Self-Induced Transparency and Electromagnetic Pulse Compression in a Plasma or an Electron Beam under Cyclotron Resonance Conditions

N. S. Ginzburg, I. V. Zotova, and A. S. Sergeev

Institute of Applied Physics, RAS, GSP-120 N. Novgorod, Russia

(Received 2 November 2010; published 22 December 2010)

Based on analogy to the well-known process of the self-induced transparency of an optical pulse propagating through a passive two-level medium we describe similar effects for a microwave pulse interacting with a cold plasma or rectilinear electron beam under cyclotron resonance condition. It is shown that with increasing amplitude and duration of an incident pulse the linear cyclotron absorption is replaced by the self-induced transparency when the pulse propagates without damping. In fact, the initial pulse decomposes to one or several solitons with amplitude and duration defined by its velocity. In a certain parameter range, the single soliton formation is accompanied by significant compression of the initial electromagnetic pulse. We suggest using the effect of self-compression for producing multigigawatt picosecond microwave pulses.

DOI: 10.1103/PhysRevLett.105.265001 PACS numbers: 52.35.Mw, 41.20.Jb

The effect of self-induced transparency (SIT) for short (in the scale of relaxation times) light pulses is well known in optics and has been studied in detail theoretically and experimentally [1–3]. Because of nonlinear effects, a dissipative medium which normally absorbs light becomes transparent to a bright, short-duration light pulse (2π pulse). As a result, the light pulse propagates without any change in shape as a "SIT soliton." Moreover, under certain conditions, nonlinear compression of the incident light pulse occurs when its duration reduces while the amplitude significantly increases [4]. Similar effects can, apparently, be observed in classical electronics when a short electromagnetic pulse propagates through a magnetized cold plasma or initially rectilinear electron beam under the cyclotron resonance condition. As it will be shown below, in this case with increasing the incident pulse amplitude the linear cyclotron absorption gives place to the pulse propagating practically without damping. Actually, the initial electromagnetic pulse transforms into one or several solitons with amplitude and duration defined by the soliton velocity. Similar to optics [4], in a certain area of parameters, the soliton formation is accompanied by significant compression of initial electromagnetic pulse. It is important to note that described effects arise only when relativistic dependence of gyrofrequency on particle energy is taken into account. As in physics of gyrotrons [5,6], this dependence is significant even for nonrelativistic energies acquired by electrons in the field of an electromagnetic pulse. Under such an assumption a cold magnetized plasma or initially rectilinear electron beam could be considered as a nonlinear resonance medium.

In principle, self-induced transparency effects should be observed for arbitrary angles between the pulse group velocity and the direction of homogeneous magnetic field: $\vec{H} = \vec{z}_0 H_0$. But, when an electromagnetic pulse propagates across a static magnetic field, this problem has the simplest description. In this case, the role of ponderomotive and recoil effects (autoresonance [7]) resulting from the wave magnetic field can be neglected.

Let us assume for simplicity that an incident electromagnetic pulse

$$\vec{E} = \vec{y}_0 \mathrm{Re}[A(x,t)\exp(i\omega t - ikx)] \tag{1}$$

propagates in a planar waveguide filled by cold plasma. Here, ω is the carrier frequency, $k = \omega/c$, $A(x,t)$ is the wave amplitude. The transverse structure of radiation corresponds to the fundamental TEM waveguide mode. The condition of cyclotron resonance between electrons and radiation propagating transversely with respect to direction of the magnetic field has the form

$$\omega \approx \omega_H^0, \tag{2}$$

where $\omega_H^0 = eH_0/m_0 c$ is electron gyrofrequency, m_0 is electron rest mass. After expansion of a linearly polarized field (1) in two circular-polarized components and subsequent averaging over the wave period the equations for particle motion can be presented as

$$\frac{dp_+}{dt} + ip_+\left(\omega - \frac{\omega_H^0}{\gamma}\right) = -i\frac{eA}{2}, \tag{3}$$

