## Observation of a Cyclotron Harmonic Spike in Microwave-Induced Resistances in Ultraclean GaAs/AlGaAs Quantum Wells

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We report the observation of a colossal, narrow resistance peak that arises in ultraclean (mobility  $\sim$ 3  $\times$  10<sup>7</sup> cm<sup>2</sup>/V s) GaAs/AlGaAs quantum wells (QWs) under millimeter wave irradiation and a weak<br>magnetic field. Such a spike is superposed on the 2nd harmonic microwave induced resistance oscillations magnetic field. Such a spike is superposed on the 2nd harmonic microwave-induced resistance oscillations (MIRO) but having an amplitude  $>300\%$  of the MIRO, and a typical FWHM  $\sim$  50 mK, comparable with the Landau level width. Systematic studies show a correlation between the spike and a pronounced negative magnetoresistance in these QWs, suggesting a mechanism based on the interplay of strong scatterers and smooth disorder. Alternatively, the spike may be interpreted as a manifestation of quantum interference between the quadrupole resonance and the higher-order cyclotron transition in well-separated Landau levels.

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Magnetotransport in quantum Hall systems under an electromagnetic wave has recently revealed unexpected new phenomena, including the microwave-induced resistance oscillations (MIRO) [[1,](#page-3-0)[2\]](#page-3-1) and zero-resistance states (ZRS) [\[3,](#page-3-2)[4](#page-3-3)]. Such discoveries have stimulated much interest in the condensed matter community [[5](#page-3-4)[–10\]](#page-3-5). Presently, the MIRO is being interpreted as resulting from either a ''displacement'' or a ''distribution'' mechanism [\[8](#page-3-6)[–10\]](#page-3-5), either of which could be responsible for a negative resistance under proper conditions, leading to periodic oscillations in inverse magnetic field,  $1/B$ . In a very-high mobility two-dimensional electron system (2DES) hosted in GaAs/AlGaAs heterostructures, the minima in MIRO can reach ZRS. A microscopic mechanism for the formation of ZRS was proposed in [\[9\]](#page-3-7), which invokes a spontaneous breaking of translational symmetry and the formation of current or electrical field domains. More recently, resistance oscillations and ZRS were observed in a nondegenerate 2DES formed on the surface of helium [\[11\]](#page-3-8). In this case transitions between Landau levels (LL) in different electrical subbands are involved. Observations of ZRS in vastly different material systems underscore the fact that irradiated 2DES is a rich system for studies of nonlinear transport where new phenomena continue to emerge.

In this Letter we report the observation of a colossal, narrow resistance peak that arises in ultraclean (mobility  $\sim$ 3  $\times$  10<sup>7</sup> cm<sup>2</sup>/V s) GaAs/AlGaAs quantum wells (QWs) under millimeter wave (MW) irradiation and a weak magnetic field. Such a spike is superposed on the 2nd harmonic of the MIRO but having amplitudes >300% of the MIRO, and a typical FWHM  $\sim$  50 mK. Such a photoconductivity (PC) peak does not follow the ''phase-shift'' pattern of MIRO [\[1](#page-3-0)[–10\]](#page-3-5) and represents a new effect in microwaveirradiated 2DES [[12](#page-3-9)]. Further analysis of its frequency  $(f_{MW})$  dependence shows that the spike occurs precisely at twice the cyclotron frequency,  $2\pi f_{\text{MW}}/\omega_C = \omega/\omega_C = 2$ , where  $\omega_C = eB/m^*$  and  $m^*$  is the effective mass of electrons in GaAs. Harmonics of the cyclotron resonance (CR) were previously observed in far-infrared absorption experiments [\[13](#page-3-10)] and theoretically interpreted in terms of the interplay of short-range scatterers and electron-electron interactions on low mobility Si metal-oxide-semiconductor field-effect transistors (MOSFETs) [[14](#page-3-11)]. In the highmobility GaAs/AlGaAs heterostructures, interaction of collective excitations with CR harmonics has been reported [\[15\]](#page-3-12). On the contrary, the PC, which is a dc response of the 2DES to electromagnetic wave excitation, is generally known to occur as MIRO (not at the exact CR harmonics). Systematic studies show a correlation between the spike and a pronounced negative magnetoresistance (NMR), both observed in our QWs, suggesting a mechanism based on the interplay of strong scatterers and smooth disorder in very-high mobility, modulation-doped  $GaAs/AlGaAs$  heterostructures [[16](#page-3-13)].

