Time Reversal of Speckle Noise

Gabriel Montaldo, Mickael Tanter, and Mathias Fink

Institut Langevin, ESPCI ParisTech, CNRS, INSERM, Université Paris VII, 10 rue Vauquelin, 75005, Paris, France

(Received 22 March 2010; revised manuscript received 27 December 2010; published 2 February 2011)

Focusing a wave in an unknown inhomogeneous medium is an open problem in wave physics. This work presents an iterative method able to focus in pulse-echo mode in an inhomogeneous medium containing a random distribution of scatterers. By performing a coherent summation of the random echoes backscattered from a set of points surrounding the desired focus, a virtual bright pointlike reflector is generated. A time-reversal method enables an iterative convergence towards the optimal wave field focusing at the location of this virtual scatterer. Thanks to this iterative time-reversal process, it is possible to focus at any arbitrary point in the heterogeneous medium even in the absence of pointlike source. An experimental demonstration is given for the correction of strongly distorted images in the field of medical ultrasound imaging. This concept enables envisioning many other applications in wave physics.

DOI: 10.1103/PhysRevLett.106.054301 PACS numbers: 43.20.+g, 42.15.Fr

In the last decade, the concept of time reversal was proven to be an elegant way to focus waves in inhomogeneous media. If a punctual source emits a pulse, the resulting wave field propagates through the medium up to the emit or receive antennas and the long-time records by this set of antennas are equivalent to the time-dependent Green's function of the pulse-emitting source. The reemission of these signals in a reversed chronology by the set of antennas, i.e., the time-reversed Green's function, will generate a converging wave that refocuses on the initial source location whatever the medium heterogeneities are [1]. The same concept can be applied in pulse-echo mode to passive sources such as reflectors illuminated by a first incident wave front emitted by the antennas. In particular, when several well resolved scatterers are located behind an aberrating layer, that distorts the incident pulsed wave front, the backscattered echoes can be recorded, time reversed, and reemitted in an iterative process. After several iterations the process converges and create a wave front that exactly focus on the brightest scatterer [2,3] and permits to assess optimal focusing from the direct estimation of the Green's function associated to this scatterer. Note that this iterative process converges only if the scatterers separation is sufficient to avoid overlap between the different beams focused on each scatterer. Focusing a wave in a medium made of a random distribution of nonresolved scatterers is a much more challenging problem that interests various communities.

To solve this problem, different adaptive focusing approaches [4–7] based on signal processing of the backscattered echoes were proposed with a limited success in terms of practical applicability. Despite its elegance, the direct application of the iterative time-reversal approach is also unable to converge towards an estimate of the Green's functions in such random media. Here, we address and solve this problem for an inhomogeneous medium containing a random distribution of scatterers and dominated by *single scattering process*; i.e., the propagation distance must be shorter than the mean free path. This kind of medium gives rise to a random backscattered field referred as the "speckle noise" in the ultrasound community. We show how to create a virtual bright reflector from a random distribution of scatterers behind an aberrating layer. The key idea consists in adding coherently the backscattered echoes coming from independent speckle realizations before time reversal. The speckle realizations are obtained by microsteering of an initial focused beam. By iterating this process, it is shown how it converges towards a virtual pointlike reflector located in the main lobe of the first illuminating beam.

The proposed concept is illustrated by an experimental example in ultrasound. An ultrasonic array made of 128 piezoelectric elements (5 MHz central frequency, 60% bandwidth) is driven by a fully programmable electronic system. The medium consists of a gelatine phantom embedding a random distribution of scatterers. Sound speed in the gelatine phantom is homogeneous (~ 1500 m/s). On top of the medium an aberrating layer strongly distorts the propagating wave field both in amplitude (6 dB distortions) and phase (3π variations). The ultrasonic array is placed in contact with the aberrating layer (see Fig. 1).

