Spin-Asymmetric Josephson Effect

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We propose that with ultracold Fermi gases one can realize a spin-asymmetric Josephson effect in which the two spin components of a Cooper pair are driven asymmetrically—corresponding to driving a Josephson junction of two superconductors with different voltages $V_\uparrow$ and $V_\downarrow$ for spin up and down electrons, respectively. We predict that the spin up and down components oscillate at the same frequency but with different amplitudes. Furthermore our results reveal that the standard interpretation of the Josephson supercurrent in terms of coherent bosonic pair tunneling is insufficient. We provide an intuitive interpretation of the Josephson supercurrent as interference in Rabi oscillations of pairs and single particles, the latter causing the asymmetry.

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When a coherent many-body system is partitioned into two subsystems, the dynamics of macroscopic observables such as the relative number of particles and relative phase is called the Josephson effect [1,2]. The external Josephson effect has been realized in superconducting junctions [3,4], superfluid ^{3}He [5] and ^{4}He [6], and in Bose-Einstein condensates (BEC) of alkali atomic gases in double-well traps [7,8]. The internal Josephson effect has been demonstrated in ^{3}He [9] and is expected to occur in spin BECs [2,10–12]. Also in the context of ultracold Fermi gases [13] the possibility of the Josephson effect has recently received theoretical interest [14–17]. In this letter we show that partitioning a system of Cooper-paired fermions so that the two components of the pair experience different potentials (this is what we mean by "spin-asymmetric" here) leads to a novel effect, namely, different-amplitude but phase-synchronized number-oscillations of the components. Although the microscopic description of the Josephson effect is based on single particle tunneling, the standard interpretation of the Josephson supercurrent is given in terms of coherent tunneling of bosons or Cooper pairs [18]. Importantly, our results show that such an interpretation is insufficient. We provide a clear, intuitive explanation of the predicted spin-asymmetric Josephson effect and a new understanding of the Josephson supercurrent as a process where not only pairs but also the spin-components separately contribute via interference.

We propose that the spin-asymmetric Josephson effect can be realized in a four-component Fermi gas in which two superfluids are coupled by radio-frequency (rf) fields [Fig. 1(a)], as in rf spectroscopy [19]. The setup is motivated by the recent realization of three-component Fermi gases [20]. Another possible, perhaps experimentally simpler, realization is a superfluid two-component ultracold Fermi gas [Fig. 1(b)] in a (spin) component-dependent double-well potential. The theoretical descriptions of the systems of Figs. 1(a) and 1(b) are identical (in this Letter we use the notation of the former). We also suggest that the spin-asymmetric effect can be realized in a *S*-*I*-*S* junction of two materials with different Zeeman splittings [21].

The setup of Fig. 1(a) corresponds to a many-body Hamiltonian $H = H_0 + H_{\rm rf}$, where $H_0 = \int d\mathbf{r}\sum_i \psi_i^\dagger(\mathbf{r}) \times (-\frac{\nabla^2}{2m} - \mu_i)\psi_i(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r}\sum_{i\neq j} U_{ij}\psi_i^\dagger(\mathbf{r})\psi_j^\dagger(\mathbf{r})\psi_j(\mathbf{r})\psi_i(\mathbf{r})$. Here, $\psi_i(\mathbf{r})$ and $\psi_i^\dagger(\mathbf{r})$ are the fermionic field operators for the internal state i and μ_i is the chemical potential (we assume $\mu_i \equiv \mu$), and U_{ij} give the interaction strengths in

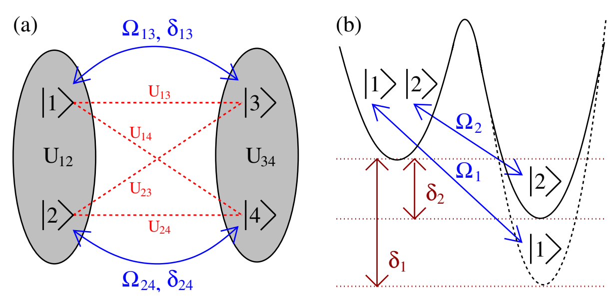

FIG. 1 (color online). Spin-asymmetric Josephson effect setup. (a) The four-component Fermi gas. Particles in internal (e.g. hyperfine) states $|1\rangle - |4\rangle$ coexist spatially. The components $|1\rangle$ and $|2\rangle$ as well as $|3\rangle$ and $|4\rangle$ form Cooper pairs due to the interactions U_{12} and U_{34}. The cross-interactions U_{13}, U_{14}, U_{23}, U_{24} are assumed weak, thus not leading to pairing. The tunneling is driven by applying rf fields to couple the states. The Rabi couplings Ω_{ij} correspond to the weak tunneling link between the two superconductors in a Josephson junction. The detunings $\delta_{ij} = \nu_{ij} - \omega_{ij}$ play the role of the voltage, eV. Remarkably, one can choose $\delta_{13} \neq \delta_{24}$ and create the analogue of a spin-dependent voltage in a Josephson junction. The states $|1\rangle$ and $|3\rangle$ ($|2\rangle$ and $|4\rangle$) must be states of the same atom to allow the rf coupling, but $|1\rangle$ and $|2\rangle$ can be e.g. ^{6}Li and ^{40}K. (b) Cooper pairs of a two-component Fermi gas in a spin-dependent double-well potential. This setup is conceptually equivalent to the system of (a) albeit the absence of cross interactions.

