

Distribution of Resonance Widths and Dynamics of Continuum Coupling

G. L. Celardo,^{1,2} N. Auerbach,^{3,4} F. M. Izrailev,^{4,5} and V. G. Zelevinsky⁴

¹*Dipartimento di Matematica e Fisica, Università Cattolica, via Musei 41, 25121, Brescia, Italy*

²*I.N.F.N., Sezione di Pavia, Italy*

³*School of Physics and Astronomy, Tel Aviv University, Tel Aviv, 69978, Israel*

⁴*NSCL and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-1321, USA*

⁵*Instituto de Física, Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla, Pue., 72570, México*

(Received 20 October 2010; published 24 January 2011)

We analyze the statistics of resonance widths in a many-body Fermi system with open decay channels. Depending on the strength of continuum coupling, such a system reveals growing deviations from the standard chi-square (Porter-Thomas) width distribution. The deviations emerge from the process of increasing interaction of intrinsic states through common decay channels; in the limit of perfect coupling this process leads to the superradiance phase transition. The width distribution depends also on the intrinsic dynamics (chaotic versus regular). The results presented here are important for understanding the recent experimental data concerning the width distribution for neutron resonances in nuclei.

DOI: 10.1103/PhysRevLett.106.042501

PACS numbers: 24.60.-k, 24.60.Dr, 24.60.Lz

Open and marginally stable quantum systems are of great current interest in relation to numerous applications in nuclear physics of exotic nuclei, chemical reactions, condensed matter, astrophysics, and quantum informatics. The general problem can be formulated as that of the signal transmission through a complicated quantum system. The complexity of theoretical description of such processes is due to the necessity of a consistent unified theory that would cover intrinsic structure, especially for many-body systems, along with cross sections of various reactions.

One of the best and historically advanced examples of the manifestation of the interplay between intrinsic dynamics and decay channels is given by low-energy neutron resonances in complex nuclei [1]. The series of these well-pronounced separated resonances were studied long ago [2–4] and later gave rise to the “Nuclear Data Ensemble” [5,6]. Interpreting these resonances as quasi-stationary levels of the compound nucleus formed after the neutron capture, agreement was found with predictions of the Gaussian orthogonal ensemble (GOE) of random matrices. With exceedingly complicated wave functions of compound states, the statistical distribution of their components is close to Gaussian. The neutron decay implements the analysis of a specific component related to the channel “neutron in continuum plus a target nucleus in its ground state.” The neutron width is proportional to the squared amplitude of this component, and the width distribution then is χ_ν^2 with $\nu = 1$ as appropriate for one channel [Porter-Thomas distribution (PTD)].

The recent experiments with improved accuracy [7] give evidence of significant deviations from the PTD so that the attempts to still use the χ_ν^2 distribution for the fit invariably require $\nu < 1$. A nonpure set of resonances, for example, a sequence of mainly s resonances contaminated by p -wave

states, would shift the distribution to a higher number of degrees of freedom, $\nu > 1$. The new result was interpreted as a consequence of an unknown nonstatistical mechanism or just a breakdown of nuclear theory as was claimed in the related article in Ref. [8].

The goal of this Letter is to point out that a correct description of unstable quantum states in a complicated many-body system naturally leads to deviations from the GOE and PTD, of the same type as observed in [7]. The random matrix theory was formulated for local statistics in a closed quantum system with a discrete spectrum governed by a very complicated Hermitian Hamiltonian. Its predictions were repeatedly checked in systems like quantum billiards [9] and their experimental embodiment in microwave cavities [10,11], and in shell-model calculations for complex atoms [12] and nuclei [13].

However, the presence of open decay channels and therefore the finite lifetime of intrinsic states unavoidably lead to new phenomena outside of the GOE framework [14,15] as it was clearly demonstrated by the first numerical simulations [16–18]. Two interrelated effects follow from the fact that we deal with unstable rather than with strictly stationary states: the level repulsion disappears at the spacings comparable to the level widths and the growing widths undergo the redistribution with the trend to collectivization and eventually formation of superradiant (short-lived) states along with the narrow (trapped) states [15,19]. The new dynamics modify the GOE predictions as well as certain features of Ericson fluctuations [20] in the regime of overlapping resonances [21]. The occurrence of a superradiant transition has been also demonstrated outside the random matrix theory framework [22].

