Testing the Chiral Magnetic and Chiral Vortical Effects in Heavy Ion Collisions

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We devise a test of the chiral magnetic and chiral vortical effects (CME and CVE) in relativistic heavy ion collisions that relies only on the general properties of triangle anomalies. We show that the ratio $R_{EB} = J_E/J_B$ of charge J_E and baryon J_B currents for CME is $R_{EB}^{\rm CME} \to \infty$ for three light flavors of quarks ($N_f = 3$), and $R_{EB}^{\rm CME} = 5$ for $N_f = 2$, whereas for CVE it is $R_{EB}^{\rm CVE} = 0$ for $N_f = 3$ and $R_{EB}^{\rm CME} = 1/2$ for $N_f = 2$. The physical world with light u, d quarks and a heavier s quark is in between the $N_f = 2$ and $N_f = 3$ cases; therefore, the ratios R_{EB} for CME and CVE should differ by over an order of magnitude providing a possibility to separate clearly the CME and CVE contributions. In both cases, there has to be a positive correlation between the charge and baryon number asymmetries that can be tested on the event-by-event basis.

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Recently, STAR [1,2] and PHENIX [3] Collaborations at the Relativistic Heavy Ion Collider reported experimental observation of charge asymmetry fluctuations. While the interpretation of the observed effect is still under intense discussion, the fluctuations in charge asymmetry have been predicted to occur in heavy ion collisions due to the chiral magnetic effect (CME) in QCD coupled to electromagnetism [4–8]. Related phenomena have been discussed in the physics of primordial electroweak plasma [9] and quantum wires [10]. The CME scenario assumes a chirality asymmetry between left- and right-handed quarks, parametrized by an axial chemical potential μ_5. Such an asymmetry can arise if there is an asymmetry between the instanton and anti-instanton events (or any topology-changing transitions in general) early in the heavy ion collision. The QCD matter is known to undergo a sharp crossover to a deconfined and chirally symmetric phase at the critical temperature of $T_c \simeq 170$ MeV [11,12]. The produced quark-gluon plasma with temperature $T \simeq (2\text{–}3)T_c$ can possess a substantial axial chemical potential $\mu_5 \sim 0.1\text{–}0.3T_c$ [7]. The chirality asymmetry, coupled to a strong magnetic field $eB \sim T_c^2$ created by the colliding ions [6,13], generates a current of electric charge. This is the CME, which is one of several effects arising from triangle anomalies in the medium.

A related effect—the emergence of a chiral current in a medium with finite baryon density, in an external magnetic field or in the presence of a vorticity the fluid—has also been discussed in the literature [14–16]. The close connection between CME and the latter effect can be established, for example, by the method of dimensional reduction appropriate in the case of a strong magnetic field [17]: the simple relations $J_V^0 = J_A^1$, $J_A^0 = J_V^1$ between the vector J_V and axial J_A currents in the dimensionally reduced $(1 + 1)$ theory imply that the density of baryon charge must induce the axial current, and the density of axial charge must induce the charge current (CME).

The CME can be derived in several ways. A heuristic explanation is as follows: magnetic field leads to the spin polarization of quarks. But since there are, say, more right-handed quarks than left-handed quarks, the quarks will preferably move along the direction of the magnetic field, leading to a current. More rigorously, if one solves the Dirac equation in external magnetic field, one finds that the lowest Landau level is chiral. When there is a chemical potential for the axial charge, some of the energy levels in the lowest Landau levels are filled, inducing a nonzero current.

One may worry that the single-particle picture based on the Dirac equation will cease working once an interaction is turned on. However, the essentially topological nature of the phenomenon guarantees the result even in the presence of interaction. In particular, in holographic models (at infinite 't Hooft coupling) the magnitude of the chiral magnetic effect [18–20] appears the same as at weak coupling [18,21–23]. The CME has been studied in lattice QCD coupled to electromagnetism, both in the quenched [24–26] and dynamical (domain wall) fermion [27] formulations.

