Bloch Oscillations of Path-Entangled Photons

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We show that when photons in N-particle path-entangled $|N, 0\rangle + |0, N\rangle$ or $N00N$ states undergo Bloch oscillations, they exhibit a periodic transition between spatially bunched and antibunched states. The period of the bunching-antibunching oscillation is N times faster than the period of the oscillation of the photon density, manifesting the unique coherence properties of $N00N$ states. The transition occurs even when the photons are well separated in space.

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When electrons in crystalline potentials are subjected to uniform external fields, classical mechanics predicts that they will exhibit Ohmic transport. Remarkably, in 1929 Bloch predicted that the quantum coherence properties of the electrons prevent their transport [1,2]. He showed that the electrons dynamically localize and undergo periodic oscillations in space. Bloch oscillations (BOs) manifest the wave properties of the electrons, and therefore appear in other systems of waves in tilted periodic potentials. BOs were observed for electronic wave packets in semiconductor supperlattices [3], matter waves in optical lattices [4], and light waves in tilted waveguide lattices [5,6] and in periodic dielectric systems [7].

In optics, BOs relate to the classical (wave) nature of light, and not to its quantum (particle) nature. Recently, quantum properties of light propagating in periodic lattices of identical waveguides have been studied, predicting the emergence of nontrivial photon correlations [8,9]. Nonclassical correlations between photon pairs were experimentally observed in periodic lattices [10], while the effect of disorder was studied in [11]. BOs of a single photon in tilted lattices were shown to follow the dynamics of coherent states [12]. Nonclassical features of BOs of photons in a two-band model were studied by Longhi, who showed that the probability to detect photon pairs in different bands oscillates nonclassically [13].

In this Letter we study theoretically the propagation of spatially entangled states in waveguide lattices which exhibit Bloch oscillations. We consider light fields initiated in a superposition of N photons in site μ' or in site ν', $|\psi\rangle = \frac{1}{\sqrt{2}}(|N\rangle_{\mu'}|0\rangle_{\nu'} + e^{-i\varphi}|0\rangle_{\mu'}|N\rangle_{\nu'})$. Such superpositions, coined $N00N$ states, exhibit fascinating quantum interference properties. $N00N$ states are considered the optimal quantum states of light for quantum metrology applications such as quantum lithography and quantum imaging [14]. Here we show that when $N00N$ states undergo BOs, the nature of the correlations between the photons oscillate between spatially bunched and antibunched states. We find that the period of the oscillations is inversely proportional to the photon number N, resembling the λ/N oscillations of $N00N$ states in Mach-Zehnder interferometers. Interestingly, the oscillation period is also inversely proportional to the initial separation of the two input sites $\mu' - \nu'$. A unique feature of the $N00N$ state BOs is that the transition between the bunched and antibunched states can happen even when the photons are separated by many lattice sites.

We consider the simplest waveguide structure which exhibits BOs, a one-dimensional lattice of single mode waveguides which are evanescently coupled. The propagation in the lattice is determined by two parameters: the phase accumulation rate in the waveguides (the propagation constant) and the tunneling rate between neighboring sites (the coupling constant) [15]. The propagation of the fields in waveguide lattices is described by the tight-binding model, and was used to demonstrate many optical analogues of solid-state phenomena [16,17]. BOs are observed when the coupling constants between all the waveguides are identical and the propagation constants depend linearly on the waveguide position [5,6]. To study the propagation of nonclassical light in such structures we quantize the fields in the lattice. Since each of the waveguides supports a single mode, the field in waveguide μ is represented by the bosonic creation and annihilation operators $a_\mu^\dagger$ and a_μ, which satisfy the commutation relations $[a_\mu, a_\nu^\dagger] = \delta_{\mu,\nu}$. The operators evolve according to the Heisenberg equations [9]:

$$-i\frac{\partial a_\mu^\dagger}{\partial z} = \mu B a_\mu^\dagger + C(a_{\mu+1}^\dagger + a_{\mu-1}^\dagger). \qquad (1)$$

