^{-1}, \kappa_2 = r \sin \theta/R,$  $\delta A_{\parallel} = -\delta \psi / R$  is the perturbation of parallel magnetic vector potential,  $R = R_0 + r \cos \theta$  is the major radius. To obtain the expression of  $\delta p_h$ , it is necessary to solve the perturbed distribution of energetic ions. By ignoring finite Larmor radius effects, the linearized gyrokinetic equations are

<span id="page-1-0"></span>
$$
\delta F_h = (\partial F_{h0}/\partial E)e \delta \phi / m_h + \delta H_h, \tag{4}
$$

$$
(\boldsymbol{v}_{\parallel}\mathbf{b}\cdot\nabla+\mathbf{v}_{d}\cdot\nabla-i\omega)\delta H_{h}=(ie/m_{h})Q_{h}(\delta\phi-\mathbf{v}_{\parallel}\delta A_{\parallel}/c),
$$
\n(5)

where  $Q_h = (\omega \partial / \partial E + \omega_{*h}) F_{h0}$ ,  $\omega_{*h} = -i(\mathbf{b} \times \nabla E)$  $\nabla \ln F_{h0}/\Omega_{ch}$   $\cdot \nabla$ ,  $\mathbf{v_d} = \left[ (v_{\parallel}^2 + v_{\perp}^2/2)/\Omega_{ch} \right] \mathbf{b} \times \boldsymbol{\kappa}$  is the magnetic drift velocity, and  $\kappa = \mathbf{b} \cdot \nabla \mathbf{b}$ . Here,  $E = v^2/2$ is the energy per unit mass,  $F_{h0}$  and  $\delta F_h$  are the equilibrium and perturbed distribution of energetic ions, respectively.  $\Omega_{ch} = eB_0/(m_h c)$  is the cyclotron frequency, e and  $m_h$  are the ion charge and mass, respectively.  $\delta \phi$  is the perturbed electrostatic potential.

<span id="page-1-1"></span>By introducing the transforms  $\delta A_{\parallel} = -ic\mathbf{b} \cdot \nabla \delta \phi^r / \omega$ and  $\delta H_h = -e/(m_h \omega)Q_h \delta \phi^r + \delta G$ ,  $\delta \phi^r = \delta \phi$  is obtained, since  $E_{\parallel} = -\mathbf{b} \cdot \nabla \delta \phi + i(\omega/c) \delta A_{\parallel} = 0$  in the ideal outer region. Then Eq. ([5\)](#page-1-0) becomes

$$
\begin{aligned} v_{\parallel}(qR)^{-1}(\partial/\partial \theta + q\partial/\partial \zeta)\delta G - v_d(r^{-1}\cos\theta\partial/\partial \theta \\ &+ ik_r\sin\theta)\delta G - i\omega\delta G \\ &= -e/(m_h\omega)Q_h v_d(im\cos\theta/r + ik_r\sin\theta)\delta\phi, \end{aligned} \tag{6}
$$

where  $v_d = (v_{\parallel}^2 + v_{\perp}^2/2)/(R\Omega_c)$  and  $ik_r = \partial/\partial r$ . In this article, only the effects of circulating energetic ions (CEI) are considered. By introducing the transform  $\delta G = \exp(im\theta - in\zeta - i\lambda_d \cos\theta) \delta g(r, \theta)$ , where  $\lambda_d =$  $k_r v_d/\omega_t$ ,  $\omega_t = v_{\parallel}/(qR)$ , Eq. [\(6\)](#page-1-1) can be written as

<span id="page-1-2"></span>
$$
\frac{\partial \delta g}{\partial \theta} + i \frac{k_{\parallel} v_{\parallel} - \omega}{\omega_t} \delta g - \frac{im v_d \cos \theta}{r \omega_t} \delta g \n- \frac{i \lambda_d v_d \sin(2\theta)}{2r \omega_t} \delta g - \frac{v_d \cos \theta}{r \omega_t} \frac{\partial \delta g}{\partial \theta} \n= - \frac{i e}{m_h \omega} Q_h \left( \frac{m v_d \cos \theta}{r \omega_t} + \lambda_d \sin \theta \right) \delta \hat{\phi} \exp(i \lambda_d \cos \theta),
$$
\n(7)