To focus a wave with an array of N emit or receive elements, the concept of time-reversal focusing consists in assessing the Green's function $g(x_n, \mathbf{r}_F, t)$ between the desired focal point $\mathbf{r}_F$ and the transducers located at position x_n. Its time-reversed version $g(x_n, \mathbf{r}_F, -t)$ corresponds to the optimal set of signals for focusing at $\mathbf{r}_F$. In the presence of some well resolved scatterers, the iterative time-reversal method consists in a first pulsed emission (for example a plane wave) insonifying a large field of view and that gives rise to backscattered signals. A time window enables selecting a portion of the signals coming from the desired region of interest. These signals are time-reversed and sent back to the medium, and the new backscattered

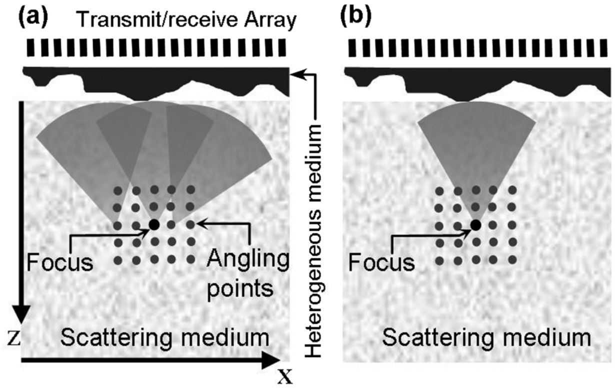

FIG. 1. Principle of time-reversal focusing in speckle noise. (a) Focused beams are emitted at different locations nearby the desired focus. (b) Backscattered signals are recorded and "steered" using time delays as if all signals were coming from the same location.

signals are recorded. By iterating this process it is possible to converge towards the Green's function $g(x_n, \mathbf{r}, t)$ of the brightest scatterer located at position $\mathbf{r}$ [3]. This method converges only if the number of scatterers selected in the temporal window is smaller than N and if the distance between scatterers is larger than the focal spot of the time-reversal mirror [3].

In this Letter, this method is extended to focus in a random distribution of scatterers. For sake of clarity the general concept is described in two steps. In a first section, it is shown how a virtual pointlike reflector can be created from the speckle noise in a simple medium with a uniform and known sound speed. The signature of this artificial pointlike reflector obtained using a coherent summation of different realizations of the backscattered "speckle" signals is used for time-reversal focusing. In a second section, it is shown how this creation of virtual pointlike reflector can be achieved in more complex heterogeneous medium.

Creating a "bright star" in a homogeneous randomly scattering medium.— Learning how to focus in a homogeneous medium whose wave speed is known can seem an anecdotical goal, as the solution is trivial. However, our goal here is different: the section aim is to show, when focusing at different locations in the focal area, how coherent summation of backscattered speckle noise can lead to reconstructing the pulsed acoustic signature of a "virtual" pointlike reflector at the desired focus (virtual stands for the fact that such real bright spot is not available in the medium).

Let us first emit a focused wave in the randomly scattering medium. As the medium wave speed c is known, the initial emitted signals (iteration 0) on each element n of the array generate a cylindrical wave front and can be written

$$e_0(x_n, t) = g_0(x_n, \mathbf{r}_F, -t) \otimes e(t), \quad (1)$$

with $g_0 = \delta(t - \tau(x_n, \mathbf{r}_F))/4\pi c \tau(x_n, \mathbf{r}_F)$ being the free space Green's function for an homogeneous medium with travel time $\tau(x_n, \mathbf{r}_F) = c^{-1}\sqrt{(x_n - x_F)^2 + z_F^2}$, $e(t)$ the time profile of the emitted pulse, $\otimes$ stands for the time convolution, and c is the wave speed. This wave front focuses at location $\mathbf{r}_F$ and is backscattered by the random distribution of scatterers. The signals $s(x_n, t)$ are then received and selected in an appropriate time window [see Fig. 2(a)].

A key idea comes from the fact that independent realizations of speckle noise coming from a single location can be constructed using different emit beams focused on several points $\mathbf{r}_i$ located nearby around the desired focal point$\mathbf{r}_F$ (lateral and axial shifts). Each beam generates different interferences of the echoes and a set of independent speckle realizations $S_i = \{s_1, s_2, \ldots, s_n\}$ is obtained. As these different backscattered signals s_i correspond to different emit focal spots, they must be "steered back" as if they were all coming from the same initial point$\mathbf{r}_F$. This "back-steering" operation is performed by applying time delays that correspond to the travel time differences for focusing at points $\mathbf{r}_i$ and$\mathbf{r}_F$. These time delays are easily calculated as the wave speed is known and the steered signals are $\tilde{s}_i(x_n, t) = s_i(x_n, t - \tau(x_n, \mathbf{r}_i) + \tau(x_n, \mathbf{r}_F))$.