Here z is the spatial coordinate along the propagation axis, C is the coupling constant, and B is the difference in the propagation constants of neighboring sites. The evolution of the creation and annihilation operators is calculated using the Green function $U_{\mu,\mu'}(z)$ of Eq. (1), $a_\mu^\dagger(z) = \sum_{\mu'} U_{\mu,\mu'}(z) a_{\mu'}^\dagger(z = 0)$ [9]. The unitary transformation $U_{\mu,\mu'}(z)$ describes the amplitude for the transition of a single photon from waveguide μ to waveguide μ'. The Green function of Eq. (1) is given by [6,18]

$$U_{\mu,\mu'}(z) = e^{i\frac{\pi}{2}(\mu'-\mu)} e^{i\frac{Bz}{2}(\mu'+\mu)} J_{\mu'-\mu}\left(\frac{4C}{B}\sin(Bz/2)\right), \qquad (2)$$

where $J_\mu(x)$ is the μth Bessel function of the first kind. Since any input state can be expressed with the creation operators $a_\mu^\dagger$ and the vacuum state $|0\rangle$, the evolution of nonclassical states can be calculated using Eq. (2). The probability to locate at site μ a photon that is injected into the lattice at site $\mu' = 0$ is given by the photon density $n_\mu = \langle a_\mu^\dagger a_\mu \rangle = |U_{\mu,\mu'=0}|^2$ and is depicted in Fig. 1(a). The photon exhibits BOs: it spreads across the lattice by tunneling between the waveguides in a pattern characterized by two peaks at the two edges of the distribution. Each peak covers approximately four waveguides and oscillates around the input site with a period $\lambda_B = 4\pi/B$. The path of the peaks marks the two branches of the BO. We note that such a double-branch pattern is not a special feature of single photons. Any state of light which is coupled to a single waveguide exhibits exactly the same photon density. However, when the light is coupled to more than one waveguide, the propagation of the photons becomes state-dependent. Rai *et al.* have shown that a single photon which is initiated in a superposition of several waveguides exhibits BOs like a coherent state [12]. Figure 1(b) shows the photon density for a single photon initiated in a superposition of two neighboring waveguides, with a relative phase $\varphi = 0$. The two paths the photon can take, starting either from waveguide $\mu' = 0$ or from waveguide $\nu' = 1$, contribute coherently to the photon density, $n_\mu = \frac{1}{2}|U_{\mu,\mu'=0} + U_{\mu,\nu'=1}|^2$. Because of this interference the photon oscillates in a single branch, exactly like a coherent beam. In contrast, when a $N00N$ state with $N > 1$ is coupled to the lattice, the photon density is identical to the photon density obtained by two incoherent beams, $n_\mu = \frac{N}{2}|U_{\mu,\mu'=0}(z)|^2 + \frac{N}{2}|U_{\mu,\nu'=1}(z)|^2$ [Fig. 1(c)].

Nonclassical features of light are probed by correlations between the photons. We focus on the probability to detect p photons in waveguide μ and $q = N - p$ photons

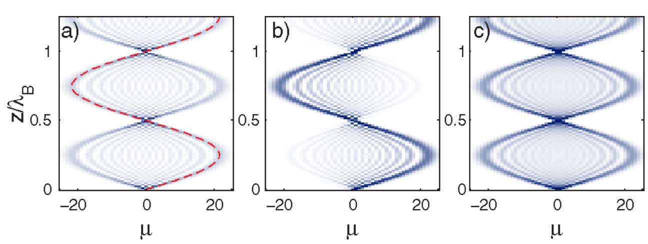

FIG. 1 (color online). (a) The photon density $n_\mu(z)$ for a single photon initiated at the waveguide $\mu' = 0$. The photon is mostly localized in two narrow peaks at the edges of the distribution. The path of each peak (red dashed line) follows a sinusoidal trajectory with a period λ_B, and marks a branch of the BO. In this example the sinusoidal trajectory has an amplitude of 24 waveguides. (b) The photon density $n_\mu(z)$ for a $N00N$ state input with $N = 1$ coupled to waveguides $\mu' = 0$ and $\nu' = 1$. The photon is located at a single branch which oscillates around the input waveguide with a period λ_B. (c) The photon density $n_\mu(z)$ for a $N00N$ input state with $N = 2$ coupled to waveguides $\mu' = 0$ and $\nu' = 1$. The photons from the $\mu' = 0$ and $\nu' = 1$ inputs add up incoherently, showing double-branch oscillations.

