Tunable Casimir Repulsion with Three-Dimensional Topological Insulators

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In this Letter, we show that switching between repulsive and attractive Casimir forces by means of external tunable parameters could be realized with two topological insulator plates. We find two regimes where a repulsive (attractive) force is found at small (large) distances between the plates, canceling out at a critical distance. For a frequency range where the effective electromagnetic action is valid, this distance appears at length scales corresponding to $1 - \epsilon(\omega) \sim (2/\pi)\alpha\theta$.

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The full experimental accessibility to micrometer and submicrometer size physics and the possibility of developing applications has turned the understanding of phenomena at these scales to be of fundamental importance. Within this scenario, the Casimir force [1] arises when two objects are placed near each other at distances of a few micrometers. In the general case of two dielectrics, the situation is well described by the theory developed by Dzyaloshinskii *et al.* [2], where the optical response of the material determines the magnitude and behavior of the force. In the simplest case of a mirror symmetric situation, a theorem ensures that the force is always attractive [3,4], resulting in a problem for nanomechanical devices. To revert the sign, one must search nonsymmetric situations, usually adding complexity to the problem. The first Casimir repulsion proposal, known as Dzyaloshinskii repulsion, was recently confirmed experimentally [5] and it involves a third dielectric medium between the plates, excluding the possibility of frictionless devices and quantum levitation. In turn, vacuum mediated proposals include magnetic versus nonmagnetic situations [6] and the use of metamaterials [7–9]. In this Letter, we report a new method for obtaining a twofold tunable Casimir repulsion. By use of the optical properties of topological insulators (TI), it is feasible to achieve all situations between repulsion to attraction by using two controllable parameters: the distance between dielectrics and the sign of the topological magnetoelectric polarizability (TMEP) θ, where the latter allows us to tune the optical properties of the mentioned materials.

TIs are characterized by a bulk insulating behavior with metallic boundary states protected by time reversal symmetry [10,11]. The topological protection of edge states ensures their stability against nonmagnetic perturbations. The 3D counterpart of this novel topological state was shown to exist in a Bi_xSb_{1-x} alloy [12] and in the stoichiometric crystals Bi_2Se_3, Bi_2Te_3, $TlBiSe_2$, and Sb_2Te_3 [13–15].

The Casimir force is intimately related to the optical properties of the two dielectric bodies [2]. For instance, consider the situation where two dielectric parallel semi-infinite bodies (labeled 1 and 2) are placed at a distance d from each other in vacuum. In this case, the Casimir energy density (CED) stored by the plates is given by [16]

$$\frac{E_c(d)}{A\hbar} = \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} \log \det[1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2k_3 d}], \qquad (1)$$

where A is the plate area, $k_3 = \sqrt{\boldsymbol{k}_\parallel^2 + \xi^2/c^2}$ is the wave vector perpendicular to the plates, $\boldsymbol{k}_\parallel$ is the vector parallel to the plates, and ξ is the imaginary frequency defined as $\omega = i\xi$. The matrices $\mathbf{R}_{1,2}$ are 2×2 reflection matrices of media 1 and 2 containing the Fresnel coefficients defined as

$$\mathbf{R} = \begin{bmatrix} R_{s,s}(i\xi, \mathbf{k}_\parallel) & R_{s,p}(i\xi, \mathbf{k}_\parallel) \\ R_{p,s}(i\xi, \mathbf{k}_\parallel) & R_{p,p}(i\xi, \mathbf{k}_\parallel) \end{bmatrix}, \qquad (2)$$

where $R_{i,j}$ describes the reflection amplitude of an incident wave with polarization i which is reflected with polarization j. The polarizations R_s (p) describe parallel (perpendicular) polarization with respect to the plane of incidence. The Casimir force per unit area on the plates is obtained by differentiating expression (1). A positive (negative) force, or equivalently a positive (negative) slope of $E_c(d)$, corresponds to attraction (repulsion) of the plates.