A quantum many-body system coupled to open decay channels can be rigorously described by an effective non-Hermitian Hamiltonian \mathcal{H} [23],

$$\mathcal{H} = H - \frac{i}{2}W. \quad (1)$$

Here H is the Hermitian Hamiltonian of the closed system that in general includes virtual (off-shell) coupling to the continuum, while the anti-Hermitian (on shell) part $(-i/2)W$ is constructed in terms of the amplitudes $\mathbf{A} = \{A_i^c\}$ coupling intrinsic states $|i\rangle$ to the open channels c ,

$$W = \mathbf{A}\mathbf{A}^T \Rightarrow W_{ij} = \sum_{c(\text{open})}^M A_i^c A_j^c. \quad (2)$$

We consider a time-reversal invariant system, when the (in general depending on running energy) amplitudes A_i^c can be taken real. The factorized form of W is an important property that follows from unitarity of the scattering matrix. The complex eigenvalues, $\mathcal{E} = E - (i/2)\Gamma$, of the Hamiltonian (1) coincide with the poles of the scattering matrix and determine the positions and the widths of the resonances in cross sections of various reactions.

The factorized matrix W has M nonzero eigenvalues, corresponding to M open channels. The matrix H has dimension N that in the nuclear case should include a large number of shell-model many-body states important for the dynamics in the energy range under consideration; in the region of neutron resonances, $N \sim 10^{5-6} \gg M$. With the trace of W equal to η , the parameter defining the dynamics is the ratio of typical “bare” widths η/N of individual states to the energy spacings D . At small value of this parameter, an open channel serves as an analyzer that singles out a specific component of the exceedingly complicated intrinsic wave function. The resonance widths in such a system obey the PTD. With widths increasing, the system moves to the regime of overlapping resonances.

Let us consider the single-channel case, $M = 1$, having in mind the s -wave elastic neutron resonances. With a high level density of intrinsic states at relevant energy, their local spectral statistics is close to the predictions of the GOE. Then, owing to the central limit theorem, the individual components of a typical intrinsic state are Gaussian distributed uncorrelated quantities, and the neutron widths, being proportional to the absolute magnitudes of those components, display the PTD. However, the correct description of the dynamics with the continuum coupling shows the limited character of this prediction. The imaginary part (2) works similar to the collective multipole forces and creates the interaction between intrinsic states through continuum. When the coupling is weak, $\kappa = \eta/ND \ll 1$, we indeed expect to see well isolated resonances with the PTD of the widths. With growing continuum coupling (increase of energy from the threshold), the deviations become more and more pronounced. At $\kappa \sim 1$, a kind of a phase transition occurs with the sharp redistribution of widths and the segregation of a superradiant state accumulating the lion’s share of the whole summed width, an analog of a giant resonance along the imaginary energy axis. The mechanism is clear from the factorized structure of W that, for $M = 1$, has only one nonzero eigenvalue

equal to the trace of W . Similar to superradiance in optics [24], this mechanism works independently of the regular or chaotic nature of intrinsic dynamics.

In the case of the GOE-type dynamics of the closed system and $M = 1$, the distribution of complex eigenvalues can be found analytically. The Ginibre ensemble [25] of complex Gaussian matrices is not applicable here. The exact result [15] is given by the Ullah distribution [26],

$$\mathcal{P}(\{E_n; \Gamma_n\}) = C_N \prod_{m < n} \frac{|\mathcal{E}_m - \mathcal{E}_n|^2}{|\mathcal{E}_m - \mathcal{E}_n^*|} \prod_n \frac{1}{\sqrt{\Gamma_n}} \exp(-NF). \quad (3)$$

Here C_N is a normalization constant; the preexponential factor describes the correlations which are reduced to the usual GOE level repulsion for stable states but for complex energies \mathcal{E}_n contain the interactions with their “electrostatic images” \mathcal{E}_n^* . Along with the Porter-Thomas factor $\Gamma^{-1/2}$, this guarantees that the widths are positive. The “equilibrium” distribution of complex energies is determined by their “free energy,”

$$F(\{E_n, \Gamma_n\}) = \sum_n \left(\frac{E_n^2}{a^2} + \frac{\Gamma_n}{2\eta} \right) + \sum_{m < n} \frac{\Gamma_m \Gamma_n}{2a^2}, \quad (4)$$

that includes the interaction between the widths, the last term in Eq. (4). The mean level spacing in the closed system is $D = 2a/N$, where $2a$ is the spectral interval of real energy. The Gaussian ensemble of the decay amplitudes is defined by the mean values $\overline{A_n} = 0$, $\overline{A_n A_m} = (\eta/N)\delta_{mn}$. Here, a regular evolution of the widths as a function of energy of the resonance is excluded as it is usually done with the rescaling to the reduced widths; we do not discuss here the way of practical rescaling that may depend on the specific nucleus. Our purpose is to show that systematic deviations from the PTD occur even for the set of reduced widths, and the effects are caused by the interaction (4).