in waveguide ν, $\Gamma_{\mu,\nu}^{(p,q)} = \frac{1}{q!p!}\langle a_\mu^{\dagger p} a_\nu^{\dagger q} a_\nu^q a_\mu^p \rangle$ [19]. For a $N00N$ state coupled to waveguides μ' and ν', the multiple detection probability is

$$\Gamma_{\mu,\nu}^{(p,q)} = \frac{1}{2}\frac{N!}{p!(N-p)!}|J_{\mu'-\mu}(\zeta)^p J_{\mu'-\nu}(\zeta)^q + e^{i\theta(z)} J_{\nu'-\mu}(\zeta)^p J_{\nu'-\nu}(\zeta)^q|^2 \quad (3)$$

Where $\zeta = 4(C/B)\sin(Bz/2)$, and $\theta(z)$ is given by

$$\theta(z) = \varphi + \tfrac{1}{2}(\pi + Bz)(\nu' - \mu')N. \quad (4)$$

Equation (3) shows that two terms contribute to the multiple detection probability: the photons arrive either from waveguide μ' or from waveguide ν'. Since the photons are indistinguishable, these two paths interfere. The phase between the two paths is proportional to Nz, indicating that the oscillation period scales like $1/N$ (see below).

In Fig. 2 we depict $\Gamma_{\mu,\nu}^{(1,1)}$, the probability to detect one photon at waveguide μ and another photon at waveguide ν, for a $N00N$ state with $N = 2$. The left column shows the evolution of the probability for a $N00N$ state with a phase $\varphi = 0$. At the beginning of the propagation ($Bz \ll \pi$), the photons follow the same path as in a periodic array of identical waveguides [9,10]. At this stage the off-diagonal terms of the probability matrix $\Gamma_{\mu,\nu}^{(1,1)}$ are much stronger than the diagonal terms, indicating that the photons exhibit antibunching: each photon takes a different branch of the oscillation. However, during the expansion period of the BO, as the photons approach the turning point, the symmetry of the two-photon probability matrix changes significantly. The diagonal terms of the matrix become more pronounced, i.e., there is a higher probability to find the two photons in the same branch of oscillations. At the turning point $z = \frac{\lambda_B}{4}$, the photons bunch: the off-diagonal terms of the probability matrix vanish, indicating that the photons are never found simultaneously at the two different branches. Remarkably, even though the photons start the propagation in spatially separated branches, at the turning point they bunch to one of the branches. Beyond this point the photon density contracts back towards the input waveguides. During this contraction the pairs again switch to an antibunched state. We note that the bunching-antibunching transition happens when the two branches of the BO are spatially separated, whereas the bunching-antibunching transition predicted in binary lattices occurs only when the photons are in the same waveguide [13]. The cycle in the symmetry of the probability matrix is observed for any phase φ of the $N00N$ state, as demonstrated Figs. 2(b) and 2(c). The phase φ sets how bunched or antibunched the photons are at the beginning of the propagation, but the period of the cycle is phase independent [see Eq. (4)].

The bunching-antibunching cycle described above can be realized experimentally by measuring the correlations at the output of lattices with identical parameters but with different