where $p_+ = (p_x + ip_y)\exp(i\omega t - ikx)$ is the transverse momentum of electrons, $\gamma = \sqrt{1 + |p_+|^2/m_0^2c^2}$ is the relativistic mass-factor. Evolution of pulse complex amplitude $A(x,t)$ is described by the following equation:

$$\frac{\partial A}{\partial x} + \frac{1}{c}\frac{\partial A}{\partial t} = -i\frac{n_0}{2m_0 c}\frac{p_+}{\gamma}, \tag{4}$$

where n_0 is unperturbed density of electrons. Under the assumption that electron energy is subrelativistic: $\gamma \approx 1 + |p_+|^2/2m_0^2c^2$ and after normalization, the self-consistent set of Eqs. (3) and (4) can be reduced to the form

$$\frac{\partial a}{\partial X} + \frac{\partial a}{\partial \tau} = p, \qquad \frac{\partial p}{\partial \tau} + ip(\Delta + |p|^2) = -a. \qquad (5)$$

Here equations for electron motion are presented as the well-known equations for nonisochronous oscillators [5,6]. In the Eqs. (5) the following dimensionless variables and parameters are used:

$$\tau = \sqrt{G}\omega t, \qquad X = \sqrt{G}\frac{\omega}{c}x, \qquad a = \frac{ieA}{2\sqrt{2}m_0 c\omega G^{3/4}},$$

$$p = \frac{p_+}{\sqrt{2}m_0 c G^{1/4}}, \qquad G = \frac{\omega_p^2}{16\pi\omega^2},$$

$\omega_p = \sqrt{4\pi e n_0/m_0}$ is the plasma frequency, $\Delta = (\omega - \omega_H^0)/\omega\sqrt{G}$ is the initial cyclotron resonance mismatch. Further, we assume that an incident pulse with amplitude a_0 and duration $T = \sqrt{G}\omega t_0$ enters the input cross section $X = 0$:

$$a|_{X=0} = a_0 \sin^2(\pi\tau/T).$$

In the initial instant of time, all electrons have zero transverse velocity:

$$p|_{\tau=0} = 0.$$

It should be noted that the above assumption of cold plasma is applicable if the transverse velocity acquired by electrons in the field of an electromagnetic pulse strongly exceeds the velocity of thermal motion.

It is important to note that the system of Eqs. (5) is valid also when an electromagnetic pulse propagates across rectilinear electron beam moving in $\vec{z}$ direction along static magnetic field. In this case, due to the absence of recoil effects longitudinal momentum of electrons is conserved: $p_{\|} = p_{\|0}$. Correspondingly, to describe the interaction with electron beam the rest mass in Eqs. (5) should be replaced by relativistic mass $m_0\gamma_{\|0}$, where $\gamma_{\|0} = \sqrt{1 + p_{\|0}^2/m_0^2c^2}$.

Results of simulations of Eqs. (5) are presented in Fig. 1 for different initial amplitudes and durations of incident electromagnetic pulse. These results confirm the analogy with the optical self-induced transparency effects. When the amplitude and duration of incident electromagnetic pulse are small the cyclotron absorption is observed [Fig. 1(a)]. This process is accompanied by a quasiperiodical energy exchange between electromagnetic pulse and electrons that is similar to Rabi oscillations of population inversion in optics [3]. Following above analogy, as the initial amplitude and duration of the microwave pulse increase, the self-induced transparency effect occurs [8] when the incident pulse propagates through plasma practically without damping [Fig. 1(b)]. As it is seen in Fig. 2, this effect has a simple explanation; viz., the leading front of an electromagnetic pulse excites transverse oscillations of electrons which then are suppressed by the trailing front. With further increasing of the amplitude and duration of incident pulse the nonlinear

