Swimmers in Thin Films: From Swarming to Hydrodynamic Instabilities

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We investigate theoretically the collective dynamics of a suspension of low Reynolds number swimmers that are confined to two dimensions by a thin fluid film. Our model swimmer is characterized by internal degrees of freedom which locally exert active stresses on the fluid. We find that hydrodynamic interactions mediated by the film can give rise to spontaneous continuous symmetry breaking (swarming), to states with either polar or nematic homogeneous order. For dipolar swimmers, the stroke averaged dynamics are enough to determine the leading contributions to the collective behavior. In contrast, for quadrupolar swimmers, details of the internal dynamics are important in determining the bulk behavior. In the broken symmetry phases, fluctuations of hydrodynamic variables destabilize order. Interestingly, this instability is not generic and depends on the length scale.

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In nature, baths of micron-scale swimmers are found to show remarkable out of equilibrium phenomena ranging from anomalous diffusion and viscosity enhancement to turbulent and swirl-like motion or self-organization into complex dissipative structures [1–5]. They act as the inspiration for man-made devices able to control and mix fluids on micron scales. The search for the design principles of such devices [6–8] remains in its infancy. However, simplified models of low Reynolds number swimmers should prove to be a useful starting point for the theoretical understanding of this class of collective phenomena.

A very simple picture [9] then of a swimmer is an internally driven vector oriented towards its direction of motion. Theoretically, the dynamics of a collection of such objects can be described, on long length and time scales, by vector and tensor equations which are natural generalizations of equilibrium liquid crystalline hydrodynamics [10]. These active fluids involve the study of conserved and broken symmetry variables that are nonequilibrium analogues [11] of Goldstone modes. Previous studies on active suspensions [12–15] have shown that fluctuations in these modes destabilize ordered states. In 3D this instability is termed generic, as it is independent of the length scale. However, these descriptions are, in a fundamental sense, phenomenological since another physical mechanism [14,15] must be invoked to generate the ordered states which have subsequently been shown to be unstable [12].

In this Letter, we introduce and study a self-contained and microscopically defined physical system in which it is possible to both generate homogeneously ordered states and to examine their stability. This is provided by a ''suspension'' of swimmers confined to two dimensions by a viscous thin film [5]. In addition, the thin film geometry is particularly accessible to experiments both from the point of view of ease of observation and external activation. We study analytically a model that directly links the collective behavior to the microscopic dynamics. It combines fluctuations, both active and passive, with the deterministic motion due to activity and hydrodynamic interactions.

We find that the purely physical coupling mediated by the thin film can give rise to local spontaneous breaking of symmetry, and we identify the possible ordered states. The system can have either nematic order, characterized by a macroscopic axis of mean orientation $\mathbf{n}$ and symmetry $\mathbf{n} \rightarrow -\mathbf{n}$, or polar order, with mean orientation axis $\mathbf{p}$ for which $\mathbf{p} \neq -\mathbf{p}$. We then examine the stability of each of the homogeneous phases to hydrodynamic fluctuations. We find that the isotropic phase is stable. On the other hand, as for the bulk 3D system, the homogeneous broken symmetry states are destabilized by the hydrodynamic modes of the system. Here, however the thin film weakens this effect and the instabilities observed are length scale dependent (i.e., not generic).

We restrict ourselves to a dilute solution, and we make use of a mean field approximation and study the one-particle distribution function, $c(\mathbf{R}, \hat{\boldsymbol{u}}, t) = \sum_i \langle \delta(\mathbf{R}_i - \mathbf{R}) \delta(\hat{\boldsymbol{u}}_i - \hat{\boldsymbol{u}}) \rangle$, the probability of finding a swimmer with average orientation $\hat{\boldsymbol{u}}$ at position $\mathbf{R}$. This satisfies a dynamic equation