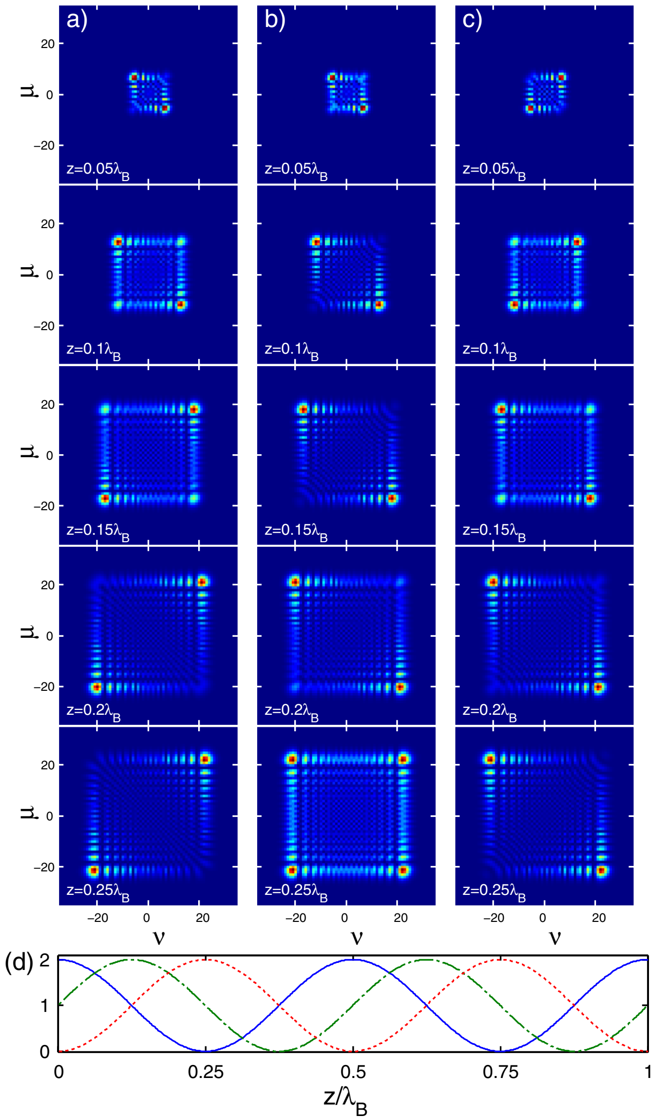

FIG. 2 (color online). Bloch oscillations of $N00N$ states with $N = 2$ coupled to two adjacent waveguides $|\psi\rangle = \frac{1}{\sqrt{2}} \times (|2\rangle_0|0\rangle_1 + e^{-i\varphi}|0\rangle_0|2\rangle_1)$. (a) The multiple detection probability $\Gamma^{(1,1)}_{\mu,\nu}$ at several propagation distances, for $\varphi = 0$. At the beginning of the propagation the two photons exhibit antibunching and are located at the two different branches of the oscillations. As the photons approach the turning point ($z = \lambda_B/4$), they bunch and are found with the highest probability in the same branch. (b) Same as (a) for $\varphi = \frac{\pi}{2}$. The photons show bunching-antibunching cycle, but in this case start the oscillation partially bunched. (c) Same as (a) and (b), for $\varphi = \pi$. Here the photons start the bunching-antibunching cycle bunched. (d) The normalized coincidence rate $\gamma^{(1,1)}(z)$ as a function of the lattice length for $\varphi = 0$ (blue solid line), $\varphi = \frac{\pi}{2}$ (green dash-dotted line), and $\varphi = \pi$ (red dotted line). The coincidence rate is calculated between the positions of the central waveguide in each branch, showing oscillations with a period $\lambda_B/2$. See [25] for a movie visualizing the propagation of (a).

propagation lengths. For each propagation length, the waveguide at the center of each oscillating branch (henceforth waveguides x and y) can be imaged on two photon-number resolving detectors. The probability to detect p photons at waveguide x and $q = N - p$ photons at waveguide y is proportional to $\Gamma^{(p,q)}_{x,y}$. When a delay is introduced between the photons that are injected to waveguide μ' and the photons injected to waveguide ν', the photons become distinguishable, as in the Hong-Ou-Mandel (HOM) experiment [20]. This corresponds to replacing the $N00N$ state with a mixed state of N photons in either one of the two input waveguides. The ratio of the detection probabilities for the $N00N$ and mixed states is given by

$$\gamma^{(p,q)} = \frac{\Gamma^{(p,q)}_{x,y}}{\frac{1}{2}\frac{N!}{p!(N-p)!}(|J^p_{\mu'-x}J^q_{\mu'-y}|^2 + |J^p_{\nu'-x}J^q_{\nu'-y}|^2)}. \quad (5)$$

Figure 2(d) shows $\gamma^{(1,1)}$ as a function of the lattice length, for $N00N$ states with $N = 2$ and $\varphi = 0, \pi/2, \pi$. When $\gamma^{(1,1)} = 0$, the photons are bunched and are never found in the two different branches of the BO; scanning the delay between the input ports of the lattice will yield a HOM dip. When $\gamma^{(1,1)} = 2$, the photons are antibunched, and a delay scan will result in a HOM peak [21]. Figure 2(d) clearly shows that the bunching-antibunching oscillations have a period of $\lambda_B/2$.

