Taming Nonlocality in Theories with Planck-Scale Deformed Lorentz Symmetry

Giovanni Amelino-Camelia,[1,2] Marco Matassa,[1] Flavio Mercati,[1,2] and Giacomo Rosati[1,2]
[1]*Dipartimento di Fisica, Università di Roma "La Sapienza", P.le A. Moro 2, 00185 Roma, EU*
[2]*INFN, Sez. Roma1, P.le A. Moro 2, 00185 Roma, EU*
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We report a general analysis of worldlines for theories with deformed relativistic symmetries and momentum dependence of the speed of photons. Our formalization is faithful to Einstein's program, with spacetime points viewed as an abstraction of physical events. The emerging picture imposes the renunciation of the idealization of absolutely coincident events, but is free from some pathologies which had been previously conjectured.

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Over the last decade there has been considerable interest in the quantum-gravity literature about the deformations of Lorentz symmetry [1] that would allow the introduction of a momentum dependence of the speed of photons

$$v = 1 - \ell p \tag{1}$$

as a relativistic law, with an observer-independent length parameter ℓ usually assumed to be roughly of the order of the Planck length. This is the most studied possibility for a "doubly-special relativity" (DSR) [1–5]. The interest it attracts is mostly due to associated descriptions of energy-momentum space, which find some support in results obtained within the loop quantum gravity approach [6] and in some models based on spacetime noncommutativity [7]. But the development of this research program must face the challenge of several indirect arguments (see, e.g., Ref. [8,9] and references therein) suggesting that a logically consistent formulation of (1) is not possible within a fully conventional description of spacetime.

The possibility of novel properties for spacetime was expected at the onset [1] of DSR research, since some aspects of the quantum-gravity problem suggest that there might be some absolute limitations to localizability of an event. But the fact it was expected does not make it any less of a challenge: what could replace the classical points of spacetime?

Those who looked at DSR research from the outside have been understandably rather puzzled (see, e.g., Ref. [10]) about some of the implications of renouncing to an ordinary spacetime picture. In particular, the recent Ref. [11] ventured to make a bold claim: even without adopting any specific formalization, using only the bare idea of the momentum dependence of the speed of photons, one could robustly estimate the nature and size of the nonlocal effects that should be produced. And, still according to Ref. [11], this could be used to constrain ℓ to $|\ell| < 10^{-58}$ m, i.e., at a level which is 23 orders of magnitude beyond the one of direct experimental bounds based on the momentum dependence of the speed of photons [12,13].

The claim reported in Ref. [11] clearly renders even more urgent for DSR research to establish what are the actual implications for nonlocality. We start by observing that the argument presented in Ref. [11] did not make use of the well-established results on DSR-deformed boosts, but rather relied on assumptions that fail to be consistently relativistic. As shown in Fig. 1, the assumptions of Ref. [11] amount to adopting *undeformed* rules of boost transformation for the coordinates of the emission points of particles but *deformed* boost transformations for the velocities of the particles. Evidently such criteria of "selective applicability" of deformed boosts cannot produce a consistently relativistic picture.

The picture proposed in Ref. [11] clearly needed to be revised. We here report a deductive result of characterization of the nonlocality produced by DSR boosts. We derive it rigorously from the formalizations of

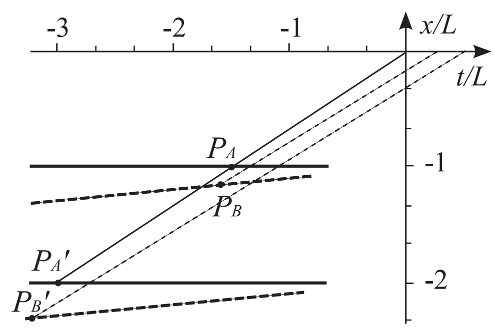

FIG. 1. In the argument of Ref. [11] a key role is played by the assumption that a photon which Alice sees emitted in P_A (from a source at a distance L from Alice) with speed v (and momentum p) should be seen by boosted Bob as a photon emitted from P_B, obtained by classical, *undeformed* boost of P_A, with speed obtained from the speed v with a *deformed* boost. In this figure we expose the logical inconsistency of such criteria of "selective applicability" of deformed boosts by allowing for a second photon which according to Alice also has speed v and is emitted from a point $P_{A'}$ such that the two photons share the same worldline: a single worldline would be mapped by a relativistic boost into two wildly different worldlines.

