Acoustic Cloaking by a Superlens with Single-Negative Materials

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We propose a specific transformation in cloaking to make an acoustic sensor undetectable, in which the cloaking shell consists of complementary media with single-negative acoustic parameters instead of double-negative ones, and is proved to be a magnifying superlens. Moreover, the acoustical parameters of the cloak are completely independent of those of the host material as well as the cloaked object. This may significantly facilitate the experimental realization of acoustic cloaks and is of fundamental importance in a wide range of acoustics, optics, and engineering applications.

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Transformation optics was recently proposed as a general technique to design functional optical devices or structures on the basis of the invariance of Maxwell's equations under coordinate transformations [1–4]. One of the most famous and intriguing examples of the designed devices is the ''invisibility cloak'' [5–9]. Based on the same principle, two-dimensional (2D) transformation acoustics was then proposed for designing the ''inaudibility cloak'' to manipulate acoustic waves in a similar manner [10]. Two schemes have been developed to yield the cloaking effect based on transformation acoustics (or transformation optics). The first scheme is to put the cloaked object in the ''hole.'' Then the incident wave will detour around the ''hidden region'' without reflection, while a fundamental limitation arises that the cloaked object cannot have communication with the outside world [5–9,11]. For solving such a difficulty, the second scheme was then presented to realize the ''external cloaking, '' in which the cloaked object is acoustically ''canceled out'' by its antiobject for the propagating waves and, meanwhile, can share information with the surroundings [12]. To realize perfect cloaking, however, the antiobject must be made of double-negative materials, which highly depend upon the parameters of the cloaked object. As a matter of fact, compared with materials with single-negative parameters, i.e., negative bulk modulus or negative mass density, the fabrication of double-negative materials necessarily involves much more theoretical and technical difficulty. Although it has been proved in theory that simultaneous negative bulk modulus and mass density can be obtained by coexistent dipolar and monopolar resonators, the working frequency range of the resulting double-negative material is much narrower than those of corresponding single-negative materials [13]. Moreover, the restriction on the choice of parameters of cloak, i.e., the antiobject, dramatically limits the potential applications of the ''external cloaking'' scheme to diverse practical situations. In this context, it is urgent to figure out a crucial question: Is there an effective way to cloak an acoustic sensor by employing only single-negative materials whose effective acoustic parameters are free from the restriction of the properties of the cloaked object? To our best knowledge, the answer still lacks despite of its significance to the feasibility as well as the efficiency of the acoustic cloaks.

In this Letter, we propose a different cloaking scheme in an attempt to solve such a difficulty. In the transformation, the cloaked object is mapped to a bulk of host material with similar shape, and the cloaking shell only consists of complementary media made of single-negative materials. Full-wave simulations by finite element method (FEM) are performed to demonstrate the cloaking properties of the designed structure. The results show that the incident acoustic wave can pass across the complementary media changelessly. As a result, the cloaked object is able to receive signals, while its presence is not sensed by the surrounding, and internal signals can also transmit to the outside environment for the purpose of detection or communication. Particularly, the acoustical parameters of the cloak, which is proved equivalent to a magnifying superlens, are totally independent of those of the cloaked object. With the feasibility of designing an acoustic cloak by single-negative materials instead of double-negative ones and the flexibility in choosing the acoustical parameters of the cloak, this scheme may significantly facilitate the experimental realization of acoustic cloaks and is of fundamental importance in a wide range of acoustics, optics, and engineering applications.

During coordinate transformation, i.e., folding a piece of space $(A + B)$ into another $(A' + B)$, a hole, which is inaccessible to the pressure field, may form. Note that the transformed space $(A' + B)$ could be a connected domain [Fig. 1(b)] or an unconnected domain [Figs. 1(c) and 1(d)]. The first scheme is to make the transformed space $(A' + B)$ a connected domain and put the cloaked object in the hole C, as shown in Fig. 1(b). Mathematically, this coordinate transformation produces a singularity, which in real space shows up as an infinite phase velocity for the wave at the boundary of the hole. Also, the cloaked region

