Entangling Different Degrees of Freedom by Quadrature Squeezing Cylindrically Polarized Modes

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Quantum systems such as, for example, photons, atoms, or Bose-Einstein condensates, prepared in complex states where entanglement between distinct degrees of freedom is present, may display several intriguing features. In this Letter we introduce the concept of such complex quantum states for intense beams of light by exploiting the properties of cylindrically polarized modes. We show that already in a classical picture the spatial and polarization field variables of these modes cannot be factorized. Theoretically it is proven that by quadrature squeezing cylindrically polarized modes one generates entanglement between these two different degrees of freedom. Experimentally we demonstrate amplitude squeezing of an azimuthally polarized mode by exploiting the nonlinear Kerr effect in a specially tailored photonic crystal fiber. These results display that such novel continuous-variable entangled systems can, in principle, be realized.

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Entanglement is one of the most fascinating features arising from quantum mechanics and is of great importance for applications in quantum optics such as quantum information processing, quantum lithography, and quantum imaging [1–5]. Most of the currently investigated systems contain entanglement in the same degree of freedom (d.o.f.), among other things, in the quadrature [6], polarization [7,8], or spatial field variables [4,9]. Recent developments in the single-photon regime have introduced so-called hybrid entanglement [10–14], which can expand the application spectrum of entangled states. The term ‘‘hybrid entanglement’’ denotes the peculiar property of quantum states to manifest nonclassical correlations between different d.o.f. of the system itself [12]. These different d.o.f. may belong either to different parties of the same quantum system, e.g., the matter part and radiation part of an atom-light interacting system [15,16], or to a single-party system, e.g., the polarization and time-bin d.o.f. of a single photon [12]. One common method in discrete space to generate hybrid-entangled states is by exciting an additional d.o.f. in one arm of a single, e.g., polarization, entangled state. We, on the other hand, concentrate on another more direct method by generating the hybrid entanglement in the nonlinear process.

In this Letter we exploit the properties of cylindrically polarized modes to investigate hybrid entanglement in continuous-variable systems, the noise properties of which are described by the electric field variables. The intriguing feature of these cylindrically polarized modes is that their spatial and polarization field variables cannot be separated even in a classical description. We theoretically show that this property leads to entanglement between the spatial and polarization d.o.f. when one of these modes is quadrature squeezed. Such hybrid-entangled systems represent a new class of yet unexplored continuous-variable entangled states. The cylindrically polarized modes utilized in our investigation already have a wide range of important applications ranging from the generation of sharper focused light beams for the usage in lithography or nonconfocal microscopy [17] to unity-efficient coupling to a single atom [18].

New concepts, such as continuous-variable hybrid entanglement, are likely to be of similar advantage for a wide range of applications in quantum optics as they already have proven to be in the discrete variables [19,20]. Furthermore, we would like to note that continuous-variable hybrid entanglement in principle can be applied to other physical systems, such as position and momentum of particles or phonon interactions in solid state physics as well.

A classical light field is commonly described by its spatial mode function as well as its polarization vector. In most cases, such a description can be carried out in a single mode picture by constructing a proper mode basis in which only one spatial mode and one polarization mode are occupied. However, as will be shown, such a basis transformation is not possible for cylindrically polarized modes. They display nontrivial structural properties which, when investigated in a quantum-mechanical picture, can yield continuous-variable hybrid entanglement. To illustrate this, consider, for example, the classical description of a radially polarized beam of light which can be written as

$$\mathbf{u}_R(x, y, z) = (\hat{x}\psi_{01} + \hat{y}\psi_{10})/\sqrt{2}, \qquad (1)$$

where the functions ψ_{nm}, $n, m \in \{0, 1\}$, are the first-order Hermite-Gauss paraxial solutions of the scalar wave

