Spatial Intermittency of Surface Layer Wind Fluctuations at Mesoscale Range

Rachel Baïle* and Jean-François Muzy†

SPE UMR 6134, CNRS, Université de Corse, Route des Sanguinaires, Vignola, 20000 Ajaccio, France

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We study various hourly surface layer wind series recorded at different sites in the Netherlands by the "Royal Netherlands Meteorological Institute." By reporting all velocity magnitude correlation coefficients, associated with the available couples of locations, as a function of their spatial distance, we find that they fall on a single curve. This curve turns out to be remarkably well described by a logarithmic shape, characteristic of continuous cascades with an intermittency coefficient $\lambda^2 \simeq 0.04$ and an integral scale $L \simeq 600$ km. Along the same line, we study the scaling properties of spatial velocity increment structure functions. This allows one to estimate the $\zeta(q)$ spectrum and to confirm an intermittent nature of mesoscale fluctuations similar to the one observed in fully developed turbulence.

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From large geostrophic motions to turbulence, atmospheric surface layer wind dynamics is a complex process involving a wide range of spatiotemporal scales. The modeling of wind speed behavior in the mesoscale range ($\sim$ 1–1000 km) is of great interest in wind power generation or in order to control pollutant dispersion. In this intermediate range of scales, the statistical properties of velocity fluctuations are less known than at larger planetary or at finer turbulent scales [1,2]. Indeed, atmospheric conditions, terrain effects, and diurnal oscillations are well known to play an important role in wind variations in the mesoscale range. Several recent empirical studies suggest that the concepts of universality and scale invariance, as usually introduced in the description of fully developed turbulence, may also be pertinent at larger scales [3–7]. However, since the observation of a $k^{-5/3}$ spectrum by Nastrom and Gage in the upper troposphere [8], there has been a debate about the nature (2D–3D) of the energy transfer and the existence of a direct or inverse cascade in this intermediate range of scales [9]. In a recent work, we have studied various time series of surface layer wind velocity and provided evidence for the intermittent nature of the wind time variations in the mesoscale range [10]. For that purpose, we have used magnitude covariance analysis, which has been shown to be a more efficient tool to study intermittency than classical scaling analysis. Along the same line, in this Letter, we would like to address directly the question of the spatial intermittency of the wind velocity field. Since the database we consider consists of synchronous wind series recorded at different locations, the spatial variations of some statistics can be explored through the way they depend on the distance of each pair of sites.

Let us begin by recalling some basic facts on intermittent fields and mainly reproduce arguments of Ref. [11] in order to introduce magnitude correlation functions. In fully developed turbulence, it is well known that the scaling behavior of longitudinal structure functions $S(q, \ell)$ departs from the dimensional prediction of Kolmogorov theory and displays multiscaling:

$$S(q, \ell) \equiv \langle|\delta v_{\parallel}(x, \ell)|^q\rangle \sim \ell^{\zeta(q)} \quad \text{for } \eta \ll \ell \ll L, \quad (1)$$

