# Extended Ginzburg-Landau Formalism for Two-Band Superconductors

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Recent observation of unusual vortex patterns in $MgB_2$ single crystals raised speculations about possible "type-1.5" superconductivity in two-band materials, mixing the properties of both type-I and type-II superconductors. However, the strict application of the standard two-band Ginzburg-Landau (GL) theory results in simply proportional order parameters of the two bands—and does not support the "type-1.5" behavior. Here we derive the extended GL formalism (accounting all terms of the next order over the small $\tau = 1 - T/T_c$ parameter) for a two-band clean $s$-wave superconductor and show that the two condensates generally have different spatial scales, with the difference disappearing only in the limit $T \to T_c$. The extended version of the two-band GL formalism improves the validity of GL theory below $T_c$ and suggests revisiting the earlier calculations based on the standard model.

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The Ginzburg-Landau (GL) approach [1], based on Landau's theory of second-order transitions, is one of the most powerful and most widely used theoretical tools of present-day physics. It constitutes a solid base for theoretical studies in fields ranging from condensed matter theory (e.g., superconductivity, superfluidity, phase transitions, and fluctuation phenomena) to particle physics and cosmology (e.g., Higgs mechanism), and other topics reviewed in Ref. [2]. It is generally believed that the GL theory accurately describes essential physics in the vicinity of the critical temperature $T_c$ (and, qualitatively, in a much wider temperature range). Surprisingly, this is not the case for two-band (and multiband) superconductors, such as magnesium diboride [3] and several iron pnictides [4], etc., where the expected difference in spatial distributions of the two Cooper-pair condensates is not captured by the standard formulation of the two-band GL formalism. As explained further, the latter problem requires development of the extended GL theory, derived to a higher order in $\tau = 1 - T/T_c$. This is the core objective of this Letter.

Recently, unconventional vortex patterns were observed in single-crystal $MgB_2$ by Bitter decoration [5] and by scanning SQUID microscopy [6]. Although $MgB_2$ is largely accepted as a type-II two-band superconductor, in Refs. [5,6] no evidence of an Abrikosov lattice was found for low vortex densities. The interpretation was offered through the intervortex potentials derived from the standard two-band GL theory of, e.g., Refs. [7–9]. Namely, for particularly chosen (different) coherence lengths $\xi_i$ and penetration depths $\lambda_i$ of the two bands ($i = 1, 2$), vortices were shown to conventionally repel each other only at short distances, while long-range *attracting* [10]. This gives rise to stripelike vortex patterns, unattainable in either type-I or type-II superconductors, which led Moshchalkov *et al.* [5] to name this behavior "the type-1.5 superconductivity."

An avalanche of theoretical works followed [11,12], based either on the standard two-band GL formalism itself or the molecular dynamics simulations using the GL-calculated intervortex potentials, racing to describe the new type of superconductivity. Brandt and Das were the first to point out that long-range vortex attraction is not necessarily a "type-1.5" property [13]. The real criticism followed, in the analysis of Kogan and Schmalian [14]. They showed that in the standard formulation of the two-band GL approach [i.e., two GL equations for two order parameters $\Delta_i(\mathbf{x})$ coupled through the Josephson interband coupling terms] there appear contributions to both order parameters of higher orders than $\tau^{1/2}$, where $\tau = 1 - T/T_c$. However, the microscopic basis for the standard GL formalism assumes that only the terms $\propto \tau^{1/2}$ are accurate, which means that the aforementioned higher-order terms are incomplete and, thus, incorrect. After removing the higher-order contributions, Kogan and Schmalian found the order parameters of two bands to be proportional to each other and can thus be characterized by a single coherence length $\xi$. As a consequence, type-1.5 superconductivity is not supported by the formalism.

It is thus of abiding fundamental interest to clarify whether the relation $\Delta_1(\mathbf{x}) \propto \Delta_2(\mathbf{x})$ is generic to two-band superconductors or it holds only in the standard GL domain (to the order $\tau^{1/2}$ in $\Delta_i$'s). To settle the above issues, we derive here the extended version of the GL formalism for a two-band clean $s$-wave superconductor, where the contributions to $\Delta_j$'s up to the order $\propto \tau^{3/2}$ are included in their full, correct form (while appearance of the higher orders is precluded).