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equation and $\hat{x}$ and $\hat{y}$ are unit vectors denoting linear polarization along the x and y axes, respectively. If we redefine $\{\hat{x}, \hat{y}\} \equiv \{\hat{e}_1, \hat{e}_2\}$ and $\{\psi_{10}, \psi_{01}\} \equiv \{v_1, v_2\}$, one can rewrite Eq. (1) as $\mathbf{u}_R(x, y, z) = \sum_{n=1}^{2} \sqrt{\lambda_n}\, \hat{e}_n v_n$ with $\lambda_1, \lambda_2 = 1/2$. Although $\mathbf{u}_R(x, y, z)$ represents a perfectly classical object, it has the same tensor-product form of the quantum state of a two-dimensional bipartite maximally entangled system with Schmidt rank [21] $K = 1/\sum_{n=1}^{2} \lambda_n^2 = 2$. Therefore, $\mathbf{u}_R(x, y, z)$ is not separable into the product of a spatially uniform polarization vector $\hat{u}$ and a singular function $f(x, y, z)$: $\mathbf{u}_R(x, y, z) \neq \hat{u} \cdot f(x, y, z)$; i.e., the polarization and spatial d.o.f. are not separable. We will refer to this intriguing classical feature as *structural inseparability.* Similar nonseparable modes have been used to violate Bell-like spin-orbit inequalities in discrete variables [22].

A closer inspection of Eq. (1) also reveals that, still in analogy with a maximally entangled quantum singlet state, it is shape invariant with respect to simultaneous polarization and spatial basis change. For example, with $\hat{e}_\pm = (\hat{x} \pm i\hat{y})/\sqrt{2}$ and $\phi_\pm = (\psi_{10} \pm i\psi_{01})/\sqrt{2}$ denoting the circular and orbital-angular-momentum polarization and spatial bases respectively, Eq. (1) can be rewritten as

$$\mathbf{u}_R(x, y, z) = (\hat{e}_+ \phi_- + \hat{e}_- \phi_+)/\sqrt{2}. \tag{2}$$

In order to show that quantum hybrid-entangled states can be generated by quadrature squeezing a cylindrically polarized optical mode, one needs to change from a classical to a quantum-mechanical description of the state. To do this, let us consider the special case of a bright squeezed azimuthally polarized mode, the state that was used in the experiment. However, it should be noted that the unique features contained in this state can also be observed for other cylindrically polarized optical modes and are, furthermore, not limited to bright states but are also present in squeezed vacuum states. The annihilation operator of the azimuthally polarized mode can be written as

$$\hat{a}_A = (-\hat{a}_{x01} + \hat{a}_{y10})/\sqrt{2}. \tag{3}$$

An azimuthally polarized mode (as well as a radially polarized mode) has a Schmidt rank of 2, i.e., that no matter which basis one chooses, be it the Hermite-Gaussian, Laguerre-Gaussian, or any other basis, one always needs two modes to describe this state. Here we choose, in analogy with Eq. (1), the Hermite-Gaussian basis to investigate the state.

The annihilation operator $\hat{a}_A$ is associated with the coherent azimuthally polarized eigenstate $|\alpha\rangle_A = \hat{D}_A(\alpha)|0\rangle$ with the displacement operator being defined as $\hat{D}_A(\alpha) = \exp(\alpha\hat{a}_A^\dagger - \alpha^*\hat{a}_A)$, α being the classical complex amplitude of the field. To squeeze this state an appropriate squeezing operator can be defined $\hat{S}_A(\zeta) = \exp[(\zeta^*\hat{a}_A^2 - \zeta\hat{a}_A^{\dagger 2})/2]$, ζ being a parameter quantifying the amount of squeezing. The bright squeezed azimuthally polarized state is then given by

$$|\alpha, \zeta\rangle = \hat{D}_A(\alpha)\hat{S}_A(\zeta)|0\rangle = \hat{U}_{x01}\left(-\frac{\alpha}{\sqrt{2}}, \frac{\zeta}{2}\right)\hat{U}_{y10}\left(\frac{\alpha}{\sqrt{2}}, \frac{\zeta}{2}\right)\hat{S}_{x01,y10}\left(-\frac{\zeta}{2}\right)|0\rangle. \tag{4}$$

We define $\hat{U}_i(\alpha/\sqrt{2}, \zeta/2) = \hat{D}_i(\alpha/\sqrt{2})\hat{S}_i(\zeta/2)$ for describing the single modes and additionally the two-mode squeezing operator

$$\hat{S}_{x01,y10}\left(-\frac{\zeta}{2}\right) = \exp[(-\zeta^*\hat{a}_{x01}\hat{a}_{y10} + \zeta\hat{a}_{x01}^\dagger\hat{a}_{y10}^\dagger)/2]. \tag{5}$$