where  $k_{\parallel} = (m - nq)/(qR)$ . For CEI, we assume the ordering  $\omega \ll \omega_t$  for tearing modes and  $q\rho_h/a \ll 1$  [ $\rho_h=$  $v_d/(q\omega_t)$ , where  $q\rho_h$  and a are the drift orbit radius of energetic ions and minor radius, respectively.  $\beta_h \sim \beta_c$  is also assumed  $(\beta_h = 8\pi p_h / B_0^2$ ,  $\beta_c = 8\pi p_c / B_0^2$ ). For the region of  $k_r \gg m/r$ , Eq. ([7\)](#page-1-2) can be expanded as

<span id="page-1-5"></span>
$$
\partial \delta g_0 / \partial \theta = -ie/(m_h \omega) Q_h \lambda_d \sin \theta \delta \hat{\phi} \exp(i \lambda_d \cos \theta), \quad (8)
$$

$$
\frac{\partial \delta g_1}{\partial \theta} + i \frac{k_{\parallel} v_{\parallel} - \omega}{\omega_t} \delta g_0 - \frac{im v_d \cos \theta}{r \omega_t} \delta g_0
$$

$$
- \frac{i \lambda_d v_d \sin(2\theta)}{2r \omega_t} \delta g_0 - \frac{v_d \cos \theta}{r \omega_t} \frac{\partial \delta g_0}{\partial \theta}
$$

$$
= -\frac{ie}{m_h \omega} Q_h \frac{m v_d \cos \theta}{r \omega_t} \delta \hat{\phi} \exp(i \lambda_d \cos \theta). \quad (9)
$$

<span id="page-1-4"></span><span id="page-1-3"></span>Equation ([8](#page-1-3)) can be solved as

$$
\delta g_0 = e/(m_h \omega) Q_h \delta \hat{\phi} \exp(i \lambda_d \cos \theta) + C_0(r). \quad (10)
$$

<span id="page-1-6"></span>Substituting Eq. ([10](#page-1-4)) into Eq. [\(9](#page-1-5)) and making the integral  $\oint d\theta/(2\pi)$ , one can get

$$
C_0(r) = -e/(m_h \omega) Q_h \delta \hat{\phi} J_0(\lambda_d), \qquad (11)
$$

where  $J_n(\lambda_d)$  is the Bessel function. From Eqs. [\(9](#page-1-5))–([11\)](#page-1-6), using the transform

$$
\delta g_1 = \sum_l \delta g_l^l e^{il\theta} + i \left( \frac{m v_d \sin \theta}{r \omega_l} - \frac{\lambda_d v_d \cos(2\theta)}{4r \omega_l} \right) C_0(r)
$$

$$
- \frac{e}{m_h \omega} Q_h \delta \hat{\phi} C_1(r), \qquad (12)
$$

one finds

$$
\delta g_1^l = -\frac{e}{m_h \omega} Q_h \delta \hat{\phi} \frac{k_{\parallel} v_{\parallel} - \omega}{\omega_t} \frac{i^l}{l} J_l(\lambda_d), \qquad l \neq 0.
$$
\n(13)

Thus, the perturbed distribution of energetic ions  $\delta F_h$ is derived. Next, only  $\delta p_{\parallel,h} = \int d^3v v_{\parallel}^2 \delta F_h$  needs to be solved, since CEI is considered here,  $\delta p_{\parallel,h} \gg \delta p_{\perp,h}$ .

<span id="page-1-8"></span>Making the integral  $\oint d\theta \exp(-im\theta + in\xi)/(2\pi)$  on Eq. ([3\)](#page-1-7), one yields

$$
- in\left(\frac{m}{n} - q\right) \frac{c}{4\pi B_0} \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d\delta \hat{A}_{\parallel}}{dr} \right) - \frac{m^2}{r^2} \delta \hat{A}_{\parallel} \right] + im\left(\frac{\partial \psi}{\partial r}\right)^{-1} \frac{d}{dr} \left(\frac{J_{\parallel 0}}{B_0}\right) \delta \hat{A}_{\parallel} + \delta K = 0,
$$
(14)

where the terms about the core pressure and the adiabatic part of energetic ion pressure, which are of order  $O(\varepsilon)$ comparing with other terms, are neglected.  $\varepsilon = r/R_0$  is the inverse aspect of ratio, and