Experimental results were obtained from 3 wafers of very-high mobility, Si modulation-doped  $Al_xGa_{1-x}As/$  $GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As QWs. All specimens were Hall bars$ (width W between 90 and 180  $\mu$ m) defined by photolithography and wet etching; high quality electrical contacts were made by  $Ge/Pd/Au$  alloy. Sample A, from which the main data will be presented, has  $x = 0.24$ , a well width  $w = 30$  nm, and is symmetrically doped with a spacer distance of  $d = 80$  nm. After a brief illumination from a red light-emitting diode the sample attained an electron density of  $n_e = 2.9 \times 10^{11}/\text{cm}^2$  and a mobility of

 $\mu \sim 3 \times 10^7$  cm<sup>2</sup>/V s at  $T = 0.3$  K. For comparison we also took data from sample R which has a very similar also took data from sample  $B$ , which has a very similar structure except that  $w = 25$  nm. It has  $n_e = 4.6 \times 10^{11}$  /cm<sup>2</sup> and  $\mu \sim 1.2 \times 10^7$  cm<sup>2</sup>/V s. Sample C in  $10^{11}/\text{cm}^2$  and  $\mu \sim 1.2 \times 10^7 \text{ cm}^2/\text{V s}$ . Sample C, in which only the regular MIRO and ZRS but not the spike which only the regular MIRO and ZRS, but not the spike, were observed, has parameters  $x = 0.30, d = 30$  nm,  $w =$ 20 nm,  $n_e = 6 \times 10^{11} / \text{cm}^2$ , and  $\mu \sim 8.6 \times 10^6 \text{ cm}^2 / \text{V s}$ .<br>The experiments were performed in a top-loading <sup>3</sup>He The experiments were performed in a top-loading  ${}^{3}$ He refrigerator with a base temperature of 0.3 K; experimental details can be found in [[1,](#page-3-0)[4\]](#page-3-3). The magnetic field was calibrated by a Gauss meter and all frequencies  $f_{\text{MW}}$  < 120 GHz were calibrated by a frequency counter. An InSb bolometer was placed directly behind a  $3 \text{ mm} \times 5 \text{ mm}$ <br>piece of OW wafer (the same as for sample A) for the piece of QW wafer (the same as for sample A) for the CR experiments.

An example of the PC is shown in Fig. [1\(a\)](#page-1-0) with  $f_{MW}$  = 103 GHz  $[\rho_{xx}^{\omega}$ , thick (red) line]; for comparison, the "dark" resistance  $\rho_{xx}^0$  (thin black line) is also shown. The coolant temperature for  $\rho_{xx}^0$  was  $T = 0.32$  K, whereas for  $\rho_{xx}^{\omega}$  it rose slightly to 0.4 K; the MW power incident on for  $\rho_{xx}^{\omega}$  it rose slightly to 0.4 K; the MW power incident on the sample is estimated to be on the order of 100  $\mu$ W,

<span id="page-1-2"></span>

<span id="page-1-0"></span>FIG. 1 (color online). (a) An example of the colossal spike in magnetoresistivity  $\rho_{xx}^{\omega}$  (hatched area) under the millimeter wave irradiation is shown for sample A. The spike is superposed on the MIRO. The trace  $\rho_{xx}^0$  without irradiation (thin black line) shows a strong NMR for magnetic field  $B < 1$  kG. The inset [dotted] (blue) line] is the second derivative of the bolometer signal, showing a sharp minimum at cyclotron resonance. (b) Photoresistivity  $\Delta \rho_{xx}^{\omega}$  [dark gray (red) line] is plotted against the inverse magnetic field,  $1/B$ , along with the calculated MIRO trace [light gray (blue) line].

similar to the case of [[4](#page-3-3)]. We notice that the  $\rho_{xx}^0$  exhibited a strong NMR with a plateau minimum between  $0.4 < B <$ 1 kG. The most prominent feature in  $\rho_{xx}^{\omega}$  is a spike (hatched area) at  $B \sim 1.2$  kG that has a magnitude as high as 300% comparing to the MIRO on the background.