Equation (4) clearly shows that $|\alpha, \zeta\rangle$ is a two-mode entangled state, where the two-mode squeezing operator [Eq. (5)] fully determines the degree of entanglement of the state. This intrabeam entanglement can be utilized by splitting the cylindrically polarized mode into its two basis modes with the help of an adequate mode splitter. By making use of the well-known polarization and spatial Stokes parameters [23–25], three different kinds of measurement sets can be performed on the two modes $\hat{a}_{x01}$ and $\hat{a}_{y10}$ (Fig. 1): (a) polarization Stokes measurements on mode $\hat{a}_{x01}$ and $\hat{a}_{y10}$, (b) spatial Stokes measurements on mode $\hat{a}_{x01}$ and $\hat{a}_{y10}$, and (c) spatial Stokes measurements on mode $\hat{a}_{x01}$ and polarization Stokes measurements on mode $\hat{a}_{y10}$ or vice versa.

Because of the two-mode squeezing operator [Eq. (5)], entanglement between modes $\hat{a}_{x01}$ and $\hat{a}_{y10}$ is present in all three cases. Especially intriguing is case (c) which shows entanglement between the spatial and polarization d.o.f. We refer to this entanglement, where different d.o.f. are measured in the two arms of the entangled state, as continuous-variable hybrid entanglement in analogy with the discrete variables.

To quantify these quantum correlations the continuous-variable Duan criterion for nonseparability can be applied to the Stokes parameters measured in the two spatially separated subsystems a and b [26,27]. For a symmetric system, i.e., $\langle\Delta\hat{X}^a\Delta\hat{Y}^a\rangle = \langle\Delta\hat{X}^b\Delta\hat{Y}^b\rangle$ with $\hat{X}$ and $\hat{Y}$ being

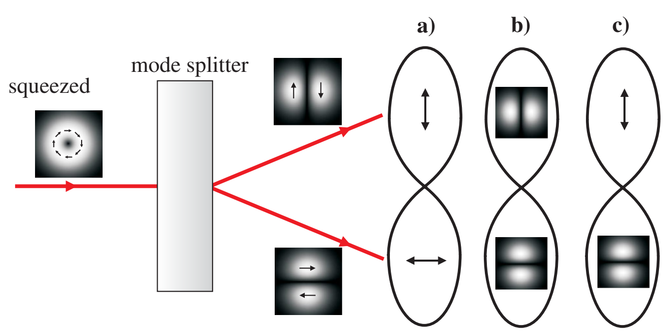

FIG. 1 (color online). Three types of entanglement are contained in a squeezed azimuthally polarized mode which can be observed by utilizing a mode splitter: (a) polarization entanglement, (b) spatial entanglement, and (c) hybrid entanglement.

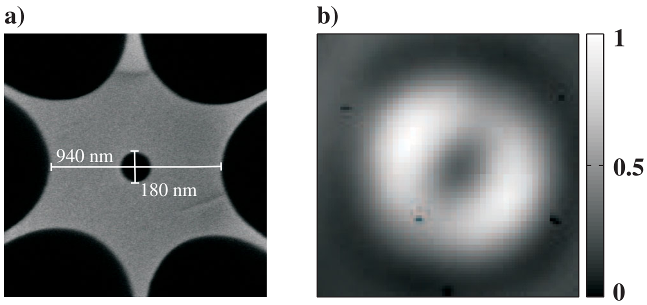

FIG. 2 (color online). (a) The specially designed photonic crystal fiber which supports the azimuthally polarized mode. (b) Normalized mode intensity profile of the azimuthally polarized mode at $\lambda = 810$ nm, measured after the fiber.