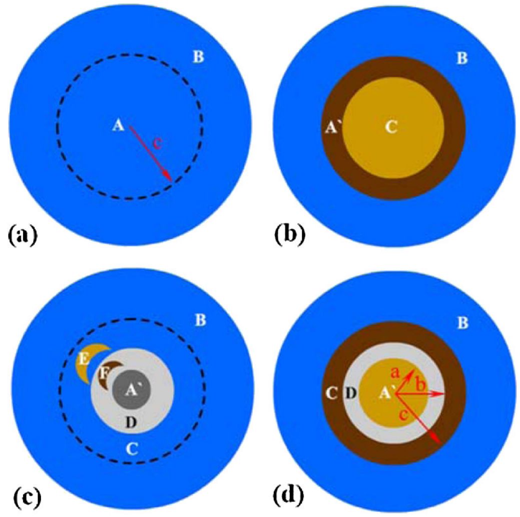

FIG. 1 (color online). (a) A bulk of host media $(A + B)$. (b) The scheme of the first mapping approach where $A \leftrightarrow A'$ and $B \leftrightarrow B$. A' is filled with the cloaking shell. C is the hole where the cloaked object is located. (c) The scheme of the second mapping approach where $A \leftrightarrow A'$, $B \leftrightarrow B$, $C \leftrightarrow D$ and $E \leftrightarrow F$. A', C, and E are filled with core material, host material, and the cloaked object, respectively. D and F are the "antiobjects" of C and E, respectively. (d) The scheme of our mapping approach where $A \leftrightarrow A'$, $B \leftrightarrow B$ and $C \leftrightarrow D$. A' is filled with the cloaked object $(0 < r' < a)$, C and D are filled with the cloaking shell made of complementary media with single-negative parameters (C: $b < r' < c$, D: $a < r' < b$).

is inaccessible to the incident wave. To overcome such difficulties, another scheme is proposed in which the transformed space $(A' + B)$ is an unconnected domain and the hole is filled with complementary media $(C + D)$ and $(E + F)$ to keep the continuity of pressure field, as shown in Fig. 1(c). As a result, the cloaked object E is acoustically "canceled out" by its antiobject F for the incident wave and can feel the surroundings. However, the antiobjects, D and F, have to be made of double-negative materials, whose acoustical parameters depend on the parameters of the cloaked object.