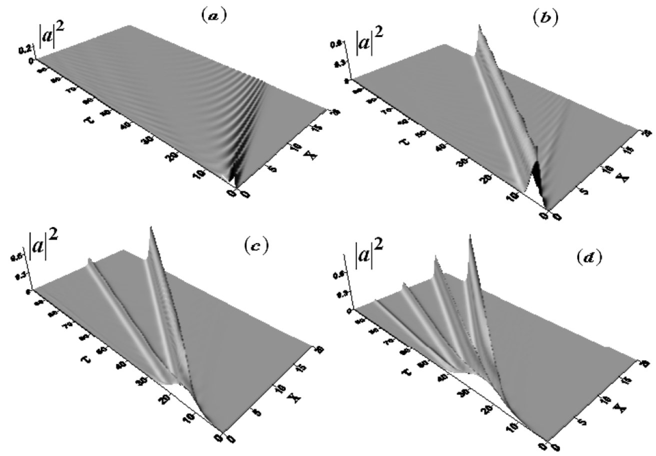

FIG. 1. Typical regimes of propagation of electromagnetic pulses in cold plasma or rectilinear electron beam under the cyclotron resonance condition $\Delta = 0$ for different values of the peak power and incident pulse duration: (a) cyclotron absorption of incident pulse ($a_0 = 0.5$, $T = 3$); (b) self-induced transparency effect ($a_0 = 0.7$, $T = 10$); (c) compression of incident electromagnetic pulse ($a_0 = 0.45$, $T = 35$); (d) decomposition of incident pulse on several solitons ($a_0 = 0.7$, $T = 40$).

pulse compression is observed. In Fig. 1(c), the peak power grows more than 3 times with simultaneous pulse shortening. In this process the energy absorption is rather low and the energy of compressed pulse amounts of 80% of the incident pulse energy.

Similar to the propagation of light pulses in the resonance two-level medium, for sufficiently large amplitude and duration the incident electromagnetic pulse decomposes to one or several solitons, e.g., localized wave packets with amplitude and duration defined by velocity of its propagation. As it is seen from Fig. 1(d), with increasing the soliton amplitude its velocity also increases.

To find analytically the solitonlike solutions of Eqs. (5) we present the electromagnetic pulse amplitude and transverse momentum in the form of a stationary wave

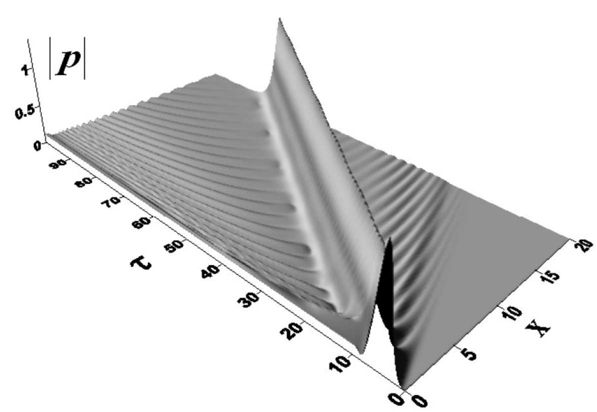

FIG. 2. Evolution of the absolute value of electron transverse momentum in the regime of self-induced transparency ($a_0 = 0.7$, $T = 10$, $\Delta = 0$).

$$a(X,\tau) = \hat{a}(\xi)e^{i\varphi(\xi)+iC\beta_S^{-1}X},\quad p(X,\tau) = \hat{p}(\xi)e^{i\psi(\xi)+iC\beta_S^{-1}X}, \tag{6}$$

where $\xi = \tau - \beta_S^{-1}X$, $\beta_s = V_s/c$ is the soliton velocity, C is the constant that defines phase shift. Taking into account (6) Eqs. (5) acquire the form

$$\frac{d\hat{a}}{d\xi} = \frac{\hat{p}}{s}\cos\chi, \qquad \frac{d\hat{p}}{d\xi} = \hat{a}\cos\chi, \qquad \frac{d\chi}{d\xi} = -\hat{\Delta} - \hat{p}^2 - \left(\frac{\hat{a}}{\hat{p}} + \frac{\hat{p}}{s\hat{a}}\right)\sin\chi, \tag{7}$$

where $\chi = \psi - \varphi$ is the phase difference, parameter $s = \beta_s^{-1} - 1$ describes the deviation of soliton velocity from velocity of light, $\hat{\Delta} = \Delta + C(s+1)/s$.