These signals correspond to independent realizations of speckle noise "virtually coming" from an identical spot at the desired focal point [see Fig. 2(b)]. In other words, it mimics an experiment where the array would focus always at the same location but the random distribution of scatterers in the focal spot was changed before each emission.

The backscattered signals in Fig. 2 clearly exhibit a wave front curvature and a strong spatial coherence between neighboring transducers. This result is explained by the van Cittert-Zernike theorem. It states that the spatial coherence of a wave field originated from an incoherent extended source is described by the Fourier transform of the intensity distribution over the source. In pulse-echo mode, it is the random distribution of scatterers, illuminated by the focused beam, that behaves as in incoherent source. Thus, the backscattered field spatial coherence is directly related to the size of the focal spot [9,10]; i.e., a sharper focus generates a spatial coherence in the backscattered wave. Here, as the emit beams are quite well

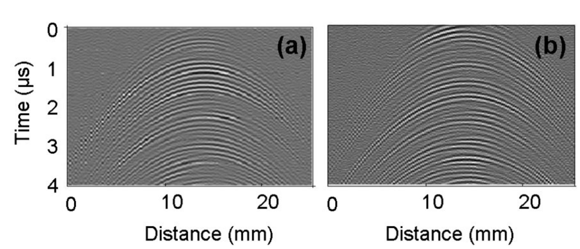

FIG. 2. (a) Observation of the backscattered signals recorded by the array elements (horizontal axis) when the initial beam is focused in a scattering phantom at point $\mathbf{r}_F$, (b) backscattered signals observed on the array when the focus is shifted at neighboring point $\mathbf{r}_i$ with delay compensation.

focused, the backscattered signals coming back from a single focal spot have similarities from one transducer to its neighbors [see Fig. 2(a)] as the wave field keeps a good spatial coherence. Figure 2(b) shows the signals backscattered when focusing at another location after back-steering in order to mimic signals coming from the desired focal spot. The wave fronts in Fig. 2(a) and 2(b) are very similar; however, a careful observation of the two figures reveals different lateral amplitude modulations and phase differences (delay) between the two figures. Such differences are easy to understand as the signals come from different random distributions of scatterers.

A simple direct summation of all speckle realizations $\tilde{s}_1$, $\tilde{s}_2, \ldots, \tilde{s}_n$ does not give a deterministic signal as seen on Fig. 3(a). Indeed, there is an unknown delay between each realization that avoids constructive temporal interferences.

Again, this arbitrary time delay can be understood due to the fact that the time profile of the signals correspond to different random distribution of scatterers leading to totally different noise signals. However, an interesting point is that this additional delay can be directly estimated from the experimental data. One only has to estimate the time at which the cross correlation between $\tilde{s}_i(x_n, t)$ and the initial emit signal $e_0(x_n, t)$ is maximized. Once delays δ_i are estimated, the backscattered signals $\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_n$ can be corrected from these delays before summation in order to interfere constructively [Fig. 3(b)]. A quasi deterministic echo results from this averaging process as it seems to originate from a pointlike reflector. Figure 3(b) results from the summation over 25 realizations obtained by focusing the beams on a 5×5 points grid surrounding the desired focal point with a λ pitch (where λ is the wavelength) in the x direction (lateral direction) and a 3λ pitch in the z direction (depth). The summed echoes

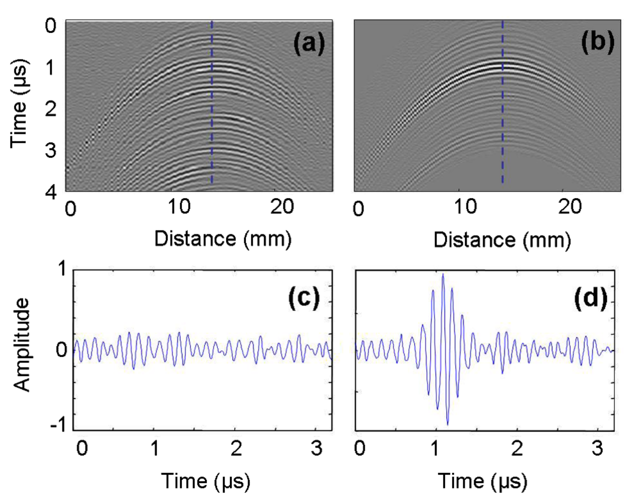

FIG. 3 (color online). Direct summation of 25 of these independent speckle realizations (a) without and (b) with delay compensation. Temporal signal in the central transducer (c) without and (d) with delay compensation.

recorded on the transducers lead to pulsed signals well compressed in time as seen in Fig. 3(d).