For very small widths, $\kappa \ll 1$, the width interaction is negligible, the first product in (3) reduces to the standard level repulsion, and the distribution (3) is factorized into the product of the GOE distribution of real energies and the PTD for the widths. While the usual Hermitian perturbation causes level repulsion and width attraction, the anti-Hermitian interaction through the continuum relaxes the level repulsion but leads to the collectivization through the common decay channel and width repulsion [27]. For $\kappa \simeq 1$, the width repulsion becomes critical, and the most probable configuration is the one where this repulsion is small because the total width is going to concentrate in a single superradiant state [15,19]. With the increase of $\kappa \gg 1$ and fixed number of open channels, the broad state becomes a smooth envelope, and we return to the set of $(N - 1)$ narrow resonances. Below we show the typical evolution of the width distribution studied in a large-scale numerical simulation.

First, we considered H as a member of the GOE [28] where the matrix elements are Gaussian random variables, $\langle H_{ij}^2 \rangle = (1 + \delta_{ij})/N$. This corresponds to the limiting case of fully chaotic intrinsic dynamics. Then, we modeled H by the two-body random ensemble (TBRE) for n fermions distributed over m single-particle states; the total number of many-body states is $N = m!/[n!(m-n)!]$; in our simulations $n = 6$, $m = 12$, $N = 924$. The TBRE can be written as $H = H_0 + V$, where the mean field part H_0 is defined by single-particle energies, ϵ_j , with a Poissonian distribution of spacings and the mean level spacing d_0 . The interaction V [29] is fixed by the variance of the two-body random matrix elements, $\langle V_{1,2;3,4}^2 \rangle = v_0^2$. While at $v_0 = 0$ we have a Poissonian spacing distribution $P(s)$ of many-body states, for $d_0 = 0$ (infinitely strong interaction, $v_0/d_0 \rightarrow \infty$), $P(s)$ is close to the Wigner-Dyson distribution typical for a chaotic system. The critical interaction for the onset of strong chaos is given [29] by $v_{\text{cr}}/d_0 \approx 2(m-n)/N_s$, where $N_s = n(m-n)[1 + (n-1) \times (m-n-1)/4]$ is the number of directly coupled many-body states in any row of the matrix H_{ij} . In our model, $v_{\text{cr}}/d_0 \approx 1/20$.

We computed the complex eigenvalues of the effective Hamiltonian, Eq. (1), for 10^3 random realizations, selecting the states at the center of the energy band where their density is almost constant. The distribution of widths, normalized to their average value, was obtained for different strengths of continuum coupling. The numerically obtained distributions were fitted, using a standard χ^2 test, with a χ_ν^2 distribution for ν degrees of freedom, following a common practice in nuclear data analysis. As a measure of quality of the fit we used the criterion $\chi_r^2 \approx 1$, where χ_r^2 is the reduced chi-square value [30]. Figure 1 shows the normalized width distribution for the GOE case with $M = 1, 2$ for different coupling to the continuum, κ . The standard distributions, PTD for $M = 1$, and $\chi_{\nu=2}^2$ for $M = 2$, are valid only when the coupling to the continuum is very weak, while strong deviations from $\chi_{\nu=M}^2$ appear, both for large and small Γ , as we increase the coupling. In Fig. 1 we also show the best fit possible for a χ_ν^2 distribution; always the corresponding value of ν is $< M$; moreover, the quality of the fit decreases as κ increases. The data in Fig. 1 also demonstrate that at the super-radiance transition, $\kappa = 1$, the tail of the widths distribution is described by a power law [see discussion in Ref. [31]] meaning that no χ_ν^2 distributions would be a good fit.

The dependence of the best fitted value of ν on the coupling strength is shown in Fig. 2 for $M = 1, 2$. Along with the data for the GOE intrinsic Hamiltonian (circles), the data for the TBRE intrinsic dynamics are shown by crosses for the case $v_0 = d_0/50$ and by squares for $v_0 = 0$ (no intrinsic chaos). Regardless of intrinsic dynamics, the best value of ν steadily decreases as κ increases. It is clear that a family of χ_ν^2 distributions is not appropriate to fit our numerical data, except for very weak continuum coupling. Indeed, the χ_r^2 criterion steeply increases with the