 

Our starting point is the BCS mean-field Hamiltonian of a two-band, $s$-wave, clean superconducting system, i.e.,

$$H^{\rm BCS} = H^c + \sum_{j=1,2} \int d^3x [\hat{\psi}^\dagger_{j\sigma}(\mathbf{x}) T_j(\mathbf{x}) \hat{\psi}_{j\sigma}(\mathbf{x}) + \hat{\psi}^\dagger_{j\uparrow}(\mathbf{x}) \hat{\psi}^\dagger_{j\downarrow}(\mathbf{x}) \Delta_j(\mathbf{x}) + \text{H.c.}], \tag{1}$$

where $j = 1, 2$ denotes each of the bands, $H^c$ is the $c$ term whose specific form (see, e.g., Ref. [7]) is not of relevance for the present investigation, $T_j(\mathbf{x})$ is the single-electron Hamiltonian, and the summation in the kinetic term is taken over the coinciding spin indices. The generalization of the mean-field self-consistency equation for two-band superconductors reads

$$\Delta_i(\mathbf{x}) = \sum_{j=1,2} g_{ij} \langle \hat{\psi}_{j\uparrow}(\mathbf{x}) \hat{\psi}_{j\downarrow}(\mathbf{x}) \rangle, \tag{2}$$

with $g_{ij}$ being the relevant coupling constants ($g_{ij} = g_{ji}$). One of the most powerful formalisms to treat the superconducting properties in the presence of a nonuniform spatial distribution of the pair condensate is the Gor'kov equations. For our study it is convenient to write these equations in the form of the Dyson equation for the $2 \times 2$-matrix band propagator $\check{\mathcal{G}}_{j\omega}$ (see, e.g., Ref. [15]):

$$\check{\mathcal{G}}_{j\omega} = \check{\mathcal{G}}^{(0)}_{j\omega} + \check{\mathcal{G}}^{(0)}_{j\omega} \check{\Delta}_j \check{\mathcal{G}}_{j\omega}, \tag{3}$$

with

$$\check{\mathcal{G}}_{j\omega} = \begin{pmatrix} \mathcal{G}_{j\omega} & \mathcal{F}_{j\omega} \\ \bar{\mathcal{F}}_{j\omega} & \bar{\mathcal{G}}_{j\omega} \end{pmatrix}, \qquad \check{\mathcal{G}}^{(0)}_{j\omega} = \begin{pmatrix} \mathcal{G}^{(0)}_{j\omega} & 0 \\ 0 & \bar{\mathcal{G}}^{(0)}_{j\omega} \end{pmatrix}, \tag{4}$$

where $\hbar\omega = \pi T(2n + 1)$ is the fermionic Matsubara frequency ($n$ is an integer and $k_B$ is set to unity) and the $2 \times 2$ matrix operator $\check{\Delta}_j$ in Eq. (3) is defined by

$$\check{\Delta}_j = \begin{pmatrix} 0 & \hat{\Delta}_j \\ \hat{\Delta}^*_j & 0 \end{pmatrix}, \qquad \langle \mathbf{x} | \hat{\Delta}_j | \mathbf{x}' \rangle = \delta(\mathbf{x} - \mathbf{x}') \Delta_j(\mathbf{x}'). \tag{5}$$

Equations (3) and (4) further give

$$\mathcal{F}_{j\omega} = \mathcal{G}^{(0)}_{j\omega} \hat{\Delta}_j \bar{\mathcal{G}}_{j\omega}, \tag{6a}$$

$$\bar{\mathcal{G}}_{j\omega} = \bar{\mathcal{G}}^{(0)}_{j\omega} + \bar{\mathcal{G}}^{(0)}_{j\omega} \hat{\Delta}^*_j \mathcal{G}^{(0)}_{j\omega} \hat{\Delta}_j \bar{\mathcal{G}}_{j\omega}, \tag{6b}$$

which makes it possible to expand $\mathcal{F}_{j,\omega}$ in powers of $\Delta_j$, when working near $T_c$. This is the well-known basis for Gor'kov's derivation of the GL theory [16].