The electromagnetic response of a dielectric, which defines the reflection matrices, is governed by Maxwell's equations derived from the ordinary electromagnetic action $S_0 = \int dx^3 dt[\epsilon \boldsymbol{E}^2 - (1/\mu)\boldsymbol{B}^2]$, with $\boldsymbol{E}$ and $\boldsymbol{B}$ the electric and magnetic fields, respectively. TIs in three dimensions are well described by adding a term of the form $S_\theta = (\alpha/4\pi^2) \int dx^3 dt \theta \boldsymbol{E} \cdot \boldsymbol{B}$, where $\alpha = 1/137$ is the fine structure constant and θ is the TMEP (axion field) [11,17]. Because of time reversal symmetry, this term is a good description of the bulk of a trivial insulator (e.g., vacuum) when $\theta = 0$ and of the bulk of a TI when $\theta = \pi$. However, the axion coupling is only a good description of both the bulk and the boundary of a TI when a time reversal breaking perturbation is induced on the surface and the system becomes fully gapped. In this situation, θ can be shown to be quantized in odd integer values of π such that $\theta = (2n+1)\pi$, where $n \in \mathbb{Z}$. The value of n is determined by the nature of the time reversal breaking perturbation, which could be controllable experimentally by covering the TI with a thin magnetic layer. In particular, positive or negative values of θ are related to different signs of the magnetization on the surface [18]. As we will demonstrate in what follows, the Casimir force is very sensitive to the value of θ, and the tunability of its sign will allow us to describe a mechanism for switching between repulsive and attractive forces.

The electromagnetic response of a system in the presence of a θ term is still described by the ordinary Maxwell equations, but the constituent relations which define the electric displacement $\boldsymbol{D}$ and the magnetic field $\boldsymbol{H}$ acquire an extra term proportional to θ [19], $\boldsymbol{D} = \epsilon \boldsymbol{E} + \alpha(\theta/\pi)\boldsymbol{B}$ and $\boldsymbol{H} = \boldsymbol{B}/\mu - \alpha(\theta/\pi)\boldsymbol{E}$. We note that Eq. (1) can be easily modified to take into account magnetoelectric couplings (this happens also in chiral metamaterials [9]). The result is the same equation with the proper reflection matrices. It is then possible to derive by means of ordinary electromagnetic theory the reflection coefficients of a TI-vacuum interface. For a TI characterized by a frequency dependent dielectric function $\epsilon(\omega)$ and a TMEP θ, the reflection coefficients will take a symmetric form [20] where the off-diagonal coefficient can be expressed as

$$R_{s,p}(i\xi, \mathbf{k}_{\parallel}, \theta) = \mathrm{sgn}(\theta) r_{sp}(i\xi, \mathbf{k}_{\parallel}, \theta), \quad (3)$$

where r_{sp} is an even function of θ. When $\theta = 0$, $R_{s,p} = 0$, leading to the usual attractive Casimir force due to the nonmixing of polarizations [21]. When $\theta \neq 0$, the reflection coefficients mix polarizations and the sign of θ plays a crucial role on the sign of the Casimir force (see below). In what follows we will consider that the surface of the TIs of the Casimir system are covered by a thin magnetic layer as shown in Fig. 1. This effectively turns the TI into a full insulator (both in the bulk and on the surface) which can be safely described with the TMEP and a dielectric function, as shown in earlier works [11,18]. Hence, to numerically compute the CED by means of (1), a model for the dielectric function is necessary (we henceforth assume $\mu = 1$). Because of the low concentration of free carriers in insulators the most general phenomenological model to describe the optical response of a dielectric is a sum of oscillators to account for particular absorption resonances [21]. When only one oscillator is considered (see, however, [22]), the dielectric function evaluated can be written as

$$\epsilon(i\xi) = 1 + \frac{\omega_e^2}{\xi^2 + \omega_R^2 + \gamma_R \xi}. \quad (4)$$

