Erratum: Einstein Equations for Generalized Theories of Gravity
and the Thermodynamic Relation $\delta Q = T\delta S$ are Equivalent
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Equation (1) had a sign error due to a sign error in Eq. (32) of Ref. [6]. It should read

$$\frac{\partial \mathcal{L}}{\partial R_{pabq}}\delta R_{pabq} = 2\frac{\partial \mathcal{L}}{\partial R_{pabq}}\nabla_p\nabla_q\delta g_{ab} - \frac{\partial \mathcal{L}}{\partial R_{pabc}}R^q{}_{abc}\delta g_{pq}. \tag{1}$$

Consequently, the correct form of Eq. (2) is

$$\sqrt{-g}\left(-\frac{\partial \mathcal{L}}{\partial g_{ab}} - 2\nabla_p\nabla_q\frac{\partial \mathcal{L}}{\partial R_{pabq}} + \frac{\partial \mathcal{L}}{\partial R_{pqra}}R^b_{pqr}\right) - \frac{1}{2}\sqrt{-g}g^{ab}\mathcal{L} = 0, \tag{2}$$

and the coefficients of $\nabla_p\nabla_q\frac{\partial \mathcal{L}}{\partial R_{pabq}}$ in Eqs. (3)–(5) should be -2.

Another unrelated sign error occurred in the inline equation in the paragraph before Eq. (12), which should read $d(W^{cd}\epsilon_{cd}) = -2\nabla_c W^{cd}\epsilon_d$. Consequently, the coefficients of the right-hand side (rhs) of Eqs. (12),(13) should be $+2/T$ and the coefficient on the rhs of Eq. (15) should be $+2$.

So, the new Eq. (15) holds if and only if

$$T^{ab} = 2\left[-2\nabla_p\nabla_q\frac{\partial \mathcal{L}}{\partial R_{p(ab)q}}\right] + g^{ab}f, \tag{3}$$

where $R_{p(ab)q}$ is symmetric in (a, b). Since

$$-2\nabla_p\nabla_q\frac{\partial \mathcal{L}}{\partial R_{p(ab)q}} = -2\nabla_p\nabla_q\frac{\partial \mathcal{L}}{\partial R_{pabq}} + \frac{\partial \mathcal{L}}{\partial R_{pqra}}R_{pqr}{}^b, \tag{4}$$

we find that Eq. (16) should read

$$T^{ab} = 2\left[-2\nabla_p\nabla_q\frac{\partial \mathcal{L}}{\partial R_{pabq}} + \frac{\partial \mathcal{L}}{\partial R_{pqra}}R_{pqr}{}^b\right] + g^{ab}f. \tag{5}$$

The conclusions of the Letter are unaffected by correcting the sign errors. We thank Dan Gorbonos for pointing to us the sign error in [6] and Joey Medved for discovering the second sign error and for useful discussions.