Cyclotron resonance was measured to determine the electron effective mass; dotted (blue) line in Fig. [1\(a\)](#page-1-0) shows the InSb bolometer signal  $d^2R_{InSh}/dB^2$ . We observed a sharp minimum at  $B = 2.422$  kG corresponding to a CR effective mass of  $m^* = 0.066m_e$ , where  $m_e$  is the free electron mass. This value is within  $\sim$ 1.5% as compared to the electron band mass in GaAs,  $m_b = 0.067m_e$ . We then use this value for calibration of  $\varepsilon = \omega/\omega_c$ . In particular, we found that the spike position  $B = 1.210 \text{ kG}$ , accurately yielding  $\omega = 2\omega_C$ . For this reason, we refer to the spike described here as the " $2\omega_C$  peak."

In Fig. [1\(b\)](#page-1-0) we plot  $\Delta \rho_{xx}^{\omega} \equiv \rho_{xx}^{\omega} - \rho_{xx}^0$  vs  $1/B$  [dark av (red) line] for  $f_{x} = 117.59$  GHz where the  $\Delta \rho_{xx}^{\omega}$  is gray (red) line] for  $f_{MW} = 117.59$  GHz, where the  $\Delta \rho_{xx}^{\omega}$  is directly measured by a double-modulation technique.  $\Delta \rho_{xx}^{\omega}$ directly measured by a double-modulation technique.  $\Delta \rho_{xx}^{\omega}$ shows a series of ZRS up to the 8th order, attesting to the high quality of data. The  $\Delta \rho_{xx}^{\omega} > 0$  part, except for the  $\epsilon = 2$  spike, is the well-known MIRO showing a periodical pattern with a phase shift  $\delta\varphi$ , of which the value depends on the order of peak  $j = \omega/\omega_c = 1, 2, 3, \dots$  [[17\]](#page-3-14). Specifically,  $\delta \varphi$  tends to be close to 0.25 for higher orders but gradually diminishes towards the major peaks  $j = 1, 2$ . The light gray (blue) line is a fit to the MIRO [[7\]](#page-3-15) by using

<span id="page-1-1"></span>
$$
\Delta \rho_{xx}^{\omega}(B) = A \int dE [n_F(E) - n_F(E + \hbar \omega)] \nu(E) \partial_E
$$
  
 
$$
\times \nu(E + \hbar \omega), \qquad (1)
$$

where A is a scaling factor for amplitude,  $n_F(E) = 1/[1 +$  $\exp(\frac{E-E_F}{k_B T})$  is the Fermi distribution function, and  $\nu(E) = \sum_{k=0}^{k} \left(\frac{eB}{k_B T}\right) \left(1 + \frac{E}{k_B T}\right) \left(\frac{1}{k_B T}\right) \left(1 + \frac{1}{k_B T}\right)$  $\sum_{\substack{(a) \text{odd } c}} \frac{(e^B}{\pi^2 h \Gamma}) / \{1 + [E - (i + \frac{1}{2}) \hbar \omega_c]^2 / \Gamma^2\}$  (LL index  $i =$  $(0, 1, 2, ...)$  is the density of states with  $\Gamma$  the LL broadening,  $E_F = 15$  meV and  $T = 0.4$  K from the experiment. For  $\epsilon > 2$  the calculated magnetic field position and the amplitude of the MIRO fit the experimental data quite well, yielding a LL line width  $\Gamma \sim 10 \mu\text{eV} \sim 120 \text{ mK}$ .<br>Remarkably a distinct  $2\omega_{\text{e}}$  peak is superposed on the Remarkably, a distinct  $2\omega_C$  peak is superposed on the MIRO predicted by Eq. ([1\)](#page-1-1), indicating that it may be of a different origin. As shown in Fig. [1](#page-1-2) as well as in Fig. [2](#page-2-0), the  $2\omega_c$  peak is extremely narrow in width, and can be characterized by a large quality factor  $Q = B/\Delta B$ . For example, for  $f = 103$  GHz the FWHM  $\Delta B \sim 0.026$  kG  $\sim$ 50 mK, leading to  $Q \sim 50$ . A similar procedure yielded  $\Gamma \sim 200 \text{ mK}$  and  $\Delta B \sim 100 \text{ mK}$  for sample B.<br>Apparently the 2000 peak becomes prominent for well-Apparently, the  $2\omega_C$  peak becomes prominent for wellseparated LLs, i.e.,  $\hbar \omega_C \gg \Gamma$ ; hence, a higher  $\mu$  or  $f_{MW}$ <br>favors the observation of this spike favors the observation of this spike.