We next study input states which exhibit correlation oscillations with shorter periods. Equation (4) suggests that the period of the oscillations in the correlation properties depends on the spacing between the input waveguides and on the number of photons in the $N00N$ state. Figure 3 shows several examples of correlation oscillations with periods shorter than $\lambda_B/2$. In Fig. 3(a) we show the propagation for a $N00N$ state with $N = 2$, where the input sites are separated by one waveguide. In this case the photons exhibit a bunching-antibunching transition with a different spatial symmetry [9]. The correlation map oscillates between a state in which the peaks are highest at the corners of the correlations matrix, to a case in which the highest probability is between the corners. The oscillation period is $\lambda_B/4$, twice the period observed for a $N00N$ state input with adjacent waveguides. Finally we calculate $\Gamma^{(N/2,N/2)}_{\mu,\nu}$ for $N00N$ states with $N = 6$ [Fig. 3(b)] and $N = 10$ [Fig. 3(c)], with adjacent input waveguides. The oscillation period is indeed λ_B/N, as predicted by Eq. (4). Within one oscillation of the single photon density, the N-photon distribution switches N times from all the photons in the same branch to photons divided equally between the two branches.

In conclusion, we studied the propagation of photonic $N00N$ states in waveguide lattices which exhibit Bloch oscillations. We found that while the photon density oscillates in the Bloch frequency, the multiple detection probability oscillates at higher frequencies. These oscillations indicate that the photons show a transition from a bunched to antibunched states, with a period that scales as $1/N$. By carefully designing the parameters of the Bloch lattice this oscillatory transition can be used to distribute bunched and antibunched states of light in an integrated and thus robust manner. To experimentally observe the