In the present study, therefore, we develop a different scheme to realize acoustic cloaking with single-negative materials instead of double-negative ones. As illustrated in Fig. 1(d), an equivalence is established between the cloaked object A' and a bulk of host media A with similar shape by coordinate transformation, and the hole $(a < r' < c)$ around the cloaked object is filled with complementary media of single-negative materials $(C + D)$. Here, the circular configuration is employed for simplicity and the transformation is operated in cylindrical coordinate. For a monochromatic acoustic wave with angular frequency ω, the acoustic equation is given in the time harmonic form, $\boldsymbol{k}(\boldsymbol{x})\nabla \cdot [\boldsymbol{\rho}^{-1}(\boldsymbol{x})\nabla p(\boldsymbol{x})] = -\omega^2 p(\boldsymbol{x})\mathbf{I}$, with $\boldsymbol{\rho}^{-1}(\boldsymbol{x})$, $\boldsymbol{k}(\boldsymbol{x})$, $\mathbf{I}$, and $p(\boldsymbol{x})$ being the inverse mass density tensor, the bulk modulus tensor, the unit tensor, and the acoustic pressure, respectively. In circular cylindrical coordinates, when a space $\boldsymbol{x}$ is mapped into another space $\boldsymbol{x}'$, the modulus tensor $\boldsymbol{k}'(r', \vartheta', z')$ and the inverse mass density tensor $\boldsymbol{\rho}'^{-1}(r', \vartheta', z')$ in the space $\boldsymbol{x}'$ are given by $\boldsymbol{k}'(r', \vartheta', z') = \det(\mathbf{H})\boldsymbol{k}(r, \vartheta, z)$ and $\boldsymbol{\rho}'^{-1}(r', \vartheta', z') = \mathbf{H}[\boldsymbol{\rho}^{-1}(r, \vartheta, z)]\mathbf{H}^T[\det(\mathbf{H})]^{-1}$ according to transformation acoustics. Here $\boldsymbol{k}(r, \vartheta, z)$ and $\boldsymbol{\rho}^{-1}(r, \vartheta, z)$ are the bulk modulus tensor and the inverse mass density tensor in space $\boldsymbol{x}$ respectively, and $\mathbf{H}$ is the Jacobian transformation matrix with components $H_{ij} = (h_{x_i'}/h_{x_j})(\partial x_i'/\partial x_j)$ with h_{x_i} and $h_{x_i'}$ being the scale factors in spaces $\boldsymbol{x}$ and $\boldsymbol{x}'$, respectively. The mapping between spaces $\boldsymbol{x}$ and $\boldsymbol{x}'$ can be written as $r = f(r')$, $\vartheta = \vartheta'$, $z = z'$ with $f(r') = r'c/a$ for $0 < r' < a$, $f(r') = r'$ for $r' > c$. It is apparent that there is an unmapped area or hole $(a < r' < c)$ in space $\boldsymbol{x}'$. Since the pressure field must be a continuum, we have to fill the hole with complementary media to keep the continuity without changing the wave when it passes by. Further consideration will reveal that the complementary media should be mapped into each other obeying coordinate transformation in $\boldsymbol{x}'$, which is given by $r' = f(r'')$, $\vartheta' = \vartheta''$, $z' = z''$ with $f(r'') = c(a/r'')$ for $a < r'' < b$, where $b = \sqrt{ac}$. For the mapping $A \leftrightarrow A'$, the scale factors are $h_r = h_z = 1$, $h_\vartheta = r = f(r') = r'(c/a)$ and $h_{r'} = h_{z'} = 1$, $h_{\vartheta'} = r'$. Let us suppose the modulus tensor and the density tensor in domain A are $\boldsymbol{k}(r, \vartheta, z) = \kappa_0\mathbf{I}$ and $\boldsymbol{\rho}(r, \vartheta, z) = \rho_0\mathbf{I}$, then the modulus tensor and the density tensor in domain A' take the form $\boldsymbol{k}'(r', \vartheta', z') = \kappa_0(a/c)^2\mathbf{I} = \kappa_0(b/c)^4\mathbf{I}$ and $\boldsymbol{\rho}'(r', \vartheta', z') = \mathrm{diag}\{\rho_0, \rho_0, \rho_0(a/b)^4\}$. For the mapping $C \leftrightarrow D$, the scale factors are $h_{r'} = h_{z'} = 1$, $h_{\vartheta'} = r' = f(r'') = c(a/r'')$ and $h_{r''} = h_{z''} = 1$, $h_{\vartheta''} = r''$. Let us suppose the modulus tensor and the density tensor in domain C are $\boldsymbol{k}'(r', \vartheta', z') = \kappa_1\mathbf{I}$ and $\boldsymbol{\rho}'(r', \vartheta', z') = -\rho_1\mathbf{I}$, then the modulus tensor and the density tensor in domain D become $\boldsymbol{k}''(r'', \vartheta'', z'') = -\kappa_1(r''/b)^4\mathbf{I}$ and $\boldsymbol{\rho}''(r'', \vartheta'', z'') = \mathrm{diag}\{\rho_1, \rho_1, \rho_1(r''/b)^4\}$. It is noticed that the bulk modulus and density in domain A' and D are isotropic under the mapping relationship. In 2D acoustic cloaking, the wave vectors have no z component, so κ_z and ρ_z is not involved in the formulation, which eventually gives $\boldsymbol{k}'(r', \vartheta') = \{\kappa_0(a/b)^4, \kappa_0(a/b)^4\}$, $\boldsymbol{\rho}'(r', \vartheta') = \mathrm{diag}\{\rho_0, \rho_0\}$ for $0 < r' < a$, $\boldsymbol{k}'(r', \vartheta') = \{-\kappa_1(r'/b)^4, -\kappa_1(r'/b)^4\}$, $\boldsymbol{\rho}'(r', \vartheta') = \mathrm{diag}\{\rho_1, \rho_1\}$ for $a < r' < b$, $\boldsymbol{k}'(r', \theta') = \{\kappa_1, \kappa_1\}$, $\boldsymbol{\rho}'(r', \vartheta') = \mathrm{diag}\{-\rho_1, -\rho_1\}$ for $b < r' < c$, $\boldsymbol{k}'(r', \theta') = \{\kappa_0, \kappa_0\}$, $\boldsymbol{\rho}'(r', \vartheta') = \mathrm{diag}\{\rho_0, \rho_0\}$ for $r' > c$ in the cloaking system. It is worth mentioning that there is in fact no restriction on the shape of the cloaked region. When the shape of the cloaked region is not circular, the material parameters of the cloak may be required to be more anisotropic [14].