In this model, ω_R is the resonant frequency of the oscillator while ω_e accounts for the oscillator strength. The damping parameter γ_R satisfies $\gamma_R \ll \omega_R$, playing therefore a secondary role. In what follows, we have rescaled all quantities in units of ω_R, leaving the quantity

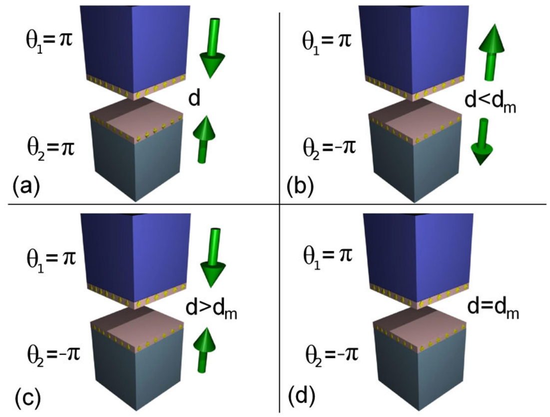

FIG. 1 (color online). Different configurations of the Casimir effect with identical TI covered with a thin magnetic layer. (a) The magnetization is of the same sign on each surface, resulting in a case where $\theta_1 = \theta_2$ giving Casimir attraction. In (b)–(d) the magnetizations have opposite signs on the surface ($\theta_1 = -\theta_2$) leading to attraction when $d > d_m$, repulsion when $d < d_m$, and to a quantum levitation configuration at d_m where the net force is zero.

$\epsilon(0) \equiv 1 + (\omega_e/\omega_R)^2$ as the only parameter of the model. A good candidate to be described by this model is the TI $TlBiSe_2$ [15]. This material has experimentally [23] (neglecting free carrier contributions and assuming high frequency transparency) $\epsilon(0) \sim 4$ and has a single resonant frequency near 56 cm^{-1}. Other TI could need more oscillators to be added in (4).

We have computed the CED between two TI plates described by the TMEP θ_1 and θ_2 and the value of the dielectric constant at zero frequency $\epsilon(0)$ by numerical evaluation of expression (1). The results are summarized in Figs. 2 and 3 where the CED is plotted against the dimensionless distance $\bar{d} \equiv d\omega_R/c$. From Fig. 2(a) it is clear that opposite signs of $\theta_{1,2}$ lead to the existence of a minimum ($\bar{d}_m$) where the net force is zero. The behavior is attractive when both signs become equal, suggesting that it is possible to tune the Casimir force by tuning the relative signs of θ, i.e., switching the magnetizations of the coverings. The existence of a minimum is analytically shown below in terms of the relative importance of the off-diagonal terms (3) against the diagonal terms and in terms of the relative sign of the TMEPs. At large distances the diagonal terms dominate and the usual Casimir attraction is recovered. At small distances, the off-diagonal terms dominate and their sign determines whether the CED approaches $\pm\infty$ (i.e., repulsive or attractive) leading to a minimum at intermediate distances if the signs of $\theta_{1,2}$ are opposite.

To prove the existence of the minimum we consider the Fresnel equations for TI obtained earlier in [20] which lead to Eq. (3) added to the following properties of the dielectric function: finite dielectric permittivity at zero frequency [$\epsilon(0) < \infty$] and high frequency transparency, $\epsilon(\omega) \rightarrow 1$