$$\partial_t c = -\boldsymbol{\nabla} \cdot \mathbf{J}_T - \boldsymbol{\mathcal{R}} \cdot \boldsymbol{\mathcal{J}}_R; \qquad \boldsymbol{\mathcal{J}}_R = -D_R \boldsymbol{\mathcal{R}} c + \boldsymbol{\Omega} c,$$
$$\mathbf{J}_T = -\mathbf{D}_T \cdot \boldsymbol{\nabla} c + (\bar{v}\, \hat{\boldsymbol{u}} + \mathbf{V}) c, \tag{1}$$

in terms of translational $\mathbf{J}_T(\mathbf{R}, \hat{\boldsymbol{u}}, t)$ and rotational $\boldsymbol{\mathcal{J}}_R(\mathbf{R}, \hat{\boldsymbol{u}}, t)$ currents, with $\boldsymbol{\mathcal{R}} := \hat{\boldsymbol{u}} \wedge \frac{\partial}{\partial \hat{\boldsymbol{u}}}$ [16]. $\mathbf{D}_T$ and D_R are, respectively, the translational (rotational) diffusion tensor (constant) of the swimmer and represent both passive and active fluctuations. The deterministic quantities are active, describing self-propulsion, $\bar{v}\, \hat{\boldsymbol{u}}$, and the translational $\mathbf{V}$ and rotational $\boldsymbol{\Omega}$ velocities induced on a swimmer due to the activity of others by hydrodynamic interactions mediated by the film.

Thin film hydrodynamics.—The film is described as an infinite incompressible two-dimensional layer of fluid with

(2D) viscosity η filling the plane $z = 0$ and coupled hydrodynamically to another incompressible bulk fluid of (3D) viscosity η_e which fills the region $z \neq 0$. To distinguish between them we indicate the three-dimensional quantities with a prime. We consider the fluid dynamics in the vanishing Reynolds number (Stokes) limit where inertia can be neglected [17]. For the in-plane quantities, given an in-plane force density $\boldsymbol{F}(x, y)$ the velocity $\boldsymbol{v}(x, y)$ and pressure $p(x, y)$ satisfy [18]

$$\eta\nabla_\perp^2 \boldsymbol{v} + \boldsymbol{\nabla}_\perp p + \boldsymbol{\sigma}_e^+ - \boldsymbol{\sigma}_e^- = -\boldsymbol{F}; \qquad \boldsymbol{\nabla}_\perp \cdot \boldsymbol{v} = 0, \quad (2)$$

where $\boldsymbol{\nabla}_\perp = (\partial_x, \partial_y)$ is the 2D gradient operator. $\boldsymbol{\sigma}_e^\pm := \eta_e \partial_z \boldsymbol{v}'|_{0^\pm}$ is the shear stress of the bulk fluid at the top or bottom of the thin film; in the external region, $z \neq 0$, the velocity $\boldsymbol{v}'(x, y, z)$ and pressure $p'(x, y, z)$ satisfy the Stokes equation, $\eta_e\nabla^2\boldsymbol{v}' + \boldsymbol{\nabla}p' = 0$, $\boldsymbol{\nabla}\cdot\boldsymbol{v}' = 0$, where $\boldsymbol{\nabla} = (\boldsymbol{\nabla}_\perp, \partial_z)$ is a 3D gradient operator. The ratio of the two- and three-dimensional viscosities introduces a length scale $s := \eta/(2\eta_e)$ that governs two asymptotic regimes. For $r \ll s$ dissipation occurs almost entirely in plane and hydrodynamic flow fields are quasi-two-dimensional while at lengths $r \gg s$ dissipation is mostly due to flow out of plane, and the hydrodynamics is similar (but not identical) to that in three dimensions.