To demonstrate the performance of the designed structure, full-wave simulations are carried out by FEM. In this Letter, the host material $(r' > c)$ is chosen to be water with the bulk modulus and the mass density to be

$\kappa_0 = 2.19$ GPa and $\rho_0 = 998$ kg/m^3, respectively. For the cloaked object $(0 < r' < a)$, the effective bulk modulus, the effective mass density and the radius are chosen as $\kappa_1 = 0.48\kappa_0$, $\rho_1 = \rho_0$, and $a = 1.0$ m, respectively. Then, the outer radii of internal and external cloaking shells can be readily determined as $b = 1.2$ m, $c = 1.44$ m, respectively. It is worth pointing out that the cloaked object is treated as a homogeneous "effective" medium whose mass density ρ_1 in fact refers to the effective mass density. Mathematically, the effective mass density ρ_1 of the cloaked object should equal ρ_0 exactly for yielding perfect cloaking effect, which can be readily observed from the transformation relations. It should be stressed, however, that the cloaking effect is robust against the variation of the effective material parameters of the cloaked object [15]. For realizing the acoustic cloaking, the acoustical parameters of the external and the internal cloaking shells must satisfy a particular relationship, but totally independent of the material parameters of host material and cloaked object [16]. As an example, the bulk modulus and mass density of the external cloaking shell $(b < r' < c)$ are set to be $10\kappa_0$ and $-\rho_0$, respectively, then the modulus and density of internal cloaking shell $(a < r' < b)$ can be determined as $-10\kappa_0(r'/b)^4$ and ρ_0 respectively.

We first investigate the case of a bare scatterer without the cloak illuminated by the plane wave. Here, the wavelength of the incident wave is assumed to be $\lambda = 0.5$ m. The pressure field around the bare scatterer is shown in Fig. 2(a). As observed, the plane wave is strongly disturbed by the scatterer, which results in backward reflection and a sharp-edged shadow. Figure 2(b) shows the cloaking effect, where the plane wave field is almost undisturbed outside the cloaking shell. It is noteworthy that the plane wave can pass through the cloaking shell without changing the shape of the wave front, which makes it capable to receive information from the outside. In order to verify the cloaking performance to different wave fronts, we further simulate the case of cylindrical wave incidence. In this case, the point source is located at $(-3.5$ m, 0) in Cartesian coordinates. Without the cloak, the waves are disturbed and shadows exist as shown in Fig. 2(c). With the cloak, however, it is seen in Fig. 2(d) that both the backscattering and shadows are remarkably suppressed, while the cylindrical wave can reach the cloaked region with the shape of the wave front unchanged. Quantitative analysis to the deformation of the field under different conditions is shown in the supplemental material [17]. The numerical results also prove that the complementary media, which is conventionally thought to consist of positive-index material ($\kappa > 0$, $\rho > 0$) and double-negative material ($\kappa < 0$, $\rho < 0$), can be made of a pair of single-negative materials ($\kappa > 0$, $\rho < 0$ and $\kappa < 0$, $\rho > 0$) as well. Under this situation, the internal shell with negative modulus can be regarded as the "antiobject" of the external shell with negative mass density.

We have also studied the case where a transmitter is put inside the cloaked region. The numerical results are given in Fig. 3 in which the transmitter sticks to the internal wall of the cloaking shell and the operating frequency is chosen as $f = 2963$ Hz. When the transmitter is a point source P, as shown in Fig. 3(a), the wave can reach the outside with the shape of the wave front unchanged. It is interesting to find out that to the outside observer, the wave appears to be produced by the point source P' which is a perfect real image of P. If the transmitter is a line source, common sense suggests that the image will simply expand according to the rules of geometrical acoustics. In Fig. 3(b), the line source L is located between $(-1.0$ m, -0.067 m) and $(-1.0$ m, 0.067 m). One can observe that the image L' is