<span id="page-2-0"></span>
$$
\delta K = -i \frac{e}{m_h} \frac{R_0}{B_0} r \left(\frac{\partial \psi}{\partial r}\right)^{-1} \int d^3 v \frac{v_{\parallel}^2}{\rho_h} Q_h [1 - J_0^2(\lambda_d)] \delta \hat{A}_{\parallel}(r), \qquad (15)
$$

where  $\delta A_{\parallel} = -ic\mathbf{b} \cdot \nabla \phi / \omega$  is used. In fact, Eq. [\(14\)](#page-1-8) is an integrodifferential equation, as

$$
F_r = [1 - J_0^2(\lambda_d)] \delta \hat{A}_{\parallel}(r)
$$
  
= 
$$
\int_{-\infty}^{\infty} dk_r [1 - J_0^2(\lambda_d)] \delta \hat{A}_{\parallel}(k_r) \exp(ik_r r).
$$
 (16)

Note that Eq. [\(15\)](#page-2-0) is only valid near the resonant surface, where  $\partial_r \delta \hat{A}_{\parallel} / \delta \hat{A}_{\parallel} \gg 1$ . In the region far from the resonant surface, the effect of CEI can be neglected, since CEI have only an adiabatic effect there [[9](#page-3-5)]. On the other hand,  $v_d \cdot E \sim 0$  in this region for tearing modes, so the energy exchange of energetic ions and tearing modes can be neglected. Furthermore, the stability criterion  $\Delta'$  is mainly determined by the behavior of  $\delta \hat{A}_{\parallel}$  near the resonant surface. Thus, we focus on the effects of energetic ions near the resonant surface in the outer region. To analyze the nonlocal effect of energetic ions, the numerical calculation about  $F_r$  is made, as shown in Fig. [1](#page-2-1). From Fig. 1, one concludes that  $F_r \sim \chi_0 \delta \hat{A}_{\parallel}(r)$  with  $\chi_0 \le 1$  near the resonant surface can be approximated. The rough explanation is that the nonlocal behavior of energetic ions due to the magnetic drift couples the regions  $\lambda_d \gg 1$  and  $\lambda_d \sim O(1)$ , and modifies the quantitative result, but it will not change the qualitative result. Thus, the approximation  $F_r \sim \delta \hat{A}_{\parallel}$  is modified. This is similar to the effect of finite thermal ion Larmor radius [[15](#page-3-11)]. Note that Cowley et al. [[15](#page-3-11)] dealt with the effect of thermal ion Larmor radius in nonideal inner region, and discussed the nonlocal response of ions across the nonideal region.

To proceed, it is assumed that the equilibrium distribution  $F = \sum_{\sigma} F_{h0}^{\sigma}$  satisfies the slowing-down model for a

<span id="page-2-1"></span>

FIG. 1. Plot of  $F_r$  and  $\delta \hat{A}_{\parallel}$  near the resonant surface at  $r_s = 0.598$  for a typical  $m/n = 2/1$  tearing mode.

population of CEI by a purely co-CEI ( $\sigma = +$ ) component or a purely counter-CEI ( $\sigma = -$ ) component, as  $F_{h0}^{\sigma} =$  $(2^{3/2} \pi m_h B_0 E_m^{\sigma})^{-1} p_h^{\sigma} E^{-3/2} \delta(\lambda) H(E - E_m)$ . Then one can obtain

$$
\delta K \sim -\chi_0 \sum_{\sigma} \sigma i \frac{2R_0 c}{B_0^2} \left(\frac{\partial \psi}{\partial r}\right)^{-1} \frac{m}{\rho_{hm}^{\sigma}} \frac{dp_h^{\sigma}}{dr} \delta \hat{A}_{\parallel},\qquad(17)
$$

where  $\rho_{hm}^{\sigma} = (2E_m^{\sigma})^{1/2}/\Omega_{\text{ch}}$  is the gyroradius of energetic ions with maxim energy  $E_m^{\sigma}$ . Thus, Eq. ([14](#page-1-8)) can be rewritten as