Using the definition of the anomalous (Gor'kov) Green function,

$$\frac{1}{\beta\hbar} \sum_\omega e^{i\omega(t'-t)} \langle \mathbf{x} | \mathcal{F}_{j\omega} | \mathbf{x}' \rangle = -\frac{1}{\hbar} \langle \mathcal{T} \hat{\psi}_{j\uparrow}(\mathbf{x}t) \hat{\psi}_{j\downarrow}(\mathbf{x}'t') \rangle,$$

one can rewrite Eq. (2) in the form

$$\Delta_1(\mathbf{x}) = \lambda_{11} n_1 R_1(\mathbf{x}) + \lambda_{12} n_2 R_2(\mathbf{x}), \tag{7a}$$

$$\Delta_2(\mathbf{x}) = \lambda_{21} n_1 R_1(\mathbf{x}) + \lambda_{22} n_2 R_2(\mathbf{x}), \tag{7b}$$

where $R_j(\mathbf{x})$ is a polynomial of $\Delta_j(\mathbf{x})$ and its spatial derivatives; $\lambda_{ij} = g_{ij} N(0)$ and $n_j = N_j(0)/N(0)$, where $N_j(0)$ is the band-dependent density of states, and $N(0) = \sum_j N_j(0)$. To construct the GL equations for a two-band superconductor, one should evaluate $R_i$ with accuracy $O(\tau^{3/2})$. This results in two equations for $\Delta_1(\mathbf{x})$ and $\Delta_2(\mathbf{x})$ coupled through the Josephson-like terms (for a clean two-band $s$-wave superconducting system, see, e.g., Refs. [7,14]; for a dirty two-band superconductor, see, e.g., Refs. [8,9]). This is where the aforementioned analysis of Kogan and Schmalian [14] is important, as such a representation of the two-band GL equations must be corrected in order to avoid the appearance of terms of orders higher than $\tau^{1/2}$ in $\Delta_j$. An appropriate correcting procedure is given in detail in Ref. [14], and results in two decoupled GL equations for $\Delta_1$ and $\Delta_2$ which exactly map on the one-band GL theory: $\Delta_1(\mathbf{x}) \propto \Delta_2(\mathbf{x})$ and both have the same coherence length unlike the expectations based on the initial formulation of the two-band GL formalism.

We now extend the GL formalism up to the order $\tau^{3/2}$ in $\Delta_j$'s by taking

$$\Delta_j(\mathbf{x}) = \Delta^{(0)}_j(\mathbf{x}) + \Delta^{(1)}_j(\mathbf{x}), \tag{8}$$

with $\Delta^{(0)}_j \propto \tau^{1/2}$ and $\Delta^{(1)}_j \propto \tau^{3/2}$. To begin with, we limit ourselves to a case of the zero-magnetic field ($\Delta_j$'s are real). Evaluating $R_j$ with accuracy $O(\tau^{5/2})$, we obtain

$$R_j = -\tilde{a}\Delta_j - \tilde{b}\Delta^3_j + \tilde{c}\Delta^5_j + \tilde{\mathcal{K}}_j \nabla^2 \Delta_j + \tilde{\mathcal{Q}}_j \nabla^2(\nabla^2 \Delta_j) - \tilde{\mathcal{L}}_j \Delta_i \boldsymbol{\nabla} \cdot (\Delta_j \boldsymbol{\nabla} \Delta_j), \tag{9}$$

with

$$\tilde{a} = -\left(\mathcal{A} + \tau + \frac{\tau^2}{2}\right), \qquad \mathcal{A} = \ln\left(\frac{2e^\Gamma \hbar\omega_D}{\pi T_c}\right),$$

$$\tilde{b} = W^2_3(1 + 2\tau), \qquad W^2_3 = \frac{7\zeta(3)}{8\pi^2 T^2_c} \quad \left(W_3 \sim \frac{1}{\pi T_c}\right),$$