We show in Fig. [2](#page-2-0) the  $2\omega_C$  peak position (in B) and its width  $(\Delta B)$  in different MW frequencies, respectively observed in samples A and B. The observations can be summarized as follows. (1) The  $2\omega_C$  peak is a generic feature from a low frequency  $f_{\text{MW}} \sim 60 \text{ GHz}$  to a high

<span id="page-2-0"></span>

FIG. 2 (color online). The frequency dependence of the spike is shown for, respectively, sample  $A$  (solid lines) and sample  $B$ (dashed lines) in  $f_{\text{MW}}$  between ~60 and ~135 GHz. It is shown that the spike amplitude and width are correlated with the sample mobility. The inset shows a linear relation between the magnetic field position of the spike and the  $f_{MW}$ , indicating that the spikes occur at  $2\omega_C = 2eB/m^*$ , with  $m^* = 0.066m_e$ .

frequency  $f_{\text{MW}} \sim 135$  GHz in both samples, (2) the peak becomes more prominent as  $f_{MW}$  increases, and (3) the peak amplitude (as compared with the MIRO amplitude), as well as its FWHM, is correlated with the sample mobility. In the inset the peak position (in  $B$ ) is plotted versus  $f_{\text{MW}}$ , which shows, again, that for both samples and in the whole MW range measured the peak is associated with  $\varepsilon = 2$  with a fitted effective mass 0.066 $m_e$ .

The amplitude of the  $2\omega_C$  peak shows a roughly linear dependence on the coolant temperature. For example, in the inset of Fig. [3](#page-2-1) we plot the amplitude  $H$  (defined as the total  $\rho_{xx}^{\omega}$  subtracted by MIRO) and found that the spike increased by a factor of 5 as T decreased from 1.5 to 0.5 K.

<span id="page-2-1"></span>

FIG. 3 (color online). Spikes measured at different temperatures are shown for sample A. Inset: The spike amplitude shows an approximately linear dependence with temperature. The line is a guide for the eye.

In summary, we have observed a new PC spike superposed on the regular MIRO. Within the experimental accuracy, the spike is found to occur precisely at  $\varepsilon = 2$ . The finding is quite surprising, for the reason that its position and amplitude, as well as line shape, do not conform to the descriptions of the presently accepted theoretical models, neither the displacement nor the distribution mechanism. Moreover, the spike can only be clearly observed under the conditions of ultraclean GaAs/AlGaAs 2DES, more stringent than MIRO and ZRS. In the following we discuss possible origins of the spike based on its phenomenology.

In the Hall bar geometry pertaining to this experiment, magnetoplasma (MP) with a wave vector  $q = \pi/W$  can be excited by microwave and contribute to PC response in addition to MIRO [\[12\]](#page-3-9). In previous work of far-infrared absorption in a grating-coupled 2DES, Batke et al. [\[15\]](#page-3-12) observed an interaction of the plasmon resonance with the second harmonic of CR and discussed the underlying mechanism in terms of nonlocality of plasmon dispersion. While we cannot rule out the role of collective plasmalike excitations (including the edge MP), we note that the  $2\omega_C$ peak can also be found in Corbino rings [[4\]](#page-3-3).

We focus now on the analysis of NMR, which are dominating features in samples  $A$  and  $B$  [inset (a) of Fig. [4\]](#page-2-2). Mirlin *et al.* [[16\]](#page-3-13) considered a two-component model of disorder in a very-high mobility, modulation-doped GaAs/ AlGaAs heterostructure containing (i) randomly distributed, dilute, hard scatterers (termed ''antidots'' here) with density  $n_S$  and radius  $a(n_S^{-1/2}) \gg a \gg k_F^{-1}$ , where  $k_F = \sqrt{2\pi n_e}$  is the Fermi vector), and (ii) smooth random<br>potential (correlation radius  $\sim d$  momentum relaxation potential (correlation radius  $\sim d$ , momentum relaxation rate  $\tau_L^{-1}$ , transport mean free path  $l_L = \nu_F \tau_L$ , where

<span id="page-2-2"></span>

FIG. 4 (color online). Inset (a): The 3 high-mobility, modulation-doped GaAs/AlGaAs samples  $(A, B, C)$  show NMR, as explained by a mechanism of interplay between strong scatterers and smooth disorder [[16](#page-3-13)]. The dashed lines are the asymptotic lines that yield the characteristic frequency  $\omega_0$  for the NMR (see the text). In addition to the  $\varepsilon = 2$  spike, extra peaks can be observed at the high-order even numbers of  $\epsilon$  ( $\epsilon$  = 4; 6; 8) in sample A. Inset (b) is a schematic for cyclotron and quadrupole transitions as discussed in the text.