It is important to establish whether the CME explanation of charge asymmetry fluctuations is the correct one. First, it would be a direct observation of a topological effect in QCD. Second, the magnitude of this effect in the chirally broken phase is expected to be much smaller, since the large pion mass in this phase prohibits the existence of the axial chemical potential μ_5. (Note that in the chirally restored phase, the thermal correction to the quark mass does not break chiral symmetry.) Hence, the observation of the CME would manifest the restoration of chiral symmetry in the medium. The effort of quantifying the charge asymmetry fluctuations in QCD matter and of examining alternative explanations and backgrounds has already begun (see, e.g., [28–39]; Ref. [40] discusses also the baryon asymmetries), and there are plans to further study this effect at RHIC, LHC, FAIR, and NICA.

Because of the importance of the question, we need to devise tests for the CME mechanism. In this letter we propose such a test. Our proposal relies on two recent findings. The first is that the matter created at RHIC behaves as an almost perfect liquid: hydrodynamic models describe the gross properties of the droplet very well (for review, see [41]). The second finding is that quantum anomalies modify the hydrodynamics of a relativistic fluid. In addition to the chiral magnetic effect, there is also a chiral vortical effect: the vorticity $\vec{\omega}$, combined with a baryon chemical potential μ_B, creates an effective magnetic field $\mu_B\vec{\omega}$. Therefore one has, in addition to the CME, a chiral vortical effect (CVE). The exact magnitude of the effect in relativistic hydrodynamics has been found in Ref. [16], but its existence has been proposed before [5]. Vorticity in heavy ion collisions is a natural consequence of the angular momentum conservation (see, e.g., [4,42–44]); the estimates of vorticity and the discussion of its role in heavy ion collisions can be found in [45].

Let us first recall the general formulae for anomalous hydrodynamics [16]. Suppose that the system under consideration has a chemical potential μ, coupled to a charge $\bar{q}\gamma^0 Bq$, where B is a flavor matrix, and an axial chemical potential μ_5, coupled to the axial charge $\bar{q}\gamma^0\gamma^5 Aq$, where A is another flavor matrix. For simplicity, we shall assume that both μ and μ_5 are much smaller than the temperature T (this assumption usually holds in relativistic heavy ion collisions). The coefficient in Eq. (1) is independent of temperature (given that the system is in the chirally symmetric phase), since the triangle anomaly can be understood as a UV phenomenon. We also assume that electromagnetism couples to the current $\bar{q}\gamma^\mu Qq$, with Q being the charge matrix. If one measures a vector current $J^\mu = \bar{q}\gamma^\mu Vq$, then the result is

$$\vec{J} = \frac{N_c\mu_5}{2\pi^2}[\mathrm{tr}(VAQ)\vec{B} + \mathrm{tr}(VAB)2\mu\vec{\omega}] \qquad (1)$$

where $\vec{B}$ and $\vec{\omega}$ are the external magnetic fields and the fluid vorticity, respectively. The two parts of the current on the right-hand side correspond to the CME and the CVE, respectively. The traces in the formula are related to the anomalous triangle diagram.

We shall consider two cases: $N_f = 3$, where u, d and s quarks are light, and $N_f = 2$ where only u and d quarks are light. In both cases, we assume A to be the unity matrix, $A = \mathbb{1}$ (which is expected if the chiral asymmetry is due to instanton events, which are flavor symmetric), and $B = (1/3)\mathbb{1}$. For $N_f = 3$, $Q = \mathrm{diag}(2/3, -1/3, -1/3)$, and for $N_f = 2$, $Q = \mathrm{diag}(2/3, -1/3)$. There are two currents that we will measure: the electromagnetic current J_E, corresponding to $V = Q$ and the baryon current J_B, corresponding to $V = B$.