Focusing through an aberrating layer.—Now, how can we extend this concept of virtual bright reflector to heterogeneous media? In particular, what happens when an aberrating layer (Fig. 1) is inserted between the transducer array and the scattering medium? Depending on the severity of the aberrations, an extended area, known as the isoplanatic patch [10], can be defined for which the Green's functions $g(x_n, \mathbf{r}, t)$ relating each point $\mathbf{r}$ to the array elements are identical after correction of the travel path differences. In optics, for example, wave-front degradations induced by atmospheric turbulence on optical beams vary with the direction of propagation of these beams. The different compensating methods of turbulence effects are therefore limited to a small field of view called the isoplanatic patch. In other words, this isoplanatic patch corresponds to the area of sky around a reference star over which high-resolution imaging is possible.

Here, we are going to take benefit on both this isoplanatic patch and the randomness of the scattering medium. Our goal is to determine the true Green's function relating the focal point to the array in the heterogeneous medium. It is clear that, when applying the process described earlier with an initial conventional focusing (cylindrical) assuming the medium to be homogeneous, the emitted signals $e_0(x_n, t)$ [Eq. (1)] are no more matched to the heterogeneous medium. So, the initial focusing beams are not perfectly focused at their desired locations and strong sidelobes can occur. However, it provides an acceptable start as a first iteration. So, 25 realizations were performed on a similar 5×5 points mesh grid as described earlier (focusing on 25 points nearby the desired focus). Then, one uses the same back-steering and averaging processing. One should here notice that the back-steering is performed assuming that the medium is homogeneous. This assumption can be done as long as the nearby focusing points remain inside the isoplanatic patch around the targeted point (region where this assumption stands). Figure 4(a) shows the result $\tilde{s}^0(x_n, t)$ of this summation. As the initial focusing beams were not optimal, this summation does not provide a wave-front that focuses through the aberrating layer on a unique point $\mathbf{r}_F$. The signal $\tilde{s}^0(x_n, t)$ consists of the combination of a coherent wave front coming from the virtual scatterer at $\mathbf{r}_F$ and an incoherent speckle noise coming from other insonified areas. This corresponds to a typical configuration where an iterative time-reversal process [1,2] enables step by step to focus only on the pointlike reflector. Here, the same concept can be applied:

The new iteration step starts replacing $e_0(x_n, t)$ by the time-reversed version of $\tilde{s}^0(x_n, t)$ in Eq. (1). The new emission signal $e_1(x_n, t) = \tilde{s}^0(x_n, -t)$ are emitted and steered in the scattering medium at the 25 positions surrounding the desired focus and the same process can

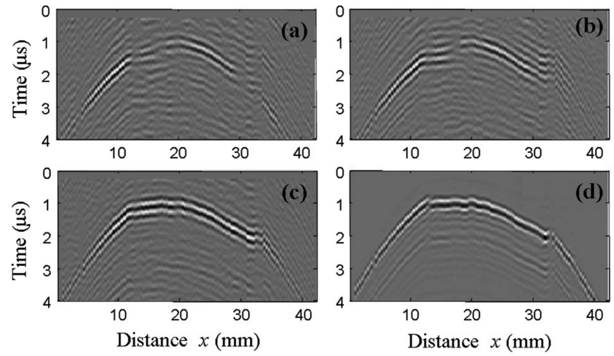

FIG. 4. (a),(b), and (c) Building of the Green's function of a virtual pointlike source in speckle noise after 1st, 2nd, and 5th iteration of steps 1–3. (d) "Reference" Green's function measured through the aberrating medium using a strong and pointlike reflector inserted at the targeted location.