 $\nu_F = \hbar k_F/m^*$  is the Fermi velocity). The mean free path for the scattering on antidots is  $l_S = v_F \tau_S \approx 1/2n_S a$ . It is assumed that  $\tau_L \gg \tau_S$ , so that the zero-B resistivity  $\rho_{xx}^0(0)$ <br>is determined by antidots  $\tau^{-1} = \tau_{xx}^{-1} + \tau_{yy}^{-1} \approx \tau_{xx}^{-1}$ . The is determined by antidots,  $\tau^{-1} = \tau_L^{-1} + \tau_S^{-1} \approx \tau_S^{-1}$ . The<br>combination of the two types of disorder induces a novel combination of the two types of disorder induces a novel mechanism leading to a strong NMR, followed by the saturation of  $\rho_{xx}^0(B)$  at a value determined by the smooth disorder disorder.

As displayed in inset (a) of Fig. [4,](#page-2-2) sample A shows an unusually deep NMR where the  $\rho_{xx}^{0}(B)$  decreases by a factor of  $\sim$  50 at  $B \sim 0.5$  kG and becomes a wide plateau. factor of  $\sim$ 50 at  $B \sim 0.5$  kG and becomes a wide plateau. For sample B it shows a steep valley at  $B \sim 0.5$  kG and then increases to a positive magnetoresistance. Such behavior can be described consistently by the above model. It is instructive here to estimate the relevant  $n<sub>S</sub>$  by a fit to the NMR. As shown by dashed lines [inset (a)] in Fig. [4](#page-2-2), the asymptotics [[16](#page-3-13)]  $\rho_{xx}(B)/\rho_{xx}(0) = 1 - (\omega_C/\omega_0)^2$  de-<br>scribes reasonably well the onset of NMR ( $\omega_C \ll \omega_0$ ) scribes reasonably well the onset of NMR ( $\omega_C \ll \omega_0$ ), where  $\omega_0 = (2\pi n_S)^{1/2} \nu_F (2l_S/l_L)^{1/4}$  is a characteristic fre-<br>quency governed by the interplay of two scattering comquency governed by the interplay of two scattering components. Using the fitted values of  $\omega_0$  and taking  $l_L/l_S \sim 50$ , 10, 5 for A, B, C, we have determined the  $n_S$ to be  $(8 \mu m)^{-2}$ ,  $(6 \mu m)^{-2}$ ,  $(2.6 \mu m)^{-2}$ , respectively. We<br>conclude that the 2D electrons in these samples experience conclude that the 2D electrons in these samples experience scatterings by dilute scatterers randomly distributed on a smooth background potential, consistent with [[16](#page-3-13)].

How the interplay of the two scattering components affects the photoconductivity remains an interesting open question. Dimitriev *et al.* [[18](#page-3-16)] have studied theoretically this regime and predicted new features in ac conductivity  $(\Delta \sigma_{\omega}^{(C)})$  and PC  $(\sigma_{\text{ph}}^{(C)})$  beyond the standard MIRO. Briefly, the authors address the non-Markovian corrections in the electron dynamics, which were ignored in the Boltzmann treatment. They found an oscillatory (in  $1/B$ ) correction  $\Delta \sigma_{ph}^{(C)} \propto \Delta \sigma_{\omega}^{(C)} \propto -\text{Re}P(\omega)/n_{S}\tau_{S}$ , where the absorption  $P(\omega)$  has a series of poles at  $\omega = j\omega_c, j = 1, 2, 3, \dots$ . In principle such effect could be at the origin of the observed spike. However, discrepancies exist, especially regarding the fact that we have only seen a singular peak at  $2\omega_C$  rather than oscillations.

In addition, a mechanism based on quantum interference could play an important role. Specifically, as depicted in inset (b) of Fig. [4,](#page-2-2) for N to  $N + 2$  transitions there could exist two possible channels: (i) due to LL mixing the dipole transition between the N and  $N + 2$  levels (line arrow), and (ii) the quadrupole resonance (dashed arrow) in the presence of a field gradient of millimeter wave. While interference between the two channels was shown [[19](#page-3-17)] to generate photocurrent at  $2\omega_C$  in high B, its effect in very-high LLs has not been addressed. Such interference effect, if confirmed by further experiments, would be the evidence for ''electromagnetically induced transparency'' in the dc transport of an ac-driven 2DES as proposed in [[20](#page-3-18)].

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