two arbitrary but conjugate quadrature operators, the criterion is given by

$$0 \leq V(\hat{S}^a_{\mu,\text{d.o.f1}} + \hat{S}^b_{\mu,\text{d.o.f.2}}) + V(\hat{S}^a_{\nu,\text{d.o.f.1}} - \hat{S}^b_{\nu,\text{d.o.f.2}}) < 4|\alpha|, \tag{6}$$

where V is the standard variance of an operator $V(\hat{X}) = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2$ and $\alpha = \text{cov}(\hat{S}^a_\mu, \hat{S}^a_\nu) = \text{cov}(\hat{S}^b_\mu, \hat{S}^b_\nu)$ is the covariance of two Stokes parameters. The inequality (6) can be evaluated for the three combinations of the Stokes parameters $(\mu, \nu) = (1, 2)$, $(\mu, \nu) = (1, 3)$, and $(\mu, \nu) = (2, 3)$. By measuring the appropriate combination of polarization or spatial Stokes parameters in arm a or b, either polarization $(\text{d.o.f.1}, \text{d.o.f.2}) = (\text{pol}, \text{pol})$, spatial $(\text{d.o.f.1}, \text{d.o.f.2}) = (\text{spa}, \text{spa})$, or hybrid-entanglement, i.e., $(\text{d.o.f.1}, \text{d.o.f.2}) = (\text{spa}, \text{pol})$ or $(\text{d.o.f.1}, \text{d.o.f.2}) = (\text{pol}, \text{spa})$, can be observed. One can then show that, for example, the Stokes parameters $\hat{S}_2$ and $\hat{S}_3$ are entangled for all combinations of (d.o.f.1, d.o.f.2):

$$V(\hat{S}^a_{2,\text{d.o.f.1}} + \hat{S}^b_{2,\text{d.o.f.2}}) + V(\hat{S}^a_{3,\text{d.o.f.1}} - \hat{S}^b_{3,\text{d.o.f.2}}) = e^{-s} \cosh s < 1. \tag{7}$$

Here V is normalized to $4|\alpha|$ and the parameter s quantifies the amount of squeezing in the cylindrically polarized mode. We would like to stress the very intriguing feature that such continuous-variable hybrid entanglement already exists in a quadrature squeezed cylindrically polarized mode. In the following we will present an experimental setup which generates such a squeezed state.

We experimentally amplitude squeeze an azimuthally polarized mode by exploiting the nonlinear Kerr effect in a fiber which directly supports this mode. In the experiment a mode-locked Ti:sapphire laser, centered at a wavelength of 810 nm and producing 170 fs pulses, acts as a light source. As a nonlinear medium a specially designed photonic crystal fiber with a core diameter of 940 nm and in its center a subwavelength hollow channel (diameter 180 nm) is chosen [Fig. 2(a)]. This specific structure allows an azimuthally polarized mode to be maintained during propagation, much like the linearly polarized eigenmodes of a polarization-preserving fiber [28,29]. To efficiently excite the desired mode inside the fiber a polarization converter (ARCoptix) is used to generate an azimuthally polarized mode which is then coupled into the fiber. The output mode of the fiber is shown in Fig. 2(b). Its asymmetric structure arises from a minor distortion in the cladding structure of the fiber. Therefore the general structure of the core is not perfectly symmetric, leading to an azimuthally polarized mode with slightly higher intensities along one axis.

In the experimental setup a Sagnac interferometer [30] consisting of a 40 cm long fiber and a beam splitter with a highly asymmetric splitting ratio of 90:10 is used [Fig. 3(a)]. The third-order nonlinear Kerr effect present in the specially designed photonic crystal fiber generates quadrature squeezing. However, for certain input energies amplitude squeezing inside the interferometer is generated which can easily be observed with a direct detection scheme [Fig. 3(b)] at the output of the Sagnac loop.

The detection system consists of a detector with sub-shot-noise resolution at a radio frequency sideband at 10.7 MHz and a high quantum efficiency silicon photodiode. A coherent beam is used to determine the quantum noise limit (QNL). We observe amplitude squeezing of the azimuthally polarized mode of 0.6 ± 0.1 dB below the QNL [Fig. 4(a)]. The squeezing was observed at pulse energies of about 14 pJ, slightly above the fiber's soliton energy.