where $\delta v_{\parallel}(x, \ell)$ is the spatial longitudinal velocity increment over a scale ℓ, $\langle . \rangle$ stands for the spatial mean, q is the order of the structure function, η is the Kolmogorov dissipation scale, and L is the integral (injection) scale. The velocity field is considered multifractal because the $\zeta(q)$ function is not linear but is curved (concave). A popular model is the so-called log-normal model according to which $\zeta(q)$ is a parabola. In the case of turbulence, such a spectrum, $\zeta(q) = q(\frac{1}{3} + 3\frac{\lambda^2}{2}) - \frac{\lambda^2 q^2}{2}$, where $\lambda^2 = \zeta'(0) \simeq 0.025$ is called the intermittency coefficient, provides a very good fit of the data [12]. The study of intermittency in fully developed turbulence has been the object of a huge literature (see, e.g., [13]). Most of the arguments leading to the observed multiscaling properties rely on the notion of a cascade process. The famous Richardson picture, according to which the energy is transferred from large eddies down to the dissipative scales, led various authors to define and study random multiplicative cascades. The simplest 1D log-normal random cascade measure [e.g., the dissipation field $\varepsilon(x)$] is defined by an iterative construction over an interval of size L (the integral scale): At each construction step n, all intervals are split in two equal parts on which the density is multiplied by two independent factors W_1 and W_2 of the same (log-normal) law and such that $E(W) = 1$ and so on. As remarked in Ref. [11], if one considers the magnitude $\omega_\ell(x)$ defined as a logarithm of the measure of each dyadic interval (of position x and scale ℓ), such a multiplicative construction becomes a simple addition of independent identically distributed Gaussian random variables along a dyadic tree. Since in the log-normal case $\omega_\ell(x)$ is Gaussian, it is fully characterized by its covariance function. In other words, the full cascade construction can be encapsulated in covariance of $\omega_\ell(x)$. It is easy to establish [11] that the (ultrametric) treelike structure implies that the covariance of $\omega_\ell(x)$ behaves, for $\ell \leq d \leq L$, as

$$\mathrm{Cov}[\omega_\ell(x), \omega_\ell(x+d)] = \lambda^2 \ln\left(\frac{L}{d}\right). \qquad (2)$$

This logarithmic covariance of the logarithm of multifractal process variations is at the heart of the notion of a continuous cascade as introduced recently [14–17]. Such a process, $V(x)$, gets rids of any grid structure and has increments that can be written as $\delta_\ell V(x) = e^{\omega_\ell(u)}\varepsilon$, where $\omega_\ell(t)$ is a Gaussian (or log-infinitely divisible) process whose covariance is given by Eq. (2) and ε is a white Gaussian noise.

As advocated in Ref. [10] (see also [18]), the switching from traditional (multi)fractal analysis, that relies on the study of structure function scaling, to magnitude correlation analysis is also interesting from a statistical point of view. Indeed, the estimation of the intermittency coefficient λ^2, defined as the curvature of the $\zeta(q)$ spectrum, is far more reliable when one uses Eq. (2) than Eq. (1). Notice that log-correlated magnitudes have interesting applications in various fields like pure mathematics [16], theoretical and statistical physics [19], mathematical finance [18], and turbulence. In this latter field, log correlations have been directly observed in Lagrangian records in a high Reynolds number turbulence experiment [20] while squared logarithmic correlations have been observed on longitudinal velocity series recorded at a given position under various experimental conditions [12]. As shown by Castaing [21] (see also [10]), when velocity variations are recorded at a fixed position, the observed magnitude fluctuations result from both Eulerian and Lagrangian dynamics. If the associated spatial and temporal variations are both governed by a random cascade process with log-correlated magnitudes, it results that the observed time correlation function should decrease as a squared logarithm. This feature has been precisely confirmed from experimental wind tunnel and jet records as reported in Ref. [12]. In a recent study that was conducted by using various wind series gathered in Corsica (France) and the Netherlands, we have shown that the wind variations in the atmospheric surface layer in a range of time scales extending from a few minutes to a few days are also characterized by squared log-correlated magnitudes [10]. We have therefore suggested the existence of a cascading process, very much like in the microscale (turbulent) range, involved in the energy transfer from synoptic scales to finer scales. Our goal in this Letter is to directly characterize this "cascade" in the spatial domain by studying a basket of synchronous wind speed time series recorded at different sites.