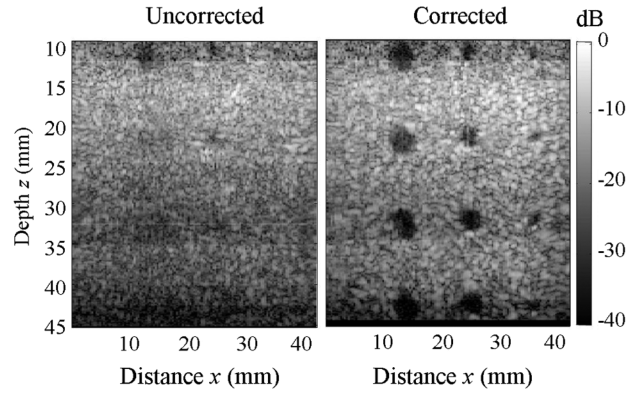

FIG. 5. (a) Conventional ultrasonic image of a tissue mimicking phantom containing hypoechogeneic inclusions surrounded by a speckle environment (b) corrected image using the Green's functions estimated by the iterative time-reversal method.

be applied: microsteering, delay, and summation in order to get a new signal $\tilde{s}^1(x_n, t)$. In Fig. 4(b) we can see the result of the first iteration in which the contribution from the virtual reflector is clearly enhanced compared to $\tilde{s}^0(x_n, t)$. This process is repeated by taking the emission signal at the kth step as $e_k(x_n, t) = \tilde{s}^{k-1}(x_n, -t)$ and at each iteration the emitted beam profile is improved. In Fig. 4(c) the signals of the 5th iteration $\tilde{s}^5(x_n, t)$ converge to a well-defined wave front that we can compare to the true Green function [Fig. 4(d)] measured by replacing the random scattering medium by a homogeneous one with a unique pointlike scatterer at the focal point.

The performance of the iterative process is related to the number of independent realizations of speckle signals that can be obtained while focusing within the isoplanatic patch. In a first approximation, it is intuitively related to the number of wavelengths inside the isoplanatic patch.

A valuable application of this adaptive focusing concept can be depicted in the field of medical ultrasonic imaging. Figure 5(a) presents a conventional ultrasonic image of a phantom containing some anechoic inclusions obtained through a strongly aberrating medium placed between the array surface and the tissue mimicking phantom. Then, the adaptive correction technique was performed for three different depths ($z = 15, 25, 35$ mm). After the estimation of the Green's functions, a new ultrasonic image can be built using the computed Green functions that correct for the aberrations. Figure 5(b) presents the ultrasonic image obtained using these corrected beams. The image quality is strongly improved and the hypoechogeneic inclusions become clearly visible.

The main limitations of this method are the single-scattering hypothesis and the difficulty to correct very strong aberrations. Both limitations are linked to an important parameter: the local ratio between the isoplanatic patch and the wavelength. On the one hand, the strong aberrations reduce the size of the isoplanatic patch compared to the wavelength and so limit the number of available independent speckle realizations. On the other hand, the presence of multiple scattering generates a speckle noise that cannot be added coherently for different speckle realizations even for very neighboring points. Indeed, in a multiple scattering medium, the size of the isoplanatic patch becomes very small and of the order of the wavelength. The ability to use time reversal focusing in a speckle environment even in the absence of pointlike source or bright reflector should offer great application opportunities in wave physics. Application of this adaptive focusing method in medical ultrasound, nondestructive testing, seismology, underwater acoustics, and radar could provide higher imaging quality.

[1] M. Fink, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **39**, 555 (1992).
[2] C. Prada and M. Fink, Wave Motion **20**, 151 (1994).
[3] Fink *et al.*, Rep. Prog. Phys. **63**, 1933 (2000).
[4] S.W. Flax and M. O'Donnell, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **35**, 758 (1988).
[5] L. Nock, G.E. Trahey, and S.W. Smith, J. Acoust. Soc. Am., **85**, 1819 (1989).
[6] J.P. Astheimer, W.C. Pilkington, and R.C. Waag, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **53**, 79 (2006).
[7] J.L. Robert and M. Fink, J. Acoust. Soc. Am. **123**, 866 (2008).
[8] R. Mallart and M. Fink, J. Acoust. Soc. Am. **90**, 2718 (1991).
[9] R. Mallart and M. Fink, J. Acoust. Soc. Am. **96**, 3721 (1994).
[10] F. Chassat, J. Opt. **20**, 13 (1989).