DSR-deformed boosts that have been proposed in the DSR literature. We succeed, where others had failed, primarily as a result of using as guidance Einstein's insight on the proper characterization of a spacetime point, to be viewed as the abstraction of an event of crossing of worldlines. This leads us to a fully relativistic characterization of the concept of locality, as a concept that pertains the coincidence of events: from a relativistic perspective the main locality issue concerns whether events that are coincident for one observer are also coincident for other observers.

We set to 1 both Planck constant $\hbar$ and the speed-of-light scale c (speed of low-momentum photons). The modulus of a spatial vector, with components W_j, is denoted by W ($W^2 = W_j W^j$). And we work in leading order in ℓ, since (1) is assumed [1,6] to be valid only for $p \ll 1/|\ell|$.

We also take on the challenge of a full (3 + 1)-dimensional analysis. Most of the previous DSR literature, including Ref. [11], is confined to (1 + 1)-dimensional frameworks, as a way to temper the complexity of dealing with deformed boosts. It is natural to expect that the nonlocality produced by DSR boosts would be already uncovered in a (1 + 1)-dimensional analysis, but our ability to characterize transverse boosts is a valuable addition, and provides further evidence of the robustness of the approach we developed. Another significant strength of our setup is that it applies to all previously considered deformations of Lorentz symmetry compatible with (1). Previous DSR studies of (1) not only failed to offer an explicit analysis of worldlines, but were also often assuming a specific ansatz for the formalization of the symmetry deformation.

The worldlines are here achieved within a Hamiltonian setup which was already fruitfully applied [14,15] to other DSR scenarios for the introduction of the second relativistic scale ℓ, but was not previously implemented for a DSR description of the speed law (1) for massless particles. We start by introducing canonical momenta conjugate to the coordinates x_j and t: $\{\Pi_j, x_k\} = -\delta_{jk}$, $\{\Omega, t\} = 1$. We must then specify a form of the DSR-deformed mass Casimir $\mathcal{C}$, which will play the role [14,15] of Hamiltonian. We have a two-parameter family of $O(\ell)$ possibilities

$$\mathcal{C} = \Omega^2 - \Pi^2 + \ell(\gamma_1\Omega^3 + \gamma_2\Omega\Pi^2), \qquad (2)$$

upon enforcing analyticity of the deformation and invariance under classical space-rotation transformations. The types of deformed boosts that were previously considered in the DSR literature are compatible with such a deformed Casimir, for some choices of γ_1, γ_2.

Hamilton's equations give the conservation of Π_j and Ω along the worldlines

$$\dot{\Pi}_j = \frac{\partial\mathcal{C}}{\partial x_j} = 0, \qquad \dot{\Omega} = -\frac{\partial\mathcal{C}}{\partial t} = 0, \qquad (3)$$

where $\dot{f} \equiv \partial f/\partial\tau$ and τ is an auxiliary worldline parameter.

The worldlines can then be obtained observing that

$$\dot{x}_j = -\frac{\partial\mathcal{C}}{\partial\Pi_j} \Rightarrow x_j(\tau) = x_j^{(0)} + (2\Pi_j - 2\ell\gamma_2\Omega\Pi_j)\tau$$
$$\dot{t} = \frac{\partial\mathcal{C}}{\partial\Omega} \Rightarrow t(\tau) = t^{(0)} + [2\Omega + \ell(\gamma_2\Pi^2 + 3\gamma_1\Omega^2)]\tau.$$

Eliminating the parameter τ and imposing the Hamiltonian constraint $\mathcal{C} = 0$ (massless case) one finds that

$$x_j = x_j^{(0)} + \frac{\Pi_j}{\Pi}(t - t^{(0)}) - \ell(\gamma_1 + \gamma_2)\Pi_j(t - t^{(0)}), \qquad (4)$$

which reproduces (1) for $\gamma_1 + \gamma_2 = 1$. Note that this derivation of worldlines compatible with (1) is insensitive to the possibility of a different DSR description for the canonical momentum Π_j and for the "momentum" p_j, intended as the DSR generalization of the concept of space-translation charge. Indeed, Π_j enters only at order ℓ and of course, since we are working in leading order, we must take $\ell\Pi_j = \ell p_j$ (while the modulus of p_j and Π_j may differ [5,16] at order ℓ).