$$\tilde{c} = W^4_5, \qquad W^4_5 = \frac{93\zeta(5)}{128\pi^4 T^4_c} \quad \left(W_5 \sim \frac{1}{\pi T_c}\right),$$

$$\tilde{\mathcal{K}}_j = \frac{W^2_3}{6} \hbar^2 v^2_j (1 + 2\tau), \qquad \tilde{\mathcal{Q}}_j = \frac{W^4_5}{30} \hbar^4 v^4_j,$$

$$\tilde{\mathcal{L}}_j = \frac{5}{9} W^4_5 \hbar^2 v^2_j, \tag{10}$$

where $\hbar\omega_D$ is the Debye energy, $\zeta(\cdots)$ is the Riemann zeta function, $\Gamma = 0.577$ is the Euler constant, and the band-dependent Fermi velocity is denoted by $v_j$. Note that, as compared to the results of Refs. [7,14], there are three new terms in Eq. (9): $\propto \Delta^5_j$, $\propto \nabla^2(\nabla^2 \Delta_j)$, and $\propto \Delta_j \boldsymbol{\nabla} \cdot (\Delta_j \boldsymbol{\nabla} \Delta_j)$. In addition, the coefficients $\tilde{a}$, $\tilde{b}$, and $\tilde{\mathcal{K}}_j$ contain extra contributions; i.e., $\tilde{a}$ is now accurate up to the order $\tau^2$ whereas $\tilde{b}$ and $\tilde{\mathcal{K}}_j$ include terms $\propto \tau$. Note also that when evaluating $\Delta_j$'s with accuracy $O(\tau^{3/2})$ [see Eq. (8)], we have $\nabla^2 \propto \tau$ in both $\Delta_j^{(0)}$ and $\Delta_j^{(1)}$.

Going back to Eq. (7), one obtains $R_1 = \Delta_1 - \lambda_{12} n_2 R_2/(\lambda_{11} n_1)$ from Eq. (7a), which can then be inserted into Eq. (7b). Similarly, $R_2$ can be expressed as $R_2 = \Delta_2 - \lambda_{21} n_1 R_1/(\lambda_{22} n_2)$ from Eq. (7b) and substituted in Eq. (7a). Such a manipulation, combined with Eq. (9), results in the following equations:

$$a_1\Delta_1 + b_1\Delta_1^3 - c_1\Delta_1^5 - \mathcal{K}_1\nabla^2\Delta_1^2 - \mathcal{Q}_1\nabla^2(\nabla^2\Delta_1) + \mathcal{L}_1\Delta_1\boldsymbol{\nabla}\cdot(\Delta_1\boldsymbol{\nabla}\Delta_1) - \gamma\Delta_2 = 0, \tag{11a}$$

$$a_2\Delta_2 + b_2\Delta_2^3 - c_2\Delta_2^5 - \mathcal{K}_2\nabla^2\Delta_2^2 - \mathcal{Q}_2\nabla^2(\nabla^2\Delta_2) + \mathcal{L}_2\Delta_2\boldsymbol{\nabla}\cdot(\Delta_2\boldsymbol{\nabla}\Delta_2) - \gamma\Delta_1 = 0, \tag{11b}$$

where

$$a_j = \frac{N(0)}{\eta}\left[\mathcal{A}_j - \eta n_j\left(\tau + \frac{\tau^2}{2}\right)\right], \qquad \eta = \lambda_{11}\lambda_{22} - \lambda_{12}^2, \tag{12}$$

with $\mathcal{A}_1 = \lambda_{22} - \eta n_1 \mathcal{A}$ and $\mathcal{A}_2 = \lambda_{11} - \eta n_2 \mathcal{A}$ ($\eta$ denotes the determinant of the $\lambda_{ij}$ matrix, where $\lambda_{12} = \lambda_{21}$). In addition, $b_j$, $c_j$, $\mathcal{K}_j$, $\mathcal{Q}_j$, $\mathcal{L}_j$ in Eqs. (11a) and (11b) are $\tilde{b}$, $\tilde{c}$, $\tilde{\mathcal{K}}_j$, $\tilde{\mathcal{Q}}_j$, $\tilde{\mathcal{L}}_j$ multiplied by $n_j N(0)$, respectively. The last terms in the left-hand side of Eqs. (11a) and (11b) are the Josephson interband coupling terms with $\gamma = \lambda_{12} N(0)/\eta$.