The viscous drag coefficient on a flat disk of radius a embedded in the film is $\gamma = 4\pi\eta/g$, with g a function of s/a [18]. On length scales large compared to a, the interaction between several disks lying in the film can be approximated using pointlike forces at their centers and the Green function $\boldsymbol{H}$ of Eq. (2), corresponding to the flow $\boldsymbol{v}(\mathbf{r}) = \boldsymbol{H}(\mathbf{r} - \mathbf{r}_0)\cdot\mathbf{f}_0$ generated by a pointlike force $\mathbf{f}_0$ at $\mathbf{r}_0$. The tensor $\boldsymbol{H}(\mathbf{r}) = \frac{s}{\eta}\int\frac{d^2k}{(2\pi)^2}e^{-i\mathbf{k}\cdot\mathbf{r}}\frac{(\mathbb{I}-\hat{\boldsymbol{k}}\otimes\hat{\boldsymbol{k}})}{sk^2+k}$ is the thin film equivalent of the Oseen tensor [16]. In the following, we work in the limit $r \gg s$ [19,20].

We consider swimmers with an average speed $\bar{v}$ moving in a direction $\hat{\boldsymbol{u}}$ which can be represented at large length scales as a time-dependent force dipole that generates an associated velocity field. We also consider swimmers for which the force dipole is zero and the leading behavior is determined by a force quadrupole. Hence the average force density of a swimmer with mean position $\mathbf{r}_\alpha$ is of the form $\mathbf{f}_\alpha(\mathbf{r}) = -\bar{f}_d\bar{L}\hat{\boldsymbol{u}}\hat{u}_i\nabla_i\delta(\mathbf{r}-\mathbf{r}_\alpha) + \frac{1}{2}\bar{f}_q\bar{L}^2\hat{\boldsymbol{u}}\hat{u}_i\hat{u}_j\nabla_i\nabla_j\delta(\mathbf{r}-\mathbf{r}_\alpha) + \cdots$, representing the dipole and quadrupole contributions, respectively, where $\bar{L}$ is the typical dimension of the swimmer.

For a concrete calculation, an explicit microscopic model of a swimmer is required and we have used a three-disk model of a swimmer [21]. Details of the model are reported in [22]. This is characterized by a minimum number (2) of degrees of freedom (of typical length l) that move in a nonreciprocal fashion in time with frequency $\omega = \frac{2\pi}{T}$ to achieve locomotion in the Stokes limit. In the following we indicate the time average over a swimmer cycle period T with an overbar $\bar{h} = \frac{1}{T}\int_0^T h(t)dt$. In the limit of small sinusoidal oscillations of amplitude d around l it is possible to obtain [21] the average self-propulsion velocity as $\bar{v} = \frac{a_0\omega d^2}{6l^2}[1 + \frac{1}{\kappa^2} - \frac{1}{(1+\kappa)^2}]$, where $a_0 := \frac{4s}{3g}$, and our convention is that the swimmer move in the same direction of $\hat{\boldsymbol{u}}$. For the average force we find $\bar{L} = l$ and $\bar{f}_d = f\frac{da_0}{12l^2}\times[(\frac{1}{\kappa}-1) + 2(\frac{1}{\kappa^2}-\kappa) + \frac{(1-\kappa)}{(1+\kappa)^2}]$, $\bar{f}_q = f\frac{da_0}{36l^2}[-1 + \frac{2}{\kappa^2} + \frac{2}{\kappa} + 2\kappa + 2\kappa^2 + 2\frac{\kappa}{(1+\kappa)^2}]$. Here $f := \gamma\omega d$ is the force scale on each degree of freedom of the swimmer. The parameter κ, which is the average ratio of the internal lengths, controls the nature of the swimmer: the dipole is positive (pusher) for $\kappa > 1$, negative (puller) for $\kappa < 1$, and zero (quadrupole) for $\kappa = 1$.