We must now enforce covariance of the worldlines under DSR-deformed boosts. The form of the correction terms introduced in (2) suggests that the type of deformed boosts considered in the DSR literature should be well suited:

$$\mathcal{N}_j = -t\Pi + x_j\Omega + \ell[\alpha_1 t\Omega\Pi_j + \alpha_2\Pi^2 x_j + \alpha_3\Omega^2 x_j + \alpha_4 x_k\Pi^k\Pi_j].$$

Note that this four-parameter family of $O(\ell)$ deformed boosts, which enforces compatibility with undeformed space rotations, includes, as different particular cases, all the proposals for deformed boosts that were put forward in this first decade of DSR research [1–5,17]. The compatibility between boost transformations and form of the Casimir is encoded in the requirement that the boost charge is conserved

$$\dot{\mathcal{N}}_j = \{\mathcal{C}, \mathcal{N}_j\} = \frac{\partial\mathcal{C}}{\partial\Omega}\frac{\partial\mathcal{N}_j}{\partial t} - \frac{\partial\mathcal{C}}{\partial\Pi^k}\frac{\partial\mathcal{N}_j}{\partial x_k} = 0, \qquad (5)$$

which straightforwardly leads to the constraints $2\alpha_2 + 2\alpha_4 = \gamma_2$ and $2\alpha_1 + 2\alpha_3 = 3\gamma_1 + 2\gamma_2$. Combining these with the requirement $\gamma_1 + \gamma_2 = 1$ derived above, we finally arrive at a three-parameter family of Hamiltonian-boost pairs

$$\mathcal{C} = \Omega^2 - \Pi^2 + \ell(2\gamma\Omega^3 + (1 - 2\gamma)\Omega\Pi^2)$$
$$\mathcal{N}_j = -t\Pi_j + x_j\Omega + \ell\alpha t\Omega\Pi_j - \ell(\gamma + \beta - 1/2)x_k\Pi^k\Pi_j + \ell x_j(\beta\Pi^2 + (1 + \gamma - \alpha)\Omega^2),$$

where $\gamma = \gamma_1/2$, $\alpha = \alpha_1$, $\beta = \alpha_2$. For any given choice of γ, α, β relativistic covariance is ensured and we have a rigorous Hamiltonian derivation of worldlines for which the speed law (1) is verified. We have so far focused on massless particles, but one also easily obtains the worldlines of particles of any mass by enforcing the Hamiltonian constraint $\mathcal{C} = m^2$:

$$x_j = x_j^{(0)} + \frac{\Pi_j}{\sqrt{\Pi^2 + m^2}}[t - t^{(0)}] - \ell\Pi_j[t - t^{(0)}]. \quad (6)$$

The covariance of these worldlines under undeformed space rotations is manifest. The covariance under γ, α, β-deformed boosts, ensured by construction, can also be verified by computing explicitly the action of an infinitesimal deformed boost with rapidity vector ξ_j ($A' = A + \xi_j\{A, \mathcal{N}^j\}$)

$$\Pi'_j = \Pi_j - \xi_j\Omega - \ell\xi_j(\beta\Pi^2 + (1 + \gamma - \alpha)\Omega^2) - \ell(1/2 - \gamma - \beta)\xi_k\Pi^k\Pi_j \quad (7)$$

$$t' = t - \xi_j x^j - \ell[\alpha t\xi_j\Pi^j + 2(1 + \gamma - \alpha)\Omega\xi_j x^j] \quad (8)$$

$$x'_j = x_j - t\xi_j + \ell(\alpha t\Omega\xi_j + 2\beta\xi_k x^k\Pi_j) - \ell(\gamma + \beta - 1/2)(\xi_k\Pi^k x_j + x_k\Pi^k\xi_j). \quad (9)$$

Using these one easily verifies that when Alice has the particle on the worldline (6) Bob sees the particle on the worldline

$$x'_j = x'^{(0)}_j + \frac{\Pi'_j}{\sqrt{m^2 + \Pi'^2}}[t' - t'^{(0)}] - \ell\Pi'_j[t' - t'^{(0)}],$$

consistently with the relativistic nature of our framework.