The data we use are hourly amplitudes and directions of horizontal wind speeds collected, 10 m above ground level, by the Royal Netherlands Meteorological Institute [22] at 27 different sites over the Netherlands from 1992/01/01 to 2008/12/31. The spatial repartition of the sites, illustrated in Fig. 1,

FIG. 1 (color online). Spatial distribution of the 27 locations over the Netherlands we use from the freely available Royal Netherlands Meteorological Institute wind series database.

allows us to access to $N_d = 351$ distances in a range from 10 to 300 km. For each site i, we denote by $V_x^i(t)$ and $V_y^i(t)$ the velocity components along, respectively, the north and west directions. Let us consider the small scale time increments (1 h) of these components: $\delta_1 V_{x,y}^i = V_{x,y}^i(t+1) - V_{x,y}^i(t)$. In Ref. [10], we have shown that the magnitudes associated with V_x and V_y are identical, and, up to additive seasonal components, one can write $\delta_1 V_x^i(t) = e^{\omega^i(t)}\epsilon_x^i(t)$ and $\delta_1 V_y^i(t) = e^{\omega^i(t)}\epsilon_y^i(t)$, where $\omega^i(t)$ is the local magnitude of velocity components of site i and ϵ_x and ϵ_y are two independent Gaussian noises. Accordingly, a surrogate of the scalar process $\omega^i(t)$ is obtained as $\omega^i(t) = \frac{1}{2}\ln[\delta_1 V_x^i(t)^2 + \delta_1 V_y^i(t)^2]$, which is less noisy than individual magnitudes $\ln[|\delta_1 V_x^i(t)|]$ or $\ln[|\delta_1 V_y^i(t)|]$. In Ref. [10], the time correlation function of the so-defined magnitude $\omega(t)$ appeared to be a "universal" squared logarithmic function (corresponding to a cascade picture in both Lagrangian and Eulerian frames) with an integral time close to $T = 5$ days.

The same kind of analysis can be reproduced in the spatial domain by considering synchronous data at different sites. If d_{ij} is the spatial distance between sites i and j, one can estimate the magnitude spatial correlation function as

$$C(d_{ij}) = \frac{1}{N}\sum_{k=1}^{N}(\omega^i[k] - \bar{\omega}^i)(\omega^j[k] - \bar{\omega}^j), \qquad (3)$$

where N is the size of the time samples ($N \simeq 1.6 \times 10^5$ for the time series we consider) and $\bar{\omega}^i = N^{-1}\sum_{k=1}^{N}\omega^i[k]$ stands for the empirical mean value of ω^i. In Fig. 2, the obtained values of the correlations are reported for the 351 pairs of sites as a function of distances. One clearly observes that all points are distributed around a single slowly decreasing function. We also see that a logarithmic cascade covariance as given in Eq. (2) provides an excellent fit of the data (solid line). We estimate the intermittency parameter $\lambda^2 \simeq 0.04$ while the integral scale is $L \simeq 600$ km. From the velocity records one can evaluate a mean traveled

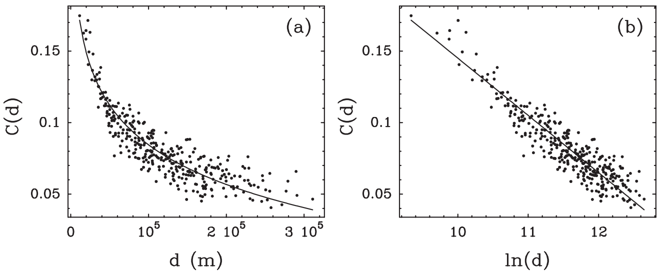

FIG. 2. Magnitude correlations as a function of the spatial distance between two sites. (a) $C(d)$ as a function of d for all 351 pairs of sites. The solid line represents the fit according to the cascade logarithmic correlation [Eq. (2)] with $\lambda^2 = 0.04 \pm 0.008$ and $L \simeq 600$ km. (b) The same plots as in (a) but in semilog representation. A linear fit of the data in this representation provides a direct estimate of λ^2 (from the slope) and L (from the intercept). From the point dispersion amplitude around each distance $\ln(d)$, we have estimated the error on λ^2 as a least square error. The error on L concerns its logarithm so this scale is estimated within roughly a factor of 2.

distance $r(t) = \sqrt{[\int_0^t V_x(u)du]^2 + [\int_0^t V_y(u)du]^2}$ as a function of time t and deduce a typical velocity $V = t^{-1}r(t) \simeq 6$ km/h [23]. The time scale associated with the spatial integral scale L is therefore close to $T = LV^{-1} = 5$ days, in full agreement with previously cited single site observations of time fluctuations of ω.