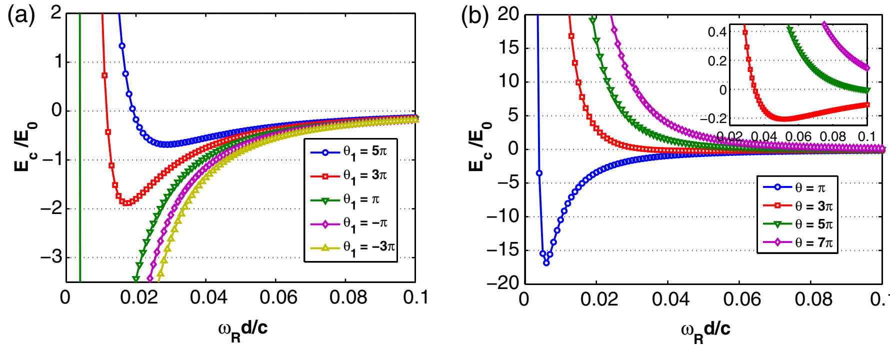

FIG. 2 (color online). Casimir energy density [in units of $E_0 = A\hbar c/(2\pi)^2(\omega_R/c)^3$] as a function of the dimensionless distance $\bar{d}$ for $\omega_e/\omega_R = 0.45$. In (a) $\theta_2 = -\pi$ is fixed. Whenever $\mathrm{sgn}(\theta_1) = -\mathrm{sgn}(\theta_2)$ a minimum $\bar{d}_m$ appears, leading to a vanishing net force on the plates. Increasing θ_1 within positive values suppresses the minimum shifting $\bar{d}_m$ towards lower values (if both signs are equal then only attractive behavior occurs). Complete repulsion is achieved when one of the TMEP is much bigger than the other. (b) The optimal situation $\theta_1 = -\theta_2 = \theta$. Different values of θ show that the minimum is enhanced when the difference between the two values is as small as possible ($\theta = \pi$). Inset: Detailed behavior around the minimum for the case $\theta = 3\pi$.

when $\omega \to \infty$. For analytical traceability we assume $\theta_1 = -\theta_2 \equiv \theta$ and that the dielectric function (4) describes the TI, although the derivation does not depend on the explicit form of the dielectric function as long as it fulfils the mentioned conditions.

In (1) we can rescale ξ and $\mathbf{k}_\parallel$ to contain $\bar{d}$, which gives an overall factor $1/\bar{d}^3$ and forces the reflection matrices to be evaluated at the rescaled frequency and momenta $\xi/\bar{d}$ and $\mathbf{k}_\parallel/\bar{d}$. Hence, $E_c(\bar{d} \to 0) \to \pm\infty$ and $E_c(\bar{d} \to \infty) \to 0$ since the behavior of $\epsilon(i\xi)$ ensures that the reflection matrices are not singular when evaluated at $i\xi/\bar{d} \to 0$ and $i\xi/\bar{d} \to \infty$.

The way the integral approaches these limits determines the sign of $E_c(\bar{d})$. For instance, if the integrand is positive at small distances and negative at large distances, necessarily a minimum exists at an intermediate distance $\bar{d}_m$. In what follows it will be shown that this is exactly what happens unless $\epsilon(0) = 1$, where both limits are positive

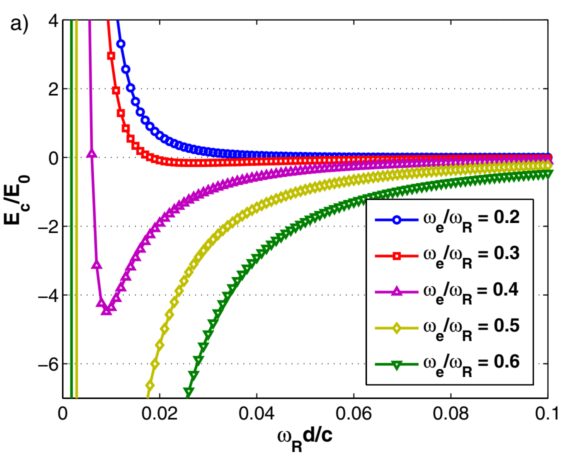

FIG. 3 (color online). Effect of parameter $\epsilon(0)$ with fixed $\theta_1 = -\theta_2 = \pi$. The effect of increasing $\epsilon(0)$ is to develop a minimum, which shifts to smaller $\bar{d}_m$ as $\epsilon(0)$ is increased.