We are now ready to exploit our technical results for a "physical" characterization of the nonlocality produced by DSR boosts. The observations we shall make on nonlocality apply equally well to all choices of γ, α, β. We notice, however, that by enforcing the condition $\alpha - \beta - \gamma = 1/2$ one has the welcome [7,15] simplification of undeformed Poisson brackets among boosts and rotations ("the Lorentz sector is classical" [7,15]). And, in particular, for the case $\gamma = 1/2$, $\alpha = 1$, $\beta = 0$, on which we focus for our graphical illustrations, the laws of transformation take a noticeably simple form:

$$\Pi'_j = \Pi_j - \xi_j(\Omega + \ell\Omega^2/2) \quad (10)$$

$$t' = t - \xi_j x^j - \ell(t\xi_j\Pi^j + \Omega\xi_j x^j) \quad (11)$$

$$x'_j = x_j - (1 - \ell\Omega)t\xi_j. \quad (12)$$

This case preserves much of the simplicity of classical boosts for what concerns boosts acting transversely to the direction of motion. We do not expect anything objectively pathological in the richer structure that other choices of γ, α, β produce for such transverse boosts. But it is nonetheless noteworthy that there are candidates for the DSR-deformed boosts that have properties as simple as codified in (11) and (12). In what follows, we shall not offer any additional comments on transverse boosts (and our figures focus on boosts along the direction of motion). But it is easy to verify using (8) and (9) [and even easier using (11) and (12)] that boosts acting transversely to the direction of motion lead to features of nonlocality that are of the same magnitude and qualitative type as the ones we visualize for boosts along the direction of motion.

Let us now move on to reconsidering the issues raised in our Fig. 1, and the shortcomings of the analysis reported in Ref. [11]. Having managed to derive constructively quantitative formulas for the action of the deformed boosts advocated in the DSR literature, we can now more definitely observe that the assumptions made for the analysis reported in Ref. [11] are inconsistent with the fact, here shown in Eqs. (8) and (9), that the deformed boosts still act, like ordinary Lorentz boosts, in a way that is homogeneous in the coordinates. A boost connects two observers with the same origin of their reference frames and, as shown in Fig. 2, the differences between DSR-deformed boosts and classical boosts are minute for points that are close to the common origin of the two relevant reference frames, but gradually grow with distance from that origin.

As shown by two of the worldlines in Fig. 3, when an observer Alice is local to a coincidence of events (the violet and a red photon simultaneously crossing Alice's worldline) all observers that are purely boosted with respect to Alice, and therefore share her origin, also describe those two events as coincident. This, in particular, addresses the "box problem" raised in Ref. [11], which concerned the possibility of a loss of objectivity of coincidences of events as witnessed by local observers: we have found that, at least in leading order in ℓ and ξ, in the DSR framework "locality," a coincidence of events, preserves its objectivity if assessed by local observers.

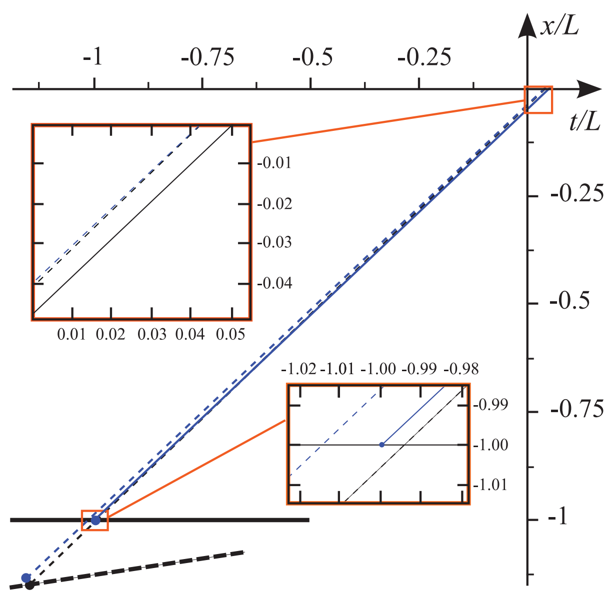

FIG. 2 (color). We here show a hard-photon worldline as seen by Alice (solid blue line), by DSR-boosted Bob (dashed blue line) and by classically boosted Bob (dashed black line). In spite of assuming (for visibility) the unrealistically huge $\Pi = 0.05/\ell$, $\xi = 0.15$, the difference between DSR boosts and undeformed boosts is minute near the origin. But according to Bob's coordinates the emission of the hard particle appears to occur slightly off the (thick) worldline of the source.