As far as standard scaling is concerned, one can also estimate the spatial structure functions [Eq. (1)] as functions of the pair distances along the same line. In turbulence several works suggest that longitudinal and transverse velocity increments show different multifractal spectra. However, this difference is small, and, given the level of noise in our data, we have checked that both quantities lead to similar results so we report only results for longitudinal structure functions which are computed as $S_{ij}(q) = N^{-1}\sum_{k=1}^N |V_\parallel^i(k) - V_\parallel^j(k)|^q$, where $V_\parallel = \|\vec{r}\|^{-1}\vec{V}\cdot\vec{r}$. According to previous magnitude correlation results, within the log-normal cascade model, one expects the following scaling:

$$S_{ij}(q) \sim d_{ij}^{\zeta(q)}, \quad \text{with} \quad \zeta(q) = q\left(H + \frac{\lambda^2}{2H}\right) - \lambda^2\frac{q^2}{2}, \tag{4}$$

where $\lambda^2 = 0.04$ and the exponent value H [such as $\zeta(1/H) = 1$] has to be determined.

In Fig. 3(a), the estimated longitudinal structure functions $S(q, d)$ for $q = 1, 3, 5$ are plotted in the log-log scale. Despite the strong noise, one clearly observes a global linear increase and a linear fit can be performed. For $q = 3$ we measure a slope close to $\zeta(3) = 1$. It thus appears that, like in fully developed turbulence, spatial wind fluctuations in the mesoscale domain are characterized by a main Kolmogorov scaling exponent $H = 1/3$. The precise nonlinear shape of the spectrum is difficult to estimate because

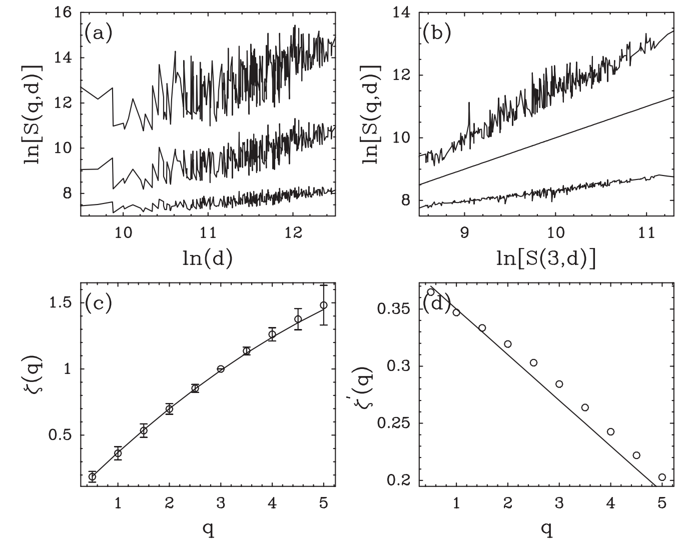

FIG. 3. Scaling of spatial structure functions. (a) $\ln[S(q, d)]$ vs $\ln(d)$ for $q = 1, 3, 5$ (from bottom to top). Plots have been shifted by arbitrary values for representation purpose. (b) Extended self-similarity: $\ln[S(q, d)]$ is plotted as a function of $\ln[S(3, d)]$. (c) Estimated $\zeta(q)$ spectrum (○) as compared to the theoretical spectrum with $\zeta_3 = 1$ and $\lambda^2 = 0.04$ (solid line). The errors have been roughly estimated from the variations of the estimated slopes in large scale and small scale ranges (d) $\zeta'(q)$ as a function of q. For a log-normal cascade, one expects a decreasing straight line of slope $\lambda^2 = 0.04$ (solid line).