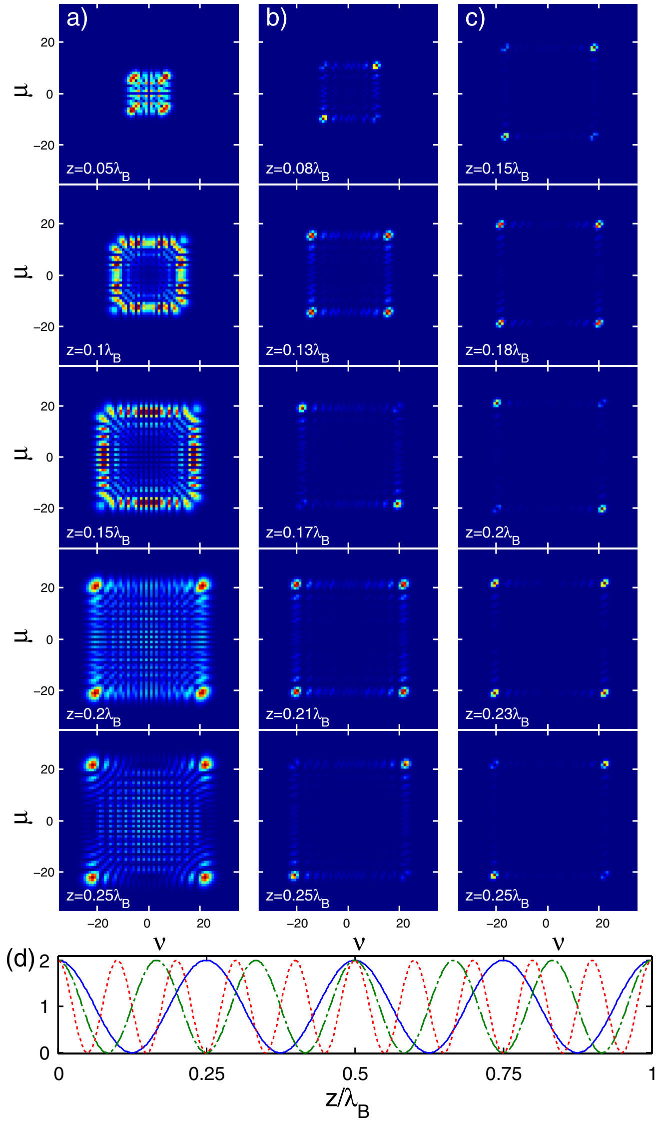

FIG. 3 (color online). Bloch oscillations of $N00N$ states with sub-$\lambda_B/2$ correlation-oscillation periods. (a) The multiple detection probability $\Gamma^{(1,1)}_{\mu,\nu}$ at several propagation distances for the input state $|\psi\rangle = \frac{1}{\sqrt{2}}(|2\rangle_{-1}|0\rangle_1 + |0\rangle_{-1}|2\rangle_1)$. The photons exhibit bunching-antibunching oscillations (see text) with a period $\lambda_B/4$. (b),(c) The multiple detection probability $\Gamma^{(N/2,N/2)}_{\mu,\nu}$ for a $N00N$ state with $N = 6$ (b) and $N = 10$ (c), injected to adjacent waveguides $|\psi\rangle = \frac{1}{\sqrt{2}}(|N\rangle_0|0\rangle_1 + |0\rangle_0|N\rangle_1)$. The oscillations of the correlation matrix are much faster, hence the probability matrix is calculated for five lattice lengths close to the turning point $z = \lambda_B/4$. (d) The normalized coincidence rate $\gamma^{(N/2,N/2)}(z)$ as a function of the lattice length for the above three cases. The period of the oscillations are $\lambda_B/4$ [(a), blue solid line], $\lambda_B/6$ [(b), green dash-dotted line], and $\lambda_B/10$ [(c), red dotted line]. See [25] for a movie visualizing the propagation of (a) and (b).

bunching-antibunching transition, we propose to perform a Hong-Ou-Mandel measurement between two waveguides at the two branches of oscillations using photon-number resolving detectors. We predict oscillations between a HOM dip and peak as a function of the propagation distance in the lattice. Recent progress in waveguide lattice fabrication [22,23], photon-number resolving detectors and photonic $N00N$ state sources [24], make such measurements in reach.

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[1] F. Bloch, Z. Phys. **52**, 555 (1929).
[2] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt-Saunders Int. Ed., Philadelphia, 1981).
[3] J. Feldmann *et al.*, Phys. Rev. B **46**, R7252 (1992); K. Leo *et al.*, Solid State Commun. **84**, 943 (1992).
[4] M. BenDahan *et al.*, Phys. Rev. Lett. **76**, 4508 (1996); S. R. Wilkinson *et al.*, Phys. Rev. Lett. **76**, 4512 (1996).
[5] R. Morandotti *et al.*, Phys. Rev. Lett. **83**, 4756 (1999).
[6] T. Pertsch *et al.*, Phys. Rev. Lett. **83**, 4752 (1999).
[7] R. Sapienza *et al.*, Phys. Rev. Lett. **91**, 263902 (2003).
[8] A. A. Rai, G. S. Agarwal, and J. H. H. Perk, Phys. Rev. A **78**, 042304 (2008).
[9] Y. Bromberg *et al.*, Phys. Rev. Lett. **102**, 253904 (2009).
[10] A. Peruzzo *et al.*, Science **329**, 1500 (2010).
[11] Y. Lahini *et al.*, Phys. Rev. Lett. **105**, 163905 (2010).
[12] A. Rai and G. S. Agarwal, Phys. Rev. A **79**, 053849 (2009).
[13] S. Longhi, Phys. Rev. Lett. **101**, 193902 (2008); S. Longhi, Phys. Rev. B **79**, 245108 (2009).
[14] J. Dowling, Contemp. Phys. **49**, 125 (2008).
[15] H. S. Eisenberg *et al.*, Phys. Rev. Lett. **81**, 3383 (1998).
[16] F. Lederer *et al.*, Phys. Rep. **463**, 1 (2008).
[17] S. Longhi, Laser Photon. Rev. **3**, 243 (2009).
[18] T. Hartmann *et al.*, New J. Phys. **6**, 2 (2004).
[19] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, U.K., 1995).
[20] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[21] B. Dayan *et al.*, Phys. Rev. A **75**, 043804 (2007).
[22] A. Politi *et al.*, Science **320**, 646 (2008); G. D. Marshall *et al.*, Opt. Express **17**, 12546 (2009).
[23] F. Dreisow *et al.*, Phys. Rev. Lett. **102**, 076802 (2009).
[24] I. Afek *et al.*, Phys. Rev. A **79**, 043830 (2009); I. Afek, O. Ambar, and Y. Silberberg, Science **328**, 879 (2010).
[25] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.105.263604 for a movie visualizing BOs of $N00N$ states.