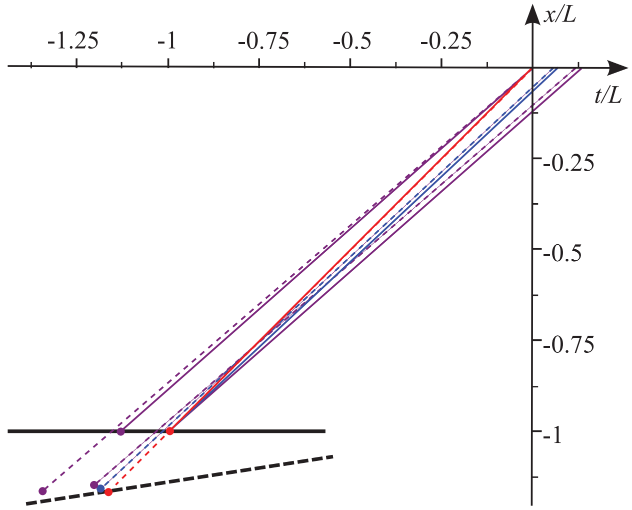

FIG. 3 (color). A case with two hard (violet lines) worldlines, with momentum $\Pi_v = 0.13/\ell$, a "semihard" (blue line) worldline with momentum $\Pi_b = \Pi_v/2$, and a ultrasoft worldline (red line, with $\Pi_r \ll 1/\ell$). According to Alice (whose lines are solid, while boosted Bob has dashed lines) three of the worldlines give a distant coincidence of events, while two of the worldlines cross in the origin.

The element of nonlocality that is actually produced by DSR-deformed boosts is seen by focusing on the "burst" of three photon worldlines also shown in Fig. 3, whose crossings establish a coincidence of events for Alice far from her origin, an aspect of locality encoded in a "distant coincidence of events." The objectivity of such distant coincidences of events is partly spoiled by the DSR deformation: the coincidence is only approximately present in the coordinates of an observer boosted with respect to Alice. But we stress that in Figs. 2 and 3 we used, for visibility, gigantically unrealistic values of photon momentum (up to $\sim 0.1/\ell$): it should nonetheless be noticed that even distant coincidence is objective up to a very good approximation, if indeed, as assumed in the DSR literature [1–5], the observer-independent length scale ℓ is as small as the Planck length ($\sim 10^{-35}$ m). On terrestrial scales one might imagine hypothetically to observe a certain particle decay with two laboratories, with a large relative boost of, say, $\xi \sim 10^{-5}$, with idealized absolute accuracy in tracking back to the decay region the worldlines of two particles that are the decay products. As one easily checks from (8) and (9), the peculiar sort of nonlocality we uncovered is of size $\xi \ell L \Pi$. Therefore, even if the distance L between the decay region and the observers is of, say, 10^4 m, and the decay products have momenta of, say, 100 GeV, one ends up with an apparent nonlocality of the decay region which is only of $\sim 10^{-19}$ m.

Another interesting case is the one of a typical observation of a gamma-ray burst, with GeV particles that travel for, say, 10^{17} s before reaching our telescopes. For two telescopes with a relative boost of $\xi \sim 10^{-4}$ the loss of coincidence of events at the source is ~ 100 m, well below the sharpness we are able to attribute [12] to the location of a gamma-ray burst.

Actually, in light of our results, two relatively boosted DSR observers should not dwell about distant coincidences, but rather express all observables in terms of local measurements. For the burst of three photons shown in Fig. 3 the momentum dependence of the speed is manifest both for Alice and Bob in the correlation between arrival times and momentum.

On the relation between boosts and simultaneity we notice that with Galileian boosts there is absolute simultaneity, while with Lorentz boosts, a deformation of Galileian boosts, simultaneity remains objective only for events occurring at the same spatial position (but independently of the distance between observer and events). We found that with one more step of deformation, the DSR boosts, simultaneity is only objective if the events are coincident according to a local observer.

Amusingly it appears that the possibility of coincident events was cumbersome already for Einstein, as shown by a footnote in the famous 1905 paper [18]: "We shall not discuss here the imprecision inherent in the concept of simultaneity of two events taking place at (approximately) the same location, which can be removed only by abstraction." We conjecture that the proper description of the quantum-gravity realm, whether or not there will be a role for DSR concepts, will impose the renunciation of the idealization of the possibility of exact and absolute coincidence of events.

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