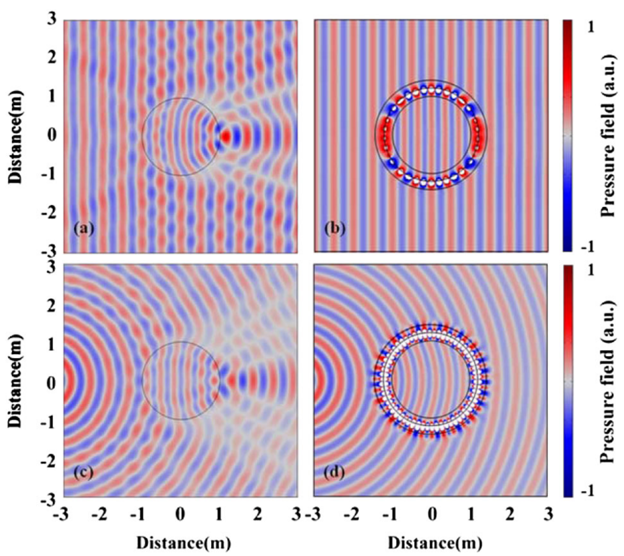

FIG. 2 (color online). Acoustic pressure field distribution for the wave incident from plane source [(a), (b)] and point source [(c), (d)] located at $(-3.5$ m, 0). In (a) and (c), a bare scatterer is directly illuminated without cloak. In (b) and (d), the bare scatterer is shielded by the double-shell cloak. In all cases, the incident wave travels from left to right and the wavelength $\lambda = 0.5$ m.

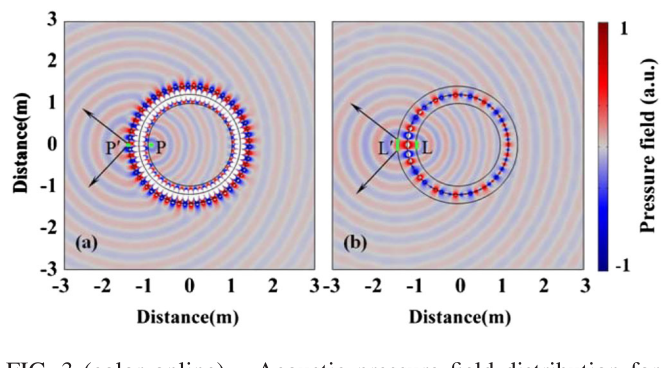

FIG. 3 (color online). Acoustic pressure field distribution for waves produced by (a) a point transmitter P located at $(-1.0$ m, 0), and (b) a line transmitter L located between $(-1.0$ m, 0.07 m) and $(-1.0$ m, -0.07 m).

located between (− 1.44 m, −0.096 m) and (− 1.44 m, 0.096 m), and the length ratio ($l_L/l_{L'} \approx 0.698$) approximately equals the ratio between the internal and the external radii ($a/c \approx 0.694$). It is noteworthy that in Figs. 2 and 3, there exist ''surface waves'' on the interface between the internal and external cloaking shells. Numerical results show that the surface wave is in fact nonpropagating along the radial direction and can only propagate along the boundaries between the complementary media, just like the ''surface plasmon-polaritons'' in electromagnetism [18]. Further consideration manifests that the ''surface waves'' can yield the unique effects of zero-phase-delay and energy compensation to the evanescent wave in the radial direction, which eventually results in the phenomena of perfect tunneling and superlens imaging [19–21].

The present scheme is sufficiently simple and efficient to encourage practical studies of experimental realization of cloaking a sensor. It is crucial to seek an efficient method to fabricate single-negative materials. In acoustics, it is well known that an air bubble in water exhibits strong monopolar resonance in response to acoustic waves, and the monopolar resonances of a periodic array of air bubbles in water give rise to a wide band gap due to the effective negative modulus. For the solid based structure, Liu *et al.* [13] proposed a periodic array of bubble-contained water spheres in epoxy matrix and proved the composite structure of effective negative modulus at the frequencies around the resonance analytically. They also constructed a periodic array of rubber-coated gold spheres in epoxy matrix and proved the composite structure of effective negative density due to the strong bipolar resonance.

In summary, we have presented a cloaking scheme based on transformation acoustics, in which the cloak is proved to be a magnifying superlens. The crucial element in our design is to establish the equivalence between the cloaked object and a bulk of host media with similar shape by coordinate transformation and fill the hole around the cloaked object with complementary media made of single-negative materials. The numerical results demonstrate that the cloaked object is able to receive signals, while its presence is not sensed by the surroundings, and internal signals can also transmit to the outside environment for the purpose of detection or communication. Moreover, the acoustical parameters of the cloak are completely independent of those of the cloaked object, which may significantly facilitate the experimental realization of acoustic cloaks and is of deep implications in the field of electromagnetism.

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