and hence long-range repulsion is obtained. Under these conditions the diagonal terms in (2) are equal for both TI (which we label r_s and r_p), and the off-diagonal terms given by (3) have opposite overall signs, but equal absolute value given by the function r_{sp}. Introducing these inside (1) the integrand reads:

$$I = \log[1 + e^{-2k_3^{(r)}}(2r_{sp}^2 - r_p^2 - r_s^2) + e^{-4k_3^{(r)}}(r_{sp}^2 - r_p r_s)^2], \tag{5}$$

where $k_3^{(r)}$ is now evaluated at the rescaled frequency and momenta just as the reflection matrices. In the limit of small distances ($\bar{d} \to 0$) and using the high frequency transparency of the dielectric function, it can be shown that $|r_s|, |r_p| \ll |r_{sp}|$ since the first are of order α^2 and the second are of order α. Hence the integrand is positive and so $E_c(\bar{d} \to 0) \to +\infty$.

Now we consider the limit of large distances ($\bar{d} \to \infty$). In this limit, the reflection coefficients take the form $r_s = [1 - \epsilon(0) - \bar{\alpha}^2]/D$ (a similar expression holds for r_p) and $r_{sp} = 2|\bar{\alpha}|/D$, where $D = 1 + \epsilon(0) + \bar{\alpha}^2 + \sqrt{\epsilon(0)}\chi$, χ is a frequency and momentum dependent function irrelevant for the present discussion and $\bar{\alpha} = \alpha\theta/\pi$. In this long distance limit, depending on the values of $\epsilon(0)$ different behaviors emerge. Since $\epsilon(0) \geq 1$ we now consider the two extreme limits, one where $\epsilon(0) = 1$ and the other with $\epsilon(0) \gg 1$.

In the limit where $\epsilon(0) \gg 1$, the condition $|r_s|, |r_p| \gg |r_{sp}|$ is always satisfied. When $\epsilon(0)$ is strictly infinity we recover the ideal case of an ordinary metal with $r_{s,p} = \pm 1$ and $r_{sp} = 0$. The integrand at large distances is a negative quantity and so $E_c(\bar{d})$ approaches zero from negative values. From the previous discussion at small distances $E_c(\bar{d}) \to +\infty$; therefore, there must be a minimum at an intermediate distance $0 < \bar{d}_m < \infty$ since the function must cross the x axis. In the unrealistic case where $\epsilon(0) = 1$, one can check that $|r_{sp}| \gg |r_s|, |r_p|$ making $E_c(\bar{d})$ always positive for all distances. In this case there is no minimum and the force is always repulsive. By this analytical analysis and when $\theta_1 = -\theta_2$ we expect that, as we increase $\epsilon(0)$ from one, a minimum develops at an intermediate distance $\bar{d}_m$. This distance shifts to lower values as we increase $\epsilon(0)$ until, at $\epsilon(0) = \infty$, we recover the metallic case where complete attraction occurs. In the case where $\theta_1 = \theta_2$ the signs inside (5) change and make the logarithm to be negative, recovering attraction at all intermediate distances (for more details see [22]). The consistency of these analytical expectations is confirmed numerically with the results shown in Figs. 2 and 3.

From these we infer that in order to enhance as much as possible the minimum, it is necessary to search for a situation where $\theta \equiv \theta_1 = -\theta_2$. The CED for different values of θ satisfying this condition are depicted in Fig. 2(b). Under these circumstances the minimum is more prominent when $\theta = \pi$, i.e., when θ takes its smallest possible value. The general analytical analysis, supported by the numerical evaluation for different parameters, suggests that a simple experimental setup (Fig. 1) could switch from complete Casimir attraction to a stable quantum levitation regime by reversing the magnetization of one of the layers covering one of the TI. In this process the system will turn from a symmetric situation where $\theta_1 = \theta_2$, resulting in attraction to a nonsymmetric situation where the optimum condition is satisfied ($\theta_1 = -\theta_2$) and a stable minimum appears.