of the noise terms. This effect can be reduced within the framework of extended self-similarity [25]. Indeed, in Eq. (4), a prefactor K_{ij}, which depends on the sites i and j, is also involved in the scaling law, and one should therefore have $\ln S_{ij}(q) = \zeta(q)\ln(d_{ij}) + q\ln K_{ij}$. If one assumes that $\zeta(3) = 1$, expressing $S_{ij}(q)$ as a function of $S_{ij}(3)$ reduces the influence of K_{ij} and leads to a better estimation of $\zeta(q)$ [26]. The reduction of the noise can be observed in Fig. 3(b), where $\ln S(q, d)$ is plotted as a function of $\ln S(3, d)$. The ζ_q function, in the range $q \in [0, 5]$, so obtained by a linear regression, is plotted in Fig. 3(c). One sees that a log-normal spectrum with an intermittency coefficient $\lambda^2 = 0.04$ provides a good representation of the data. This is confirmed in Fig. 3(d), where we have plotted the estimated derivative of $\zeta(q)$ as a function of q: As expected in the log-normal case, the function roughly decreases linearly with a slope very close to the value we obtained in former magnitude correlation analysis.

In summary, we have reported empirical evidence that the spatial velocity fluctuations in the atmospheric surface layer share many features with the microscale fully developed turbulence regime: They are characterized by scaling properties with a scaling exponent $H \simeq 1/3$ and by an intermittent behavior which mainly manifests through the slow decay of magnitude correlation functions. We have found an intermittency coefficient $\lambda^2 = 0.04$ larger than the one usually observed in turbulence [12]. Our findings fully confirm previous results obtained by using single site time fluctuations and allow one to rationalize recent observations about the strongly non-Gaussian nature of wind velocity statistics at time scales separating the mesoscale from the microscale regimes [6,7]. In Ref. [27], the authors observed results similar to ours on horizontal velocity statistics using lidar data (i.e., an exponent close to $H = 1/3$ and an intermittency coefficient close to 0.04) [28].

As far as the integral scale is concerned, it is noteworthy that the value $L \simeq 600$ km (or $T \simeq 5$ days as measured in Ref. [10]) precisely corresponds to the wave number observed by Nastrom and Gage at the beginning of the $k^{-5/3}$ range [8]. Such a characteristic scale has also been observed in wind velocity correlation functions [31] or in scaling properties of wind data or rainfalls [5,32]. It is usually associated with front spatiotemporal dynamics [1,2]. Since the energy transfer mechanism in the atmosphere at the mesoscale range is still a matter of debate (see [9] and references therein), it is rather difficult to definitely interpret our results. It appears, however, that our observations preclude a 2D inverse cascade picture (as proposed by Lilly [33]) and suggest a direct cascading picture as advocated by various authors [3,9]. A last fundamental issue related to our findings concerns the fact that most of the large scale "turbulent" features of the atmosphere could be observed close to the surface. Note, however, that, as observed in laboratory experiments (as, e.g., in Ref. [34]), strong shear effects close to boundaries preserve the cascade structure but may change the intermittent properties of the flow.