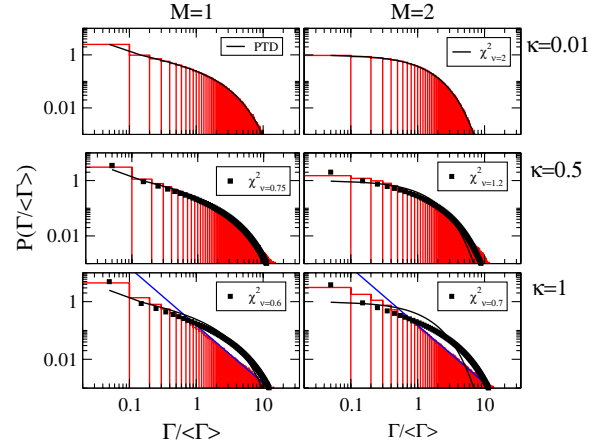


FIG. 1 (color online). Normalized distribution of the widths for one open channel, $M = 1$ (left panels), and for $M = 2$ (right panels) for the GOE intrinsic Hamiltonian and different continuum coupling strength, κ . Numerical results are given by histograms; the expected PTD for $M = 1$ and $\chi_{\nu=2}^2$ distribution for $M = 2$ are shown by a smooth curve in all panels. Full squares stand for the best fit to a χ_ν^2 distribution. In the lowest panels, a straight line shows the power law $P(\Gamma/\langle\Gamma\rangle) \propto (\Gamma/\langle\Gamma\rangle)^{-2}$. In this Figure, the resonances in the energy interval ± 0.5 for 10^3 different ensemble realizations were considered.

coupling strength [see insets in Fig. 2]. For weak internal chaos, the departure from the $\chi_{\nu=M}^2$ distribution is stronger than for chaotic intrinsic dynamics, even at weak continuum coupling. The absence of intrinsic chaos and corresponding level repulsion implies a stronger sensitivity to continuum coupling.

When analyzing empirical neutron s -wave resonances, one of the main difficulties is the p -wave contamination. In order to analyze this problem we considered the case of two nonequivalent channels, with 10% of the states coupled to the additional channel by a smaller coupling

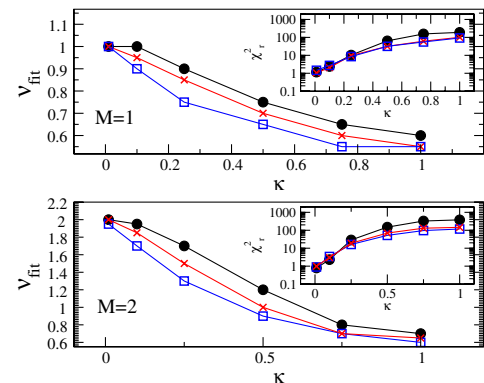


FIG. 2 (color online). Best fitted value of ν versus the coupling strength to the continuum, κ . Full circles refer to GOE intrinsic Hamiltonian, crosses stand for TBRE with $v_0 = d_0/50$, and squares for TBRE with $v_0 = 0$ (no intrinsic chaos); the number of channels is one (upper panel) and two (lower panel). The insets show the reduced χ_r^2 as a function of κ .

strength. When another channel is included, the best fit value of ν at weak coupling is always larger than 1; again this value decreases as the continuum coupling grows. We also observed the evolution of the level spacing distribution $P(s)$ along the real axis, which manifests the decrease of repulsion at spacings comparable with the level widths [17]; for example, at $\kappa = 0.5$, $P(0) \approx 0.2$.

We made an approximate estimate of the coupling κ from experimental data given in [7]. From Fig. 1 of this reference one can see that there are about 40 s -wave resonances between 8 and 12 keV. A rough evaluation gives $\langle \Gamma \rangle / D = 0.03$, which implies $\kappa \approx 0.05$. From Fig. 2, one can already notice a deviation from PTD for such a small κ .

To summarize, the width distributions were analyzed for a system with $M = 1, 2$ open channels as a function of the continuum coupling strength. As this coupling increases, the best fit value of ν for a χ^2_ν distribution decreases below M , in accordance with recent experimental findings [7]. At the same time, the fit quality becomes poor, showing that the standard PTD (and in general any χ^2_ν distribution) is applicable only for extremely narrow resonances. The low-energy neutron resonances in a heavy nucleus correspond to the very beginning of the process of width collectivization. However, already here the deviations from the (GOE \times PTD) factorized distribution are noticeable. These deviations are more pronounced for regular intrinsic dynamics than for chaotic intrinsic dynamics. Therefore, the interpretation of the width as a strength of the pure neutron component in the compound wave function fails due to the coupling through continuum that has to be accounted for in a proper statistical description. The phenomenon under discussion is of general nature and it may influence all processes of signal transmission through a quantum system.

While preparing this Letter, a preprint appeared [32] dealing with a complementary aspect of the problem discussed here, namely, a possible influence of single-particle resonances upon the energy dependence of the widths.

We are grateful to F. Borgonovi, Y. Fyodorov, W. Mittig, V. Sokolov, and A. Volya for discussions and support. V. Z. acknowledges NSF Grant No. PHY-0758099. F.I. acknowledges VIEP BUAP Grant No. EXC08-G.

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