Proceeding in the manner similar to that of Ref. [14], we now group the terms of the same order in Eqs. (11a) and (11b). Keeping only terms of the order $\tau^{1/2}$ in both equations we find

$$\left(\frac{a_1 a_2}{\gamma} - \gamma\right)_{\tau^0} = 0, \tag{13}$$

where $(\mathcal{B})_{\tau^k}$ denotes the term in the expression $\mathcal{B}$ of the order $\tau^k$, with $k$ an integer. Equation (13) allows one to evaluate $T_c$ in the two-band superconducting system and is reduced to $\mathcal{A}_1\mathcal{A}_2 = \lambda_{12}^2$, which recovers Eq. (17) from Ref. [14].

Further, when collecting the terms proportional to $\tau^{3/2}$ in Eqs. (11a) and (11b), we find

$$\alpha\Delta_j^{(0)} + \beta_j[\Delta_j^{(0)}]^3 - K\nabla^2\Delta_j^{(0)} = 0, \tag{14}$$

where

$$\alpha = \left(\frac{a_1 a_2}{\gamma} - \gamma\right)_{\tau}, \qquad K = \left(\frac{\mathcal{K}_1 a_2 + \mathcal{K}_2 a_1}{\gamma}\right)_{\tau^0},$$
$$\beta_1 = \left(\frac{b_1 a_2 + a_1^3 b_2/\gamma^2}{\gamma}\right)_{\tau^0}, \qquad \beta_2 = \beta_1|_{1\leftrightarrow 2}, \tag{15}$$

where $\beta_2$ is obtained from the expression for $\beta_1$ by replacing indices of $a_j$'s and $b_j$'s ($1 \leftrightarrow 2$). Equation (14) is the correct formulation of the standard GL approach for the two-band $s$-wave clean superconducting system, as found in Ref. [14]. Using Eq. (13), we indeed obtain from Eqs. (14) and (15) that

$$[\Delta_1^{(0)}(\mathbf{x})/\Delta_2^{(0)}(\mathbf{x})]^2 = \mathcal{A}_2/\mathcal{A}_1, \tag{16}$$

which follows from the scaling $\beta_1/\beta_2 = \mathcal{A}_1/\mathcal{A}_2$.

Now, taking the terms of order $\tau^{5/2}$ in Eqs. (11a) and (11b), we arrive at

$$\Delta_j^{(1)}(\alpha + 3\beta_j[\Delta_j^{(0)}]^2) - K\nabla^2\Delta_j^{(1)} = F(\Delta_j^{(0)}) + F_j(\Delta_j^{(0)}), \tag{17}$$

with

$$F(\varphi) = \sigma\varphi + S\nabla^2\varphi + Y\nabla^2(\nabla^2\varphi), \tag{18}$$

and

$$F_j(\varphi) = \rho_j\varphi^3 + \chi_j\varphi^5 + U_j\varphi\boldsymbol{\nabla}\cdot(\varphi\boldsymbol{\nabla}\varphi) + V_j\nabla^2\varphi^3 + Z_j\varphi^2\nabla^2\varphi. \tag{19}$$

Equation (17) is the first main result of this Letter. It includes all contributions to order $\tau^{3/2}$ to $\Delta_j$'s. Coefficients $\sigma$, $S$, and $Y$ in Eq. (18) are given by

$$\sigma = -\left(\frac{a_1 a_2}{\gamma} - \gamma\right)_{\tau^2}, \qquad S = \left(\frac{\mathcal{K}_1 a_2 + \mathcal{K}_2 a_1}{\gamma}\right)_{\tau},$$
$$Y = \left(\frac{\mathcal{Q}_1 a_2 + \mathcal{Q}_2 a_1 - \mathcal{K}_1\mathcal{K}_2}{\gamma}\right)_{\tau^0}, \tag{20}$$