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*baile@univ-corse.fr
†muzy@univ-corse.fr

[1] Y. L. Lin, *Mesoscale Dynamics* (Cambridge University Press, Cambridge, England, 2007).
[2] J. R. Holton, *An Introduction to Dynamic Meteorology* (Elsevier Academic, Burlington, 2004).
[3] S. Lovejoy, D. Schertzer, and J. D. Stanway, Phys. Rev. Lett. **86**, 5200 (2001).
[4] M. K. Lauren, M. Menabde, A. W. Seed, and G. Austin, Bound.-Lay. Meteorol. **90**, 21 (1999).
[5] R. G. Kavasseri and R. Nagarajan, IEEE Trans. Circuits Syst. I **51**, 2255 (2004).
[6] F. Bottcher, S. Barth, and J. Peinke, Stoch. Environ. Res. Risk Assess. **21**, 299 (2007).
[7] M. Kholmyansky, L. Moriconi, and A. Tsinober, Phys. Rev. E **76**, 026307 (2007).
[8] G. Nastrom and K. Gage, J. Atmos. Sci. **42**, 950 (1985).
[9] E. Gkioulekas and K.-K. Tung, J. Low Temp. Phys. **145**, 25 (2006).
[10] J. F. Muzy, R. Baïle, and P. Poggi, Phys. Rev. E **81**, 056308 (2010).
[11] A. Arneodo, E. Bacry, S. Manneville, and J. F. Muzy, Phys. Rev. Lett. **80**, 708 (1998).
[12] J. Delour, J. F. Muzy, and A. Arneodo, Eur. Phys. J. B **23**, 243 (2001).
[13] U. Frisch, *Turbulence* (Cambridge University Press, Cambridge, England, 1995).
[14] J. F. Muzy, J. Delour, and E. Bacry, Eur. Phys. J. B **17**, 537 (2000).
[15] F. Schmitt and D. Marsan, Z. Phys. B **20**, 3 (2001).
[16] J. Barral and B. B. Mandelbrot, Probab. Theory Relat. Fields **124**, 409 (2002).
[17] E. Bacry and J. F. Muzy, Commun. Math. Phys. **236**, 449 (2003).
[18] E. Bacry, A. Kozhemyak, and J. F. Muzy, J. Econ. Dyn. Control **32**, 156 (2008).
[19] Y. V. Fyodorov and J. P. Bouchaud, J. Phys. A **41**, 372001 (2008).
[20] N. Mordant, J. Delour, E. Leveque, A. Arneodo, and J. F. Pinton, Phys. Rev. Lett. **89**, 254502 (2002).
[21] B. Castaing, Eur. Phys. J. B **29**, 357 (2002).
[22] See http://www.knmi.nl/samenw/hydra/index.html.
[23] We have checked that, for each series, this large time mean velocity is greater than the one associated with the fluctuating (diffusive) part of the velocity field [24]
[24] R. Baïle, Ph.D. thesis, Université de Corse, 2010.
[25] R. Benzi, S. Ciliberto, R. Tripiccione, C. Baudet, F. Massaioli, and S. Succi, Phys. Rev. E **48**, R29 (1993).
[26] Notice that the nonlinear part of $\zeta(q)$ can also be obtained by considering $\ln S_{ij}(q) - \frac{q}{3} \ln S_{ij}(3)$ as a function of d_{ij}. We have checked that this leads to the same results.
[27] S. Lovejoy, A. F. Tuck, S. J. Hovde, and D. Schertzer, J. Geophys. Res. **114**, D07111 (2009).
[28] The fact that we find an intermittency coefficient larger than the value usually reported for the 3D isotropic turbulence has to be considered cautiously. Indeed, even in turbulence, various studies report different values of this exponent [29,30]
[29] A. Praskovsky and S. Oncley, Fluid Dyn. Res. **21**, 331 (1997).
[30] P. Kestener and A. Arneodo, Phys. Rev. Lett. **93**, 044501 (2004).
[31] D. P. Wylie, B. B. Hinton, R. Howland, and R. J. Lord, Mon. Weather Rev. **113**, 849 (1985).
[32] V. Gupta and E. Waymire, J. Geophys. Res. **95**, 1999 (1990).
[33] D. Lilly, J. Atmos. Sci. **40**, 749 (1983).
[34] G. Ruiz-Chavarria, S. Ciliberto, C. Baudet, and E. Lévêque, Physica (Amsterdam) **141D**, 183 (2000).