The interaction between two such swimmers, which is the origin of $\mathbf{V}$, $\boldsymbol{\Omega}$ in Eq. (1), is complex as each one is characterized by periodic internal dynamics whose cycles may also have different phases [23]. Here we will restrict ourselves to swimmers that all have the same phase [23], and the limit where the typical separation r between the swimmer centers $\mathbf{R}^A$ and $\mathbf{R}^B$ is much larger than the typical dimension l of the object. Dynamical quantities depending on hydrodynamic interactions, such as translational $\mathbf{v}_{2b} := \dot{\mathbf{R}}^\alpha - \bar{v}\hat{\boldsymbol{u}}^\alpha$ and angular $\boldsymbol{\omega}_{2b} := \hat{\boldsymbol{u}}^\alpha \wedge \dot{\hat{\boldsymbol{u}}}^\alpha$ velocities felt by each swimmer, for $\alpha = A, B$ are described to good approximation by the first few terms of the expansion in spherical harmonics [15,23] of the tensor $\boldsymbol{H}$. To leading order, for swimmer A we find $\mathbf{v}_{2b} = \frac{s\Theta^{(2)}}{2\pi\eta}\{(3\mathcal{B}^2-1)\frac{\hat{\boldsymbol{r}}}{r^2} - \mathcal{B}\frac{\hat{\boldsymbol{u}}^B}{r^2}\} + \mathcal{O}(1/r^3)$ and $\boldsymbol{\omega}_{2b} = \frac{s\Theta^{(2)}}{2\pi\eta}\times\{3(\mathcal{A} + 2\mathcal{B}\mathcal{C} - 5\mathcal{A}\mathcal{B}^2)\frac{\hat{\boldsymbol{u}}^A\wedge\hat{\boldsymbol{r}}}{r^3} + (3\mathcal{A}\mathcal{B} - \mathcal{C})\frac{\hat{\boldsymbol{u}}^A\wedge\hat{\boldsymbol{u}}^B}{r^3}\} + \mathcal{O}(1/r^4)$. Here $\mathcal{A} := (\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{u}}^A)$, $\mathcal{B} := (\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{u}}^B)$, $\mathcal{C} := (\hat{\boldsymbol{u}}^A\cdot\hat{\boldsymbol{u}}^B)$, where $\hat{\boldsymbol{u}}^A$ and $\hat{\boldsymbol{u}}^B$ are swimmer orientations. $\mathbf{r}$ is the separation vector and $\Theta^{(2)}$ is related to the time-dependent force dipole and scales as fl. We neglect the effect of interactions on the internal dynamics and so do not address synchronization effects [20].

The collective dynamics is obtained by a coarse-graining procedure, first in time and then in space. The velocities $\boldsymbol{\Omega}$, $\mathbf{V}$ in Eqs. (1) are obtained from the two body velocities for swimmers with positions $\mathbf{R}$ and $\mathbf{R}'$ and orientations $\hat{\boldsymbol{u}}$ and $\hat{\boldsymbol{u}}'$ as

$$\begin{pmatrix}\boldsymbol{\Omega}\\ \mathbf{V}\end{pmatrix}(\mathbf{R}, \hat{\boldsymbol{u}}) = \int_{\hat{\boldsymbol{u}}', \mathbf{R}'}\begin{pmatrix}\bar{\boldsymbol{\omega}}_{2b}\\ \bar{\mathbf{v}}_{2b}\end{pmatrix}(\mathbf{R} - \mathbf{R}', \hat{\boldsymbol{u}}, \hat{\boldsymbol{u}}')c(\mathbf{R}', \hat{\boldsymbol{u}}') \quad (3)$$

where, as before, the overbar denotes time average over the swimmer cycle. Order parameters such as local density ρ, polarization $\mathbf{P}$, and nematic orientation tensor $\mathbb{S}$ are defined as moments of c:

$$\begin{pmatrix}\rho\\ \mathbf{P}\\ \mathbb{S}\end{pmatrix}(\mathbf{R}, t) = \int_{\hat{\boldsymbol{u}}}\begin{pmatrix}1\\ \hat{\boldsymbol{u}}\\ (\hat{\boldsymbol{u}}\otimes\hat{\boldsymbol{u}} - \mathbb{I}/2)\end{pmatrix}c(\mathbf{R}, \hat{\boldsymbol{u}}, t). \quad (4)$$