<span id="page-2-2"></span>
$$
\left(\frac{m}{n} - q\right) \frac{\varepsilon}{q} \left[ \frac{d}{dr} \left( r \frac{d \delta \hat{A}_{\parallel}}{dr} \right) - \frac{m^2}{r} \delta \hat{A}_{\parallel} \right] \n- \frac{m}{n} \left( \frac{dJ_{\parallel 0}}{dr} - \chi_0 \sum_{\sigma} \frac{\sigma}{\rho_{hm}^{\sigma}} \frac{d\beta_{h}^{\sigma}}{dr} \right) \delta \hat{A}_{\parallel} = 0,
$$
\n(18)

where the variables are normalized as  $t \to \tau_A t$ ,  $\mathbf{x} \to a\mathbf{x}$ ,  $\mathbf{B} \to B_0 \mathbf{B}, \beta_h^{\sigma} = 8 \pi p_h^{\sigma}/B_0^2$ , and  $\tau_A$  is the Alfven time. The last term denotes the part of perturbed parallel current  $\delta J_{\parallel,2}$ due to CEI. As mentioned above, this term comes from the diamagnetic drift current due to the anisotropic pressure. By the magnetic curvature coupling, the magnetic drift of energetic ions can affect the perturbed parallel current, so the parallel current is not a flux function anymore. The effects of CEI also depend on the toroidal circulating direction. Thus, it can be considered that CEI affect tearing modes through the exchange of momentum. Next, the effects of CEI on the stability criterion  $\Delta'$  of tearing modes will be derived based on Eq.  $(18)$  $(18)$  $(18)$ . It can be found that the form of Eq. [\(18\)](#page-2-2) is similar to outer equation of classical tearing modes [[16](#page-3-12)]. As done in Ref. [\[16](#page-3-12)], the stability criterion without considering conducting wall can be derived approximately as

$$
\Delta' = -\frac{\pi \alpha}{r_s} \cot[\pi(\sqrt{m^2 + \alpha} - m)], \qquad (19)
$$

$$
\alpha = \alpha_0 + \alpha_h, \tag{20}
$$

<span id="page-2-3"></span>where  $\alpha_0 = -q^2(r_s)[\mathbf{e} dq(r_s)/dr]^{-1} dJ_{\parallel 0}/dr, \qquad \alpha_h =$  $\chi_0 q^2(r_s)[\varepsilon dq(r_s)/dr]^{-1} \sum_\sigma \sigma(\rho^\sigma_{hm})^{-1} d\beta^\sigma_h/dr$ .  $\alpha_0$  is a critical parameter determining the value of  $\Delta'$  without the effects of energetic ions. As pointed out in Ref. [[16\]](#page-3-12), Eq. ([19](#page-2-3)) is only valid for  $0 < \alpha < 2m + 1$ , since it is derived from the leading order expansion near the singular layer for the solution of Eq. [\(18\)](#page-2-2). For the complete solution, it is needed to solve Eq. ([18\)](#page-2-2) by numerical calculation. From the expression of  $\alpha_h$ , one can see that the effects of energetic ions are proportional to its momentum. For the typical tokamak like JT-60U, the main parameters are  $B \approx 3.5T$ ,  $R \approx 3.2m, \ \varepsilon \simeq 1/4, \ \omega_A \simeq 8 \times 10^6/s, \ q_0 \simeq 0.8, \ s \simeq 0.4,$ where  $\omega_A$  is the Alfvén frequency, s is the magnetic shear at  $r_s$ .  $\chi_0 = 0.5$  is taken. Considering  $m = 2$ ,  $n = 1$  mode,  $r_s \Delta' = 0$  for  $\alpha_0 = 2.25$  without energetic ions. It is assumed that 200 keV ions formed by co-CEI (or counter-CEI) inject into plasmas, then  $\rho_{hm} \approx 0.018m$ .  $d\beta_h/dr < 0$ and  $d\beta_h/dr \sim -\beta_h/r$  are also assumed. Then  $\Delta'$  against

<span id="page-3-13"></span>

FIG. 2. The stability criterion  $r_s \Delta^t$  against  $\sigma d\beta_h/dr$ , where  $d\beta_h/dr < 0$  and  $d\beta_h/dr \sim -\beta_h/r$  are assumed.