To conclude, for this appealing situation to be experimentally accessible one has to search for realizations of ω_R where the typical distances between the plates are at least of order 0.1–1 μm. For a frequency range where the axion Lagrangian is valid [17], the minimum is expected to appear at a position where diagonal and off-diagonal terms are similar in magnitude, i.e., length scales corresponding to $1 - \epsilon(\omega) \sim \frac{2}{\pi}\alpha\theta$. Low values of $\epsilon(0)$ (typically less than 10) favor this situation since the minimum is realized at larger distances. While the electromagnetic parameters of TI are still not well characterized, a low $\epsilon(0)$ could be achieved by using thin films or by air injection which will lower the bulk dielectric response. For $TlBiSe_2$ we estimate from numerical integration that the minimum of the CED appears at a distance of $d = 0.1$ μm and with a CED of the same order as for the usual metal-vacuum-metal system at 1 μm, hence being still experimentally accessible. We must note, however, that this estimation requires a high TMEP value ($\theta \sim 10\pi$) in order to shift the minimum to observable distances. Therefore the proposed effect is on the verge of experimental accessibility and should encourage experimental efforts to attain full optical characterization of TI. We stress here that the only effect of the magnetic coating is to gap the surface states. We have estimated the parasitic magnetic forces between the magnetic layers following [24]. The dipole-dipole interaction is of the order of attoN at distances of 50 nm and the magnetic Casimir force [24] is $\sim$1 fN, much smaller than the force described here which is of the order of 5 pN. The proposed effect could also be explored in other magnetodielectric materials such as Cr_2O_3, which can be described by a higher axion coupling [25]. However, these materials induce more general magnetoelectric couplings [26] which we will consider in a future work.

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[1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[2] I. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, Adv. Phys. **10**, 165 (1961).
[3] O. Kenneth and I. Klich, Phys. Rev. Lett. **97**, 160401 (2006).
[4] C. P. Bachas, J. Phys. A **40**, 9089 (2007).
[5] J. Munday, F. Capasso, and V. A. Parsegian, Nature (London) **457**, 170 (2009).
[6] T. H. Boyer, Phys. Rev. A **9**, 2078 (1974).
[7] F. S. S. Rosa, D. A. R. Dalvit, and P. W. Milonni, Phys. Rev. Lett. **100**, 183602 (2008).
[8] U. Leonhardt and T. G. Philbin, New J. Phys. **9**, 254 (2007).
[9] R. Zhao *et al.*, Phys. Rev. Lett. **103**, 103602 (2009).
[10] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005).
[11] X. L. Qi, T. L. Hughes, and S. C. Zhang, Phys. Rev. B **78**, 195424 (2008).
[12] D. Hsieh *et al.*, Nature (London) **452**, 970 (2008).
[13] Y. L. Chen *et al.*, Science **325**, 178 (2009).
[14] Y. Xia *et al.*, Nature Phys. **5**, 398 (2009).
[15] T. Sato *et al.*, Phys. Rev. Lett. **105**, 136802 (2010).
[16] M. T. Jaekel and S. Reynaud, J. Phys. I (France) **1**, 1395 (1991).
[17] A. M. Essin, J. E. Moore, and D. Vanderbilt, Phys. Rev. Lett. **102**, 146805 (2009).
[18] X.-L. Qi *et al.*, Science **323**, 1184 (2009).
[19] F. Wilczek, Phys. Rev. Lett. **58**, 1799 (1987).
[20] M.-C. Chang and M.-F. Yang, Phys. Rev. B **80**, 113304 (2009).
[21] G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Rev. Mod. Phys. **81**, 1827 (2009).
[22] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.106.020403 for details.
[23] C. L. Mitsas and D. I. Siapkas, Solid State Commun. **83**, 857 (1992).
[24] P. Bruno, Phys. Rev. Lett. **88**, 240401 (2002).
[25] F. W. Hehl *et al.*, Phys. Lett. A **372**, 1141 (2008).
[26] A. M. Essin *et al.*, Phys. Rev. B **81**, 205104 (2010).