while the coefficients in Eq. (19) read

$$\rho_1 = -\left(\frac{b_1 a_2 + a_1^3 b_2/\gamma^2}{\gamma}\right)_{\tau},$$
$$\chi_1 = \left(\frac{c_1 a_2 - 3a_1^2 b_1 b_2/\gamma^2 + a_1^5 c_2/\gamma^4}{\gamma}\right)_{\tau^0},$$
$$U_1 = -\left(\frac{\mathcal{L}_1 a_2 + a_1^3\mathcal{L}_2/\gamma^2}{\gamma}\right)_{\tau^0}, \qquad V_1 = \left(\frac{b_1\mathcal{K}_2}{\gamma}\right)_{\tau^0},$$
$$Z_1 = 3\left(\frac{a_1^2\mathcal{K}_1 b_2}{\gamma^3}\right)_{\tau^0}, \tag{21}$$

and $\rho_2$, $\chi_2$, $U_2$, and $V_2$ are obtained from Eq. (21) by replacing $1 \leftrightarrow 2$ in all relevant indices.

Now, if the terms $F_j(\Delta_j^{(0)})$ were absent in Eq. (17), we would obtain that $\Delta_1^{(1)}(\mathbf{x})$ is proportional to $\Delta_2^{(1)}(\mathbf{x})$ and, furthermore, the ratio $\Delta_1^{(1)}(\mathbf{x})/\Delta_2^{(1)}(\mathbf{x})$ would be identical to $\Delta_1^{(0)}(\mathbf{x})/\Delta_2^{(0)}(\mathbf{x})$ given by Eq. (16). However, in the presence of $F_j(\Delta_j^{(0)})$, this is no longer the case, as not all terms appearing in $F_j(\Delta_j^{(0)})$ support the above scaling of the order parameters. In particular, let us consider the term $\rho_j[\Delta_j^{(0)}]^3$. This term could support the scaling only if the ratio $\rho_1/\rho_2$ is equal to $\mathcal{A}_1/\mathcal{A}_2$. From Eq. (21) we find

$$\frac{\rho_1}{\rho_2} = \frac{\mathcal{A}_1}{\mathcal{A}_2}\,\frac{2(n_1\mathcal{A}_2^2 + n_2\mathcal{A}_1^2) - \eta n_1 n_2(\mathcal{A}_2 + 3\mathcal{A}_1)}{2(n_1\mathcal{A}_2^2 + n_2\mathcal{A}_1^2) - \eta n_1 n_2(\mathcal{A}_1 + 3\mathcal{A}_2)}, \tag{22}$$

which means that $\rho_1/\rho_2 \neq \mathcal{A}_1/\mathcal{A}_2$ and, consequently,

$$[\Delta_1^{(1)}(\mathbf{x})/\Delta_2^{(1)}(\mathbf{x})]^2 \neq \mathcal{A}_2/\mathcal{A}_1. \tag{23}$$

Moreover, as seen from the structure of Eq. (17), it is clear that $\Delta_1^{(1)}(\mathbf{x})$ is not at all proportional to $\Delta_2^{(1)}(\mathbf{x})$. We hereby arrive at our main conclusion, i.e., the band order parameters $\Delta_1(\mathbf{x})$ and $\Delta_2(\mathbf{x})$ are not proportional to each other when extending the Ginzburg-Landau formalism to terms in $\Delta_j$'s proportional to $\tau^{3/2}$ (beyond the standard terms $\propto \tau^{1/2}$). This means that the band coherence lengths are in general different, and this difference disappears only in the limit $T \to T_c$.

For completeness, we give here several remarks about a generalization of the extended two-band GL formalism to the case of a nonzero magnetic field (inclusion of a magnetic field will not affect any of the above conclusions). Such a generalization is not straightforward because in the first step one needs to go beyond the eikonal approximation adopted by Gor'kov for the normal state Green function (see, e.g., the textbook [17]). This task assumes extensive calculations with numerous details that are not suitable for a Letter. Therefore, we include here only the final result, while preserving the full derivation for a separate publication:

$$\langle \mathbf{x}|\tilde{\mathcal{G}}_{j\omega}^{(0)}|\mathbf{x}'\rangle = e^{(ie/\hbar c)\int_{\mathbf{x}'}^{\mathbf{x}}\mathbf{A}(\mathbf{r})d\mathbf{r}}\left\{1 + \frac{e^2}{24m^2c^2}\mathbf{B}^2(\mathbf{x})\left[\frac{\partial^2}{\partial\omega^2} + \frac{i}{\hbar}m(\mathbf{x}-\mathbf{x}')_\perp^2\frac{\partial}{\partial\omega}\right]\right\}\langle \mathbf{x}|\mathcal{G}_{j\omega}^{(0)}|\mathbf{x}'\rangle, \tag{24}$$

where $\tilde{\mathcal{G}}_{j\omega}^{(0)}$ is the normal state Green function in the presence of a magnetic field; the integration in the exponent is taken along a straight line connecting $\mathbf{x}$ and $\mathbf{x}'$; $(\mathbf{x}-\mathbf{x}')_\perp$ is the component of the vector perpendicular to $\mathbf{B}(\mathbf{x}) = \mathrm{rot}\mathbf{A}(\mathbf{x})$. As follows from Eq. (24), the corrections to the Gor'kov approximation are gauge invariant and of order $\tau^2$ ($\mathbf{A} \propto \tau^{1/2}$ and $\mathbf{B} \propto \tau$). In particular, Eq. (9) in the presence of a magnetic field reads

$$\begin{aligned} R_j = {} & \left[-\tilde{a} + \frac{W_3^2}{3}\hbar^2\Omega^2(\mathbf{x})\right]\Delta_j - \tilde{b}\Delta_j|\Delta_j|^2 + \tilde{c}\Delta_j|\Delta_j|^4 \\ & + \tilde{\mathcal{K}}_j\mathbf{D}^2\Delta_j + \tilde{\mathcal{Q}}_j\left[(\mathbf{D}^2)^2 + \frac{4m^2\Omega^2(\mathbf{x})}{\hbar^2} + \frac{4ie}{3\hbar c}\mathrm{rot}\mathbf{B}(\mathbf{x})\mathbf{D}\right]\Delta_j - \frac{\tilde{\mathcal{L}}_j}{5}[4|\Delta_j|^2\mathbf{D}^2\Delta_j \\ & + 3\Delta_j^*(\mathbf{D}\Delta_j)^2 + 2\Delta_j|\mathbf{D}\Delta_j|^2 + \Delta_j^2(\mathbf{D}^2\Delta_j)^*], \end{aligned} \tag{25}$$

where $\mathbf{D} = \boldsymbol{\nabla} + 2\pi i\mathbf{A}(\mathbf{x})/\Phi_0$ ($\Phi_0$ is the superconducting flux quantum) and $\Omega = |e|B(\mathbf{x})/mc$, with $B(\mathbf{x}) = |\mathbf{B}(\mathbf{x})|$.

As a final note, we state that our approach differs from the theory of a local superconductor in a slow varying magnetic field, used in Refs. [18,19] (the so-called generalized Ginzburg-Landau-Gor'kov equations). The approach developed in the latter papers assumes that the gradients of the order parameter are small but the order parameter itself can be close to its value at zero temperature. Instead, we extended the two-band Ginzburg-Landau formalism up to the order $\tau^{3/2}$ (in $\Delta_i$'s). This requires us to accurately select the necessary terms on the basis of the proper scaling with $\tau$ of $\Delta_i$'s and their spatial derivatives. The same holds for the magnetic field and its spatial derivatives, which, contrary to Refs. [18,19], requires one to go beyond the eikonal approximation of Gor'kov [see Eq. (24)].

In summary, by developing the extended GL formalism for a two-band superconductor (i) we improved the validity of the Ginzburg-Landau theory at temperatures away from $T_c$, (ii) we showed that the two position dependent order parameters in a two-band superconductor are generally not proportional to each other, thus their spatial scales are decoupled—contrary to conclusions of the standard GL formalism, and (iii) we developed a useful tool for further theoretical studies of two-band superconductivity, which also commands revisiting many earlier works based on the incomplete formulation of the two-band GL formalism.

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