 $\sigma d\beta_h^{\sigma}/dr$  is shown in Fig. [2.](#page-3-13) It can be seen explicitly that the effects of CEI on  $\Delta'$  are dramatic. For counter-CEI,  $\Delta'$ increases with  $\beta_h$ , and can become positive. For co-CEI,  $\Delta'$  decreases with  $\beta_h$ , and can be reduced to negative or more negative.  $\Delta'$  as the key measure of the plasma free energy is essential to the stability of classical tearing modes, and the onset and evolution of NTMs. If  $\Delta' > 0$ , the classical tearing mode is unstable, can provide the seed islands for NTMs [\[17\]](#page-3-14), and may be one of the onset mechanisms of spontaneous NTMs [[18](#page-3-15)]. If  $\Delta' < 0$ , the classical tearing mode is stable, and the island growth of NTMs will be suppressed. Actually, some recent experiments in JET and DIII-D indicated that NTMs are triggered when  $\Delta'$  becomes positive [\[1](#page-3-1),[5](#page-3-8),[19](#page-3-16)]. In the experiment in DIII-D [\[5](#page-3-8)], it was shown that the change of  $\Delta'$  may be a conceptual explanation for the onset mechanism of NTMs, the increased coinjected neutral beam reduces  $\Delta'$ , and the  $m/n = 2/1$  NTM onset thresholds are increased. Another two experiments  $[1,19]$  $[1,19]$  in JET and DIII-D also showed that  $2/1$  modes growth may start as a classical tearing mode when  $\Delta' > 0$ in some cases and the stability of tearing modes relates to  $\beta_{\text{frac}}(\beta_{\text{frac}} = \beta_h/\beta)$  attributed to the energetic ions. Recently, the simulation presented by Takahashi et. al.[\[8\]](#page-3-4) indicated that tearing modes can be stabilized by the interaction between outer region and trapped energetic ions if  $\beta_{\text{frac}}$  is large enough. Therefore, tearing modes can be affected dramatically by CEI.

In conclusion, the effects of CEI on tearing modes depend on the toroidal circulating direction, and have a great relation with the momentum of energetic ions. For co-CEI, energetic ions can reduce the value of  $\Delta'$ , and stabilize the classical tearing modes; then it may suppress the island growth of NTMs. For counter-CEI, energetic ions can destabilize the classical tearing modes, and may trigger the spontaneous NTMs. For the balanced tangential neutral beam injection, energetic ions have no effects. In this article, it is concluded that tearing modes can be stabilized by co-CEI if  $\beta_h$  is large enough and the growth of tearing modes can be enhanced by counter-CEI; this is similar to the effects of CEI on resistive internal kink mode [\[11\]](#page-3-17). The present results predict that the trigger of spontaneous NTMs by classical tearing modes may be avoided by co-CEI produced by neutral beam injection. For example, choosing some energy of co-CEI to make  $\alpha_0 + \alpha_h \rightarrow 0$ , the value of  $\Delta'$  can be reduced to maxnegative value  $-2m/r_s$ . Then, classical tearing mode is stable, and the spontaneous NTMs may not happen, or island growth of NTMs may be suppressed. Thus, our analysis suggests that it is possible to suppress island growth by co-CEI with appropriate energy. Here, we only considered the effects of energetic ions on  $\Delta'$  for  $W < q \rho_h$ . If  $W \ge q \rho_h$ , energetic ions may dominantly interact with inner region of tearing modes, and have little effect on  $\Delta'$ . In this case, the interaction between energetic ions and tearing modes should be reconsidered.