Homogeneous ordered states.—These are states in which c and its moments defined above do not vary with position (denoted by c^0, ρ^0, $\mathbf{P}^0$, $\mathbb{S}^0$, respectively). Under these conditions, the mean field velocities are $\boldsymbol{\Omega}^0 = \beta_0\hat{\boldsymbol{u}}\wedge\mathbf{P}^0 + \beta_1\hat{\boldsymbol{u}}\wedge\mathbb{S}^0\cdot\hat{\boldsymbol{u}}$ and $\mathbf{V}^0 = \beta_T\mathbf{P}^0$, where the coefficients β_0, β_1, and β_T are averaged quantities that depend on the microscopic details of each swimmer [24]. To leading

order $\beta_0 \approx f\frac{1}{(16)^2\lambda^3}\frac{s}{\eta}\frac{d}{l}(\kappa^2-1)$ and $\beta_1 \approx f\frac{s}{\eta}\frac{1}{4\lambda}\frac{a_0 d}{l^2}\times \frac{1}{(1+\kappa)^2}[\frac{1}{3}(\kappa-1)+\frac{5}{12}(\kappa^2-\frac{1}{\kappa})+\frac{1}{6}(\kappa^3-\frac{1}{\kappa^2})]$. They are both positive for $\kappa>1$ (pushers) and negative for $\kappa<1$ (pullers). Inserting the expressions of $\mathbf{\Omega}^0$ and $\mathbf{V}^0$ in Eq. (1), taking the time derivative of Eq. (4), we obtain dynamic equations for the moments of c^0. Density is conserved; hence, $\partial_t\rho^0=0$. The others are

$$\partial_t \mathbb{S}^0_{ab} = -4D_R\mathbb{S}^0_{ab} + \beta_0\left(P^0_a P^0_b - \frac{\delta_{ab}}{2}(\mathbf{P}^0)^2\right) + \frac{\beta_1}{2}\rho^0\mathbb{S}^0_{ab};$$
$$\partial_t \mathbf{P}^0 = -D_R\mathbf{P}^0 + \frac{\beta_0}{2}\rho^0\mathbf{P}^0 + \left(\frac{\beta_1}{2}-\beta_0\right)\mathbb{S}^0\cdot\mathbf{P}^0. \quad (5)$$

Analysis of Eq. (5) shows that, when β_0 or β_1 are positive, the system can undergo a bifurcation, that signals the appearance of order. Hence pushers can give rise to order whereas pullers cannot. The conditions $-D_R+\rho^0\beta_0/2=0$ and $-4D_R+\rho^0\beta_1/2=0$ define two critical lines in the space of parameters ρ^0 and f, above which the instability occurs [25]. The ratio $\beta_1/(4\beta_0)$ determines if the I-N or I-P transition occurs at lower density (see Fig. 1 where we have chosen parameters for which I-N occurs first). For pullers β_0 and β_1 are negative and contribute to enhance the noise (diffusion) in the system. For quadrupolar swimmers, these leading order terms vanish, and in addition to higher order terms one must keep track of internal mode dynamics of the swimmers. The leading order terms in β_0 are negative, so enhance diffusion while in β_1 they are positive and promote order. Hence, for this particular microscopic prescription, we find no polar phase. We find $\beta_1 \approx f\frac{s}{\eta}\frac{a_0 d^2}{l^3}\frac{7}{384\lambda^3}[1+\frac{d}{l}(2\pi+1)]$ [20]. A plot of the critical line in this case is shown in the inset of Fig. 1. Clearly, this transition occurs at higher densities than dipolar swimmers since it is due to higher order terms.

We now discuss fluctuations in hydrodynamic variables about isotropic and ordered states for dipole swimmers. As in many active systems, we find that their effect is to destabilize order [12]. In the following we consider the deviations of the fields from their homogeneous values given by $\delta\rho=\rho-\rho^0$, similarly for $\mathbf{P}$ and $\mathbb{S}$. We introduce Fourier transforms in the usual way as $f(\mathbf{r})=\int_{\mathbf{k}}\frac{1}{(2\pi)^2}e^{-i\mathbf{k}\cdot\mathbf{r}}\tilde{f}(\mathbf{k})$, $\tilde{f}(\mathbf{k})=\int_{\mathbf{r}}e^{i\mathbf{k}\cdot\mathbf{r}}f(\mathbf{r})$.