Eqs. (7) possess two integrals of motion

$$\hat{p} = \sqrt{s}\hat{a}, \qquad \sin\chi = -\frac{\hat{\Delta}\sqrt{s}}{2} - s^{3/2}\frac{\hat{a}^2}{4}. \tag{8}$$

These integrals allow us to reduce Eqs. (7) to a single equation for electromagnetic field intensity $I = \hat{a}^2$ which in the case $\hat{\Delta} = 0$ can be presented in the form

$$\frac{dI}{d\xi} = \frac{2I}{\sqrt{s}}\sqrt{1 - \frac{s^3I^2}{16}}. \tag{9}$$

This equation has a solitonlike solution, e.g., a stationary localized wave propagating without profile modification

$$I = \frac{4}{s^{3/2}}\operatorname{sech}\frac{2\xi}{\sqrt{s}}. \tag{10}$$

Correspondingly, the pulse amplitude can be defined as

$$\hat{a}(X,\tau) = \frac{2}{s^{3/4}}\operatorname{sech}^{1/2}\frac{2}{\sqrt{s}}\left(\tau - \frac{X}{\beta_s}\right). \tag{11}$$

According to (10) and (11) the soliton amplitude and accumulated energy

$$W = \int_{-\infty}^{+\infty}\hat{a}^2(\tau')d\tau' = \frac{2\pi}{s}$$

increase with simultaneous shortening of its duration with decreasing of the parameter s, e.g., with decreasing the difference between the soliton velocity and the velocity of light. These conclusions are in good agreement with results of numerical simulations of time-domain Eqs. (5). It should be noted that optical SIT solitons possess a similar dependence of amplitude and duration on propagation velocity. According to simulations soliton solutions occur in the finite range of initial detuning which approximately determinates by condition $\Delta T \leq 2\pi$, where T is the soliton duration.

From a practical point of view, the studied effects can be used for nonlinear compression of microwave superradiant (SR) pulses. Basic mechanisms and experiments on generation of such pulses are described in [9–11]. Let us estimate the possibility of compression of a Ka-band SR pulse with peak power 1.2 GW and duration of 300 ps (FWHM). Assume that the above pulse propagates through cold plasma with the density of $5 \times 10^{13}\ \mathrm{cm}^{-3}$ in the resonant 10 kG magnetic field. For the transverse aperture of the incident SR pulse $2\ \mathrm{cm}^2$ the peak power density achieves $600\ \mathrm{MW/cm}^2$. These physical parameters correspond to the normalized ones: $G \approx 0.06$, $a_0 \approx 0.5$, $T \approx 35$ that were close to simulations presented in Fig. 1(c). From the above modeling we can estimate the peak power of the compressed pulse as $\sim$4 GW with duration $\sim$60 ps (Fig. 3). The compression length is about 10–15 cm. Thus, the studied effect can be considered as an attractive method for producing electromagnetic pulses with the ultrahigh peak power and ultrashort duration. Note that by varying plasma density it is possible to adjust required parameters of the input pulse: its amplitude and pulse duration scale as $A_0 \sim n_0^{3/4}$ and $t_0 \sim n_0^{-1/2}$.

Similar self-induced transparency and pulse compression effects can be realized in the case of interactions of microwave pulses with initially rectilinear high-current electron beams immersed in a guiding magnetic field. Recently, SR pulse amplification with simultaneous shortening has been observed experimentally in the process of Cherenkov-type interaction with a quasicontinuous electron beam [12]. We plan to use the same laboratory setup for experimental studies of the effects of electromagnetic pulse self-compression. The dielectric-loaded waveguide used in [12] will be replaced by the regular metallic waveguide and the magnetic field strength will correspond to the cyclotron resonance value. It should be noted that in difference with [13] where cyclotron mechanism of picosecond microwave pulses amplification have been studied on a beam of gyrating electrons in linear (small signal) regime our case includes nonlinear processes when initial pulse amplitude is rather high and nontrivial pulse transformation can occur even for initially rectilinear electron