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[\\*h](#page-0-1)scai@mail.ustc.edu.cn

- <span id="page-3-1"></span><span id="page-3-0"></span>[1] T. C. Hender et al., [Nucl. Fusion](http://dx.doi.org/10.1088/0029-5515/44/7/010) 44, 788 (2004).
- <span id="page-3-2"></span>[2] T. C. Hender et al., Nucl. Fusion 47[, s128 \(2007\).](http://dx.doi.org/10.1088/0029-5515/47/6/S03)
- <span id="page-3-3"></span>[3] E. Poli et al., Phys. Plasmas 15[, 032501 \(2008\).](http://dx.doi.org/10.1063/1.2890771)
- <span id="page-3-8"></span>[4] M. García-Muñoz et al., Nucl. Fusion 49[, 085014 \(2009\).](http://dx.doi.org/10.1088/0029-5515/49/8/085014)
- [5] R.J. Buttery *et al.*, *Phys. Plasmas* **15**[, 056115 \(2008\)](http://dx.doi.org/10.1063/1.2894215).
- [6] V. S. Marchenko and V. V. Lutsenko, [Phys. Plasmas](http://dx.doi.org/10.1063/1.1407283) 8, [4834 \(2001\)](http://dx.doi.org/10.1063/1.1407283).
- <span id="page-3-9"></span>[7] C.C. Hegna and A. Bhattacharjee, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.63.2056) 63, [2056 \(1989\)](http://dx.doi.org/10.1103/PhysRevLett.63.2056).
- <span id="page-3-4"></span>[8] R. Takahashi, D. P. Brennan, and C. C. Kim, [Phys. Rev.](http://dx.doi.org/10.1103/PhysRevLett.102.135001) Lett. 102[, 135001 \(2009\).](http://dx.doi.org/10.1103/PhysRevLett.102.135001)
- <span id="page-3-5"></span>[9] L. Chen, R. B. White, and M. N. Rosenbluth, [Phys. Rev.](http://dx.doi.org/10.1103/PhysRevLett.52.1122) Lett. 52[, 1122 \(1984\).](http://dx.doi.org/10.1103/PhysRevLett.52.1122)
- <span id="page-3-7"></span>[10] F. Porcelli, [Plasma Phys. Controlled Fusion](http://dx.doi.org/10.1088/0741-3335/33/13/009) 33, 1601 [\(1991\)](http://dx.doi.org/10.1088/0741-3335/33/13/009).
- <span id="page-3-17"></span>[11] S. Wang, T. Ozeki, and K. Tobita, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.88.105004) 88, [105004 \(2002\).](http://dx.doi.org/10.1103/PhysRevLett.88.105004)
- <span id="page-3-6"></span>[12] G.J. Kramer et al., Nucl. Fusion 40[, 1383 \(2000\)](http://dx.doi.org/10.1088/0029-5515/40/7/308).
- [13] C. Angioni et al., [Plasma Phys. Controlled Fusion](http://dx.doi.org/10.1088/0741-3335/44/2/305) 44, 205 [\(2002\)](http://dx.doi.org/10.1088/0741-3335/44/2/305).
- <span id="page-3-10"></span>[14] G. F. Chew, M. L. Goldberger, and F. F. Low, [Proc. R. Soc.](http://dx.doi.org/10.1098/rspa.1956.0116) A 236[, 112 \(1956\).](http://dx.doi.org/10.1098/rspa.1956.0116)
- <span id="page-3-11"></span>[15] S. C. Cowley, R. M. Kulsrud, and T. S. Hahm, [Phys. Fluids](http://dx.doi.org/10.1063/1.865841) 29[, 3230 \(1986\).](http://dx.doi.org/10.1063/1.865841)
- <span id="page-3-12"></span>[16] Ding Li, [Phys. Plasmas](http://dx.doi.org/10.1063/1.872781) **5**, 1231 (1998).
- <span id="page-3-14"></span>[17] H. Reimerdes, O. Sauter, T. Goodman, and A. Pochelon, Phys. Rev. Lett. 88[, 105005 \(2002\).](http://dx.doi.org/10.1103/PhysRevLett.88.105005)
- <span id="page-3-15"></span>[18] D.P. Brennan, S.E. Kruger, T.A. Gianakon, and D.D. Schnack, Nucl. Fusion 45[, 1178 \(2005\)](http://dx.doi.org/10.1088/0029-5515/45/9/018).
- <span id="page-3-16"></span>[19] R. J. Buttery et al., in Proceedings of the 22nd IAEA Fusion Energy Conference, Geneva, Switzerland, IT/P6- 8 (IAEA, Geneva, Switzerland, 2008).