For CME, we get for the charge current (up to an overall factor of $N_c\mu_5\vec{B}/(2\pi^2)$ which is common for both charge and baryon currents)

$$J_E^{\mathrm{CME}} \sim \tfrac{2}{3}(N_f = 3) \quad \text{or} \quad \tfrac{5}{9}(N_f = 2) \qquad (2)$$

and for the baryon current

$$J_B^{\mathrm{CME}} = 0(N_f = 3) \quad \text{or} \quad \sim \tfrac{1}{9}(N_f = 2). \qquad (3)$$

For CVE, the results are (up to the overall factor $N_c\mu_5\mu\vec{\omega}/\pi^2$)

$$J_E^{\mathrm{CVE}} = 0(N_f = 3) \quad \text{or} \quad \sim \tfrac{1}{3}(N_f = 2); \qquad (4)$$

$$J_B^{\mathrm{CVE}} \sim 1(N_f = 3) \quad \text{or} \quad \sim \tfrac{2}{3}(N_f = 2). \qquad (5)$$

In the SU(3) case, the CME and CVE lead to completely different currents: the CME contributes only to the electromagnetic current and the CVE contributes only to the baryon current. In the SU(2) case, the separation is less clean, but the ratio of J_B/J_E still differs by a factor of 10. Note that the estimates above do not depend on the temperature (as long as it is above the chiral phase transition) since they originate from anomalies.

Let us now discuss the implications of our calculation in heavy ion collisions. It is known that the baryon chemical potential of the produced fireball depends on the collision energy: at smaller $\sqrt{s}$, μ is larger. Thus the CVE should be more important at lower energies. According to the computation above, J_B/J_E becomes larger as one lowers the energy of the collision. Moreover, since the symmetry arguments suggest that the magnetic field and the vorticity of the fluid have to be aligned, our results show that the two vectors $\vec{J}_B$ and $\vec{J}_E$ should point in the same direction.

We can now formulate our predictions. In addition to the charge separation, there must be a baryon number separation. The two effects are positively correlated on the event-by-event basis, and the relative importance of baryon number separation increases as one lowers the energy of the collision. Our predictions can be summarized as follows: (a) There should be a baryon number separation of the same sign as the electric charge separation; (b) the ratio between the baryon asymmetry and charge asymmetry should increase as the center of mass energy is lowered; (c) the magnitude of the ratio of charge and baryon asymmetries allows us to discriminate between the CME and CVE mechanisms.

As our calculation depends on very few assumptions about the properties of the quark-gluon plasma beside the existence of the initial chirality imbalance, the predictions above can be viewed as a nontrivial test for the CME explanation of the charge asymmetry fluctuations at

RHIC. *A priori*, the charge asymmetry and baryon asymmetry do not have to be correlated, but this correlation must exist if CME is the mechanism underlying charge asymmetry fluctuations.

Let us discuss the uncertainties of our predictions. Our computation of the ratio of charge and baryon currents depends only on the general properties of the triangle anomalies, and so should be robust. The experimental study of baryon asymmetry would ideally require the measurement of all produced baryons and antibaryons within a certain (symmetric) rapidity interval. Since this may not be feasible, our prediction would have to be supplemented by an assumption about the relative contributions of protons, neutrons and hyperons (and the corresponding antibaryons). It is highly desirable to perform a reliable evaluation of the absolute values of currents and asymmetries, and not just of their ratios. This computation would require a quantitative control over the magnitude of vorticity in the produced quark-gluon fluid and a treatment of the time evolution of vorticity and of magnetic field; such a study could be performed by methods of relativistic magnetohydrodynamics taking account of triangle anomalies. The hydrodynamic equations of a system with anomalies, in an arbitrary external electromagnetic field, have been derived in Ref. [16].

To summarize, we propose to test the CME and CVE in heavy ion collisions by the event-by-event study of correlations between the electric charge and baryon number asymmetries. We have evaluated the ratios of electric charge and baryon number asymmetries for CME and CVE mechanisms; our calculations depend only on the general properties of triangle anomalies.

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