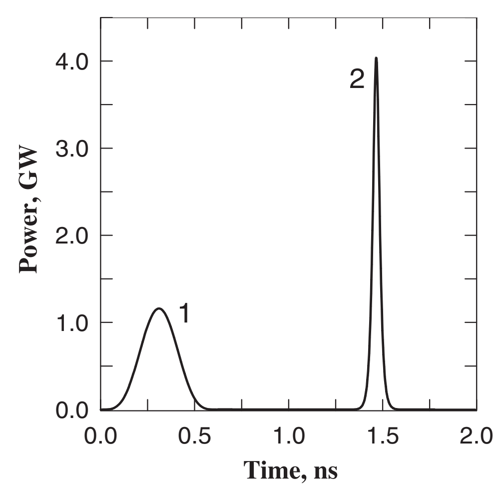

FIG. 3. Simulation of compression of a microwave superradiance pulse: (1) input pulse, (2) output compressed pulse.

beam. We should emphasize that similar to gyrotrons [5,6], transverse wave propagation considered in this Letter is the most feasible for experimental testing of the discussed nonlinear effects due to reduction of sensitivity to the spread of electron beam parameters. Electrons injected from an equipotential cathode have the same energy and gyrofreqency but posses a finite spread over transverse and longitudinal velocities. But in the case of transverse propagation this spread does not have an effect on cyclotron resonance condition (2) and the conditions of soliton formation.

Note in conclusion that alongside analysis of short pulse propagation and transformation in the case of interaction with electron beams under the normal Doppler effect, it is interesting to study such phenomena in the case of pulse-beam interaction under the anomalous Doppler effect [14] (when the velocity of electrons exceeds the radiation phase velocity). As an optical analogy to the last case one should consider the light pulse propagation through an inverted two-level medium that, as is known, is accompanied by simultaneous pulse amplification and compression [3].

This work was supported in part by Russian Foundation for Basic Researches under Grants 08-02-01059. The authors are grateful to G. Nusinovich for useful discussions.

[1] S. L. McCall and E. L. Hahn, Phys. Rev. Lett. **18**, 908 (1967).
[2] S. L. McCall and E. L. Hahn, Phys. Rev. **183**, 457 (1969).
[3] L. Allen and J. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975).
[4] H. M. Gibbs and R. E. Slasher, Appl. Phys. Lett. **18**, 505 (1971).
[5] A. V. Gaponov, M. I. Petelin, and V. K. Yulpatov, Izv. VUZov Radiofizika **10**, 1414 (1967).
[6] G. S. Nusinovich, *Introduction to Physics of Gyrotons* (J. Hopkins Univ. Press, Baltimore, MD, 2004).
[7] A. A. Kolomensky and A. N. Lebedev, Sov. Phys. JETP **44**, 259 (1963).
[8] Note that for magnetized plasma more complicated effect of electromagnetically induced transparency was studied in A. G. Litvak and M. D. Tokman, Phys. Rev. Lett. **88**, 095003 (2002).. This effect manifests itself as a formation of a ‘‘transparency window’’ for a monochromatic signal wave within the zone of resonance absorption under the action of a powerful pump wave and can be considered as a classical analogy of the λscheme in a quantum three-level system [E. Harris, Phys. Today **50**, No. 7, 36 (1997)].
[9] N. S. Ginzburg, A. S. Sergeev, and I. V. Konoplev *et al.*, Phys. Rev. Lett. **78**, 2365 (1997).
[10] N. S. Ginzburg, Yu. V. Novozhilova, and A. S. Sergeev, *et al.*, Phys. Rev. E **60**, 3297 (1999).
[11] S. D. Korovin, A. A. Eltchaninov, and V. V. Rostov *et al.*, Phys. Rev. E **74**, 016501 (2006).
[12] M. I. Yalandin, A. G. Reutova, and M. R. Ulmaskulov *et al.*, JETP Lett. **91**, 553 (2010).
[13] H. J. Kim, E. A. Nanni, and M. A. Shapiro *et al.*, Phys. Rev. Lett. **105**, 135101 (2010).
[14] V. L. Ginzburg and I. N. Frank, Dokl. Akad. Nauk SSSR **56**, 583 (1947).