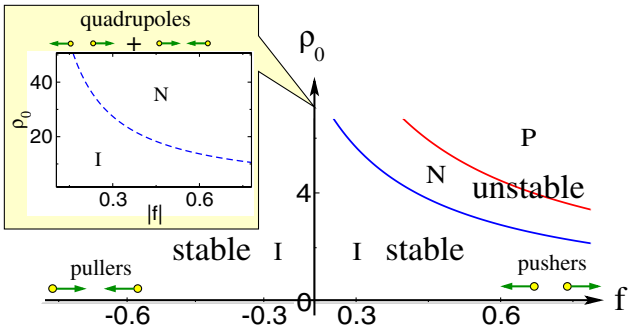

FIG. 1 (color online). Phase diagram showing homogeneous states and their hydrodynamic stability. Isotropic-nematic and nematic-polar transitions for dipolar swimmers. The transitions can occur only for pushers. Force f is measured in arbitrary units. Parameters are set to $\frac{s}{\eta}=D_R=1$, $d=3a_0$, $l=3d$, $\lambda=2$, and $\kappa=1.2$. In the inset is shown the I-N transition for quadrupolar swimmers (same parameters except $\kappa=1$).

Isotropic state.—In the homogeneous isotropic state $\rho=\rho^0$, $\mathbf{P}^0=\mathbb{S}^0=0$, and ρ is the only hydrodynamic variable. Variables $\delta\tilde{P}_\parallel := \delta\tilde{\mathbf{P}}\cdot\hat{\boldsymbol{k}}$, splay $\delta\tilde{\mathbb{S}}_{\parallel\parallel}$, and bend $(\delta\tilde{\mathbb{S}}_{\perp\parallel})_a := \delta\tilde{\mathbb{S}}_{bc}\hat{k}_c(\delta_{ab}-\hat{k}_a\hat{k}_b)$ show diffusive behavior. From the resulting set of equations, density fluctuations are linearly stable [14].

Polar state.—In the homogeneous polar state $\rho=\rho^0$, $\mathbf{P}=\mathbf{P}^0$, and the hydrodynamic variables are ρ and the director $\hat{\boldsymbol{P}}$. The orientation tensor is slaved to $\mathbf{P}$, and given by $\mathbb{S}=S_P P^2(\hat{\boldsymbol{P}}\hat{\boldsymbol{P}}-\frac{\mathbb{I}}{2})$ where S_P is determined by Eq. (5). The magnitude P is not a hydrodynamic variable and relaxes to a constant value on microscopic time scales. We set $P=1$ in the following and study linear perturbations around this state. Decomposing $\hat{\boldsymbol{k}}=\cos\phi\hat{\boldsymbol{P}}^0+\sin\phi\hat{\boldsymbol{k}}_\perp$, with $\hat{\boldsymbol{k}}_\perp\cdot\hat{\boldsymbol{P}}^0=0$, to leading order in k we find that fluctuations in density and director are coupled via splay $(\hat{\boldsymbol{k}}_\perp\cdot\delta\tilde{\hat{\boldsymbol{P}}})$, giving a growing mode Γ with real part $\mathrm{Re}(\Gamma)\sim k\frac{sS_P\bar{\Theta}^{(2)}}{8\eta}\times\cos2\phi(2+\cos2\phi)$ and imaginary part, which determines the propagation speed, $\mathrm{Im}(\Gamma)\sim\pm k\frac{\bar{v}}{\sqrt{2}}|\sin\phi|$, as shown in Fig. 2(a). When $\mathrm{Re}(\Gamma)$ is positive, fluctuations grow exponentially, signalling an instability of the ordered state due to hydrodynamic interactions, analogous to those found in 3D [12]. However unlike those, the growth rate here scales as k in the $k\to0$ limit.