As has been shown, the squeezing of the azimuthally polarized mode implies that we have generated a continuous-variable hybrid entangled state. It should be

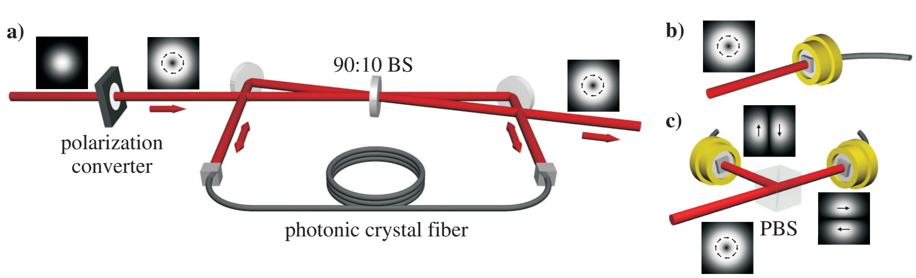

FIG. 3 (color online). (a) The experimental setup of the Sagnac loop to generate amplitude squeezing in the azimuthally polarized mode [beam splitter (BS)]. (b) Direct detection is used to measure the amplitude squeezing. (c) The detection system to measure amplitude correlations between the horizontally polarized TEM_{01} mode and the vertically polarized TEM_{10} mode [polarizing beam splitter (PBS)].

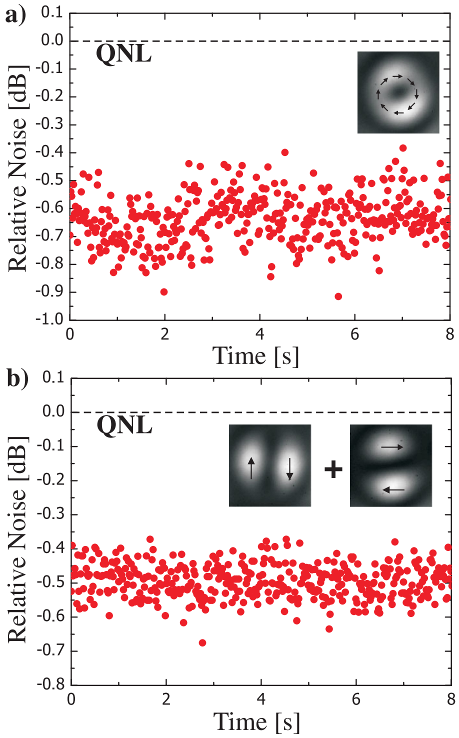

FIG. 4 (color online). (a) A quantum noise reduction of 0.6 dB is observed in the azimuthally polarized mode. (b) Anticorrelations of 0.5 dB below the QNL of the horizontally polarized TEM_{01} and vertically polarized TEM_{10} have been measured. The insets illustrate the measured spatial modes and the arrows indicate the polarization of the state.

noted that the observed quadrature squeezing is a necessary but not sufficient condition to fully verify continuous-variable hybrid entanglement. The full verification of the hybrid entanglement by observing the violation of the Duan criterion for nonseparability [Eq. (6)] is part of future investigations. However, a fundamental feature of this entanglement is the intensity anticorrelations between the horizontally polarized TEM_{01} and vertically polarized TEM_{10} mode as stated in Eq. (3). These can be measured with the detection system illustrated in Fig. 3(c). It consists of a polarizing beam splitter (PBS) and two intensity detectors with carefully balanced amplifiers. The PBS acts as a mode splitter, dividing the azimuthally polarized mode into a horizontally polarized TEM_{01} and a vertically polarized TEM_{10} mode which impinge on the two detectors. By taking the sum signal of the detectors the anticorrelations are determined, while the QNL is measured with a coherent beam. Anticorrelations of 0.5 ± 0.1 dB below the QNL have been observed [Fig. 4(b)]. This strongly indicates the existence of entanglement between the horizontally polarized TEM_{01} mode and the vertically polarized TEM_{10}.

In conclusion, a novel kind of state, namely, a continuous-variable hybrid-entangled state, has been presented. This state has the peculiar attribute that its quantum hybrid entanglement is based on a structural inseparability of the classical state itself. It has been theoretically shown that such a highly complex state can be generated in a remarkably convenient manner by squeezing a cylindrically polarized mode. Experimentally a quadrature squeezed azimuthally polarized mode has been demonstrated, showing that the generation of hybrid-entangled states is, in principle, feasible.

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