Nematic state.—A similar analysis can be performed around the homogeneous nematic state (finite ρ^0, $\mathbb{S}^0$). In this case hydrodynamic variables are density, ρ, and nematic orientation tensor, $\mathbb{S}=S(\hat{\boldsymbol{n}}\otimes\hat{\boldsymbol{n}}-\frac{\mathbb{I}}{2})$. Again the magnitude S relaxes fast and in the following will be set to $S=1$. To leading order in k fluctuations of density $\delta\tilde{\rho}$ and nematic director $\delta\tilde{\hat{\boldsymbol{n}}}$ are decoupled and splay fluctuations have a real mode $\Gamma\sim k\frac{s\bar{\Theta}^{(2)}}{4\eta}\cos2\phi(2+\rho^0\cos2\phi)$. For angles above $\pi/4$ destabilize the order, as shown in Fig. 2(b). For quadrupolar swimmers the relevant terms describing hydrodynamic interactions scale as k^2 instead of k. Their analysis is not reported here [20].

It is instructive to compare our results with previous studies of swimmers in a 3D fluid [12,14,15]. Our analysis starts from a microscopic model and "integrates out" the fluid degrees of freedom to see the effect on the other hydrodynamic modes. Alternatively, one may perform a phenomenological analysis of an ordered state as in [12], replacing the Stokes equation by Eq. (2); this yields qualitatively the same hydrodynamic instabilities that we have presented above. In essence, the thin film model changes the Fourier spectrum of the hydrodynamic kernel from k^2 to $k+sk^2$, which in the limit $sk\ll1$ reduces to $\approx k$. This is the origin of the different scaling of the instability here. In another contrast, hydrodynamic interactions of

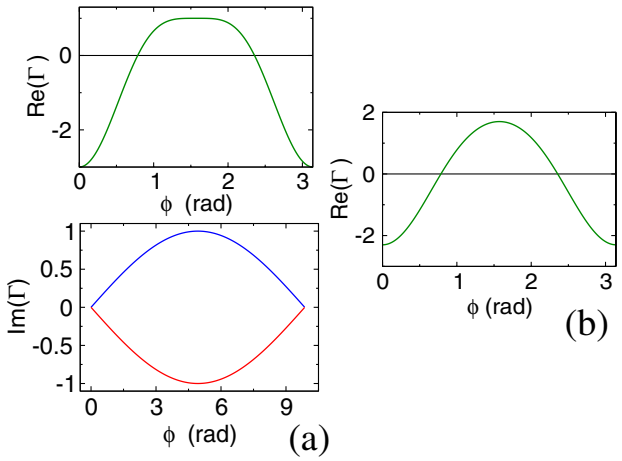

FIG. 2 (color online). Angular dependence of the growing mode Γ in the splay fluctuations around homogeneous order for pushers. A positive sign indicates instability. (a) Polar state. $\mathrm{Re}(\Gamma)$ is measured in units of $k(sS_P|\bar{\Theta}^{(2)}|)/(8\eta)$ and $\mathrm{Im}(\Gamma)$ in units of $k\bar{v}/\sqrt{2}$. (b) Nematic state. $\mathrm{Re}(\Gamma)$ is measured in units of $k(s|\bar{\Theta}^{(2)}|)/(4\eta)$ and we have used $\rho^0 = 0.3$.

simplified far-field models of swimmers cannot lead to homogeneous order (swarming) in 3D [15]. This can be ascribed to a mathematical cancellation that occurs performing angular integrals of spherical harmonics generated by the hydrodynamic kernels. In the thin film limit considered here, this is circumvented by the confinement of the swimmer directors to two dimensions. In addition, the instability generated by activity in the film is "soft" in the sense that it scales with $k \to 0$ in comparison to the 3D case where it is independent of k (hard) to leading order [12]. We can conclude that in comparison to 3D, hydrodynamic interactions between swimmers in a thin film favor order but are still not strong enough to overcome the destabilizing effect of activity. Finally, we note that our approach is complementary to models where simple phenomenological rules of interaction between swimmers are used to study aspects of collective behavior [11,25,26]. In contrast, here we "derive" such rules from a particular microscopic model using a coarse-graining procedure which is valid under a precise set of conditions.

The question of how to characterize the system beyond the instability [14,26] remains open. This highlights a fundamental theoretical issue. The instabilities of homogeneous ordered states seen here and in other nonequilibrium active systems illustrate the limits of the Landau-Ginzburg framework [12,15], which has been so successful in the study of phase transitions in equilibrium systems. It might be that alternative approaches, such as describing the homogeneous state as dynamical rather than stationary, or the development of a formalism in which hydrodynamic and non-hydrodynamic variables are treated on the same level, have to be considered.

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[1] X.-L. Wu and A. Libchaber, Phys. Rev. Lett. **84**, 3017 (2000).
[2] C. Dombrowski *et al.*, Phys. Rev. Lett. **93**, 098103 (2004); I. Tuval *et al.*, Proc. Natl. Acad. Sci. U.S.A. **102**, 2277 (2005).
[3] I. H. Riedel, K. Kruse, and J. Howard, Science **309**, 300 (2005).
[4] K. C. Leptos *et al.*, Phys. Rev. Lett. **103**, 198103 (2009).
[5] A. Sokolov *et al.*, Phys. Rev. Lett. **98**, 158102 (2007).
[6] R. Dreyfus *et al.*, Nature (London) **437**, 862 (2005).
[7] P. Dhar *et al.*, Nano Lett. **6**, 66 (2006).
[8] A. Ghosh and P. Fischer, Nano Lett. **9**, 2243 (2009).
[9] T J. Pedley and J. O. Kessler, Annu. Rev. Fluid Mech. **24**, 313 (1992).
[10] P. G. de Gennes and J. Prost, *The Physics of Liquid Crystals* (Oxford University Press, New York, 1995).
[11] T. Vicsek *et al.*, Phys. Rev. Lett. **75**, 1226 (1995).
[12] R. A. Simha and S. Ramaswamy, Phys. Rev. Lett. **89**, 058101 (2002).
[13] I. S. Aranson *et al.*, Phys. Rev. E **75**, 040901 (2007).
[14] D. Saintillan and M. J. Shelley, Phys. Rev. Lett. **100**, 178103 (2008); Phys. Fluids **20**, 123304 (2008).
[15] A. Baskaran and M. C. Marchetti, Proc. Natl. Acad. Sci. U.S.A. **106**, 15 567 (2009).
[16] M. Doi and S. Edwards, *The Theory of Polymer Dynamics* (Clarendon, Oxford, 1993).
[17] L. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1987), 2nd ed.
[18] P. G. Saffman and M. Delbruck, Proc. Natl. Acad. Sci. U.S.A. **72**, 3111 (1975); P. G. Saffman, J. Fluid Mech. **73**, 593 (1976); D. K. Lubensky and R. E. Goldstein, Phys. Fluids **8**, 843 (1996).
[19] A. J. Levine, T. B. Liverpool, and F. C. MacKintosh, Phys. Rev. Lett. **93**, 038102 (2004); Phys. Rev. E **69**, 021503 (2004).
[20] M. Leoni and T B. Liverpool (unpublished).
[21] A. Najafi and R. Golestanian, Phys. Rev. E **69**, 062901 (2004); R. Golestanian and A. Ajdari, Phys. Rev. E **77**, 036308 (2008).
[22] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.105.238102 for a more detailed description.
[23] G P. Alexander, C M. Pooley, and J. M. Yeomans, J. Phys. Condens. Matter **21**, 204108 (2009).
[24] To obtain these we perform integrals which require a regularization at small distances. This cutoff is naturally provided by the swimmer size λl, for $\lambda \geq 2$.
[25] F. Peruani, A. Deutsch, and M. Bar, Eur. Phys. J. Special Topics **157**, 111 (2008).
[26] H. Chaté *et al.*, Phys. Rev. E **77**, 046113 (2008).