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the s-wave contact potential approximation. We assume that U_{12} and U_{34} lead to pairing and set $\hbar = 1$.

In the rotating wave approximation [22] the tunneling coupling between states $|i\rangle$ and $|j\rangle$ is given by the Rabi frequency Ω_{ij} with the detuning $\delta_{ij} = \nu_{ij} - \omega_{ij}$. Here, ν_{ij} is the frequency of the field and ω_{ij} is the resonance frequency of the hyperfine transition. The effect of the electromagnetic field on the system is then described by $H_{\rm rf} = \frac{\delta_{13}}{2}\int d\mathbf{r}[\psi_1^\dagger(\mathbf{r})\psi_1(\mathbf{r}) - \psi_3^\dagger(\mathbf{r})\psi_3(\mathbf{r})] + \frac{\delta_{24}}{2}\times\int d\mathbf{r}[\psi_2^\dagger(\mathbf{r})\psi_2(\mathbf{r}) - \psi_4^\dagger(\mathbf{r})\psi_4(\mathbf{r})] + \Omega_{13}\int d\mathbf{r}\psi_1^\dagger(\mathbf{r})\psi_3(\mathbf{r}) + \mathrm{H.c.} + \Omega_{24}\int d\mathbf{r}\psi_2^\dagger(\mathbf{r})\psi_4(\mathbf{r}) + \mathrm{H.c.}$ In analogy to the usual Josephson junctions, the states $|1\rangle$ and $|2\rangle$ correspond to spin up and down electrons in the left-side superconductor, $|3\rangle$ and $|4\rangle$ on the right. The detunings δ_{13} and δ_{24} play the role of the voltage.

The essential new feature in atomic gases is the possibility to set $\delta_{13} \neq \delta_{24}$, which in the case of the Josephson junction corresponds to spin-dependent voltages. Note that this is different from superconductor-ferromagnet-superconductor (S-F-S) structures [23] in which the spin-active barrier coupling plays the crucial role. Though in our case also the couplings could be different, only the spin-asymmetric potential is relevant for our predictions. Moreover, one might consider ferromagnetic superconductors [24] in a junction as a related system but those materials have most likely an exotic ground state which does not fit our description.

We now determine the transition rates (i.e., particle currents) between states $|1\rangle$ and $|3\rangle$, $I_{13}(t) \equiv \langle \dot{N}_1\rangle$, and between states $|2\rangle$ and $|4\rangle$, $I_{24}(t) \equiv \langle \dot{N}_2\rangle$. We calculate a self-consistent linear response with respect to $H_{\rm rf}$ (valid when the number of transferred particles is small compared to the total particle number) for the system of Fig. 1 with the aid of the Kubo formula and the Kadanoff-Baym method [25]. We obtain the currents

$$\begin{aligned} I_{13}(t) &= I_{13}^S + I_{13}^C \sin[(\delta_{13} + \delta_{24})t + \varphi], \\ I_{24}(t) &= I_{24}^S + I_{24}^C \sin[(\delta_{13} + \delta_{24})t + \varphi]. \end{aligned} \tag{1}$$

Here, I^S is the standard single particle (quasiparticle) current that occurs only for detunings δ_{ij} above the excitation gap 2Δ. The initial phase of the Josephson current is φ. The critical Josephson currents I^C become, in a spatially homogeneous case and in the BCS description,

$$I_{13}^C = 2|\Omega_{13}\Omega_{24}\Pi_{\mathcal{F}}(\mathbf{p} = \mathbf{0}, \delta_{24} + i\eta^+)|, \tag{2}$$

$$I_{24}^C = 2|\Omega_{13}\Omega_{24}\Pi_{\mathcal{F}}(\mathbf{p} = \mathbf{0}, \delta_{13} + i\eta^+)|, \tag{3}$$

where $\Pi_{\mathcal{F}}(\mathbf{p}, \omega) = \frac{1}{\beta V}\sum_{\mathbf{q},\chi}\mathcal{F}_{12}(\mathbf{q}, \chi)\mathcal{F}_{34}^*(\mathbf{q} - \mathbf{p}, \chi - \omega)$. Here, V is the volume and $\beta = 1/(k_B T)$ (T is temperature and k_B the Boltzmann constant). $\mathcal{F}_{12} = \mathcal{F}_{21}^*$ ($\mathcal{F}_{34} = \mathcal{F}_{43}^*$) is the anomalous mean field Matsubara Green's function for the superfluid of components $|1\rangle$ and $|2\rangle$ ($|3\rangle$ and $|4\rangle$). For details see supplementary material [26].

The striking result is that the critical current I_{13}^C in Eq. (2) depends only on δ_{24}, and similarly I_{24}^C in Eq. (3) only on δ_{13} (this is not limited to the BCS regime; it remains true whenever pairing correlations exist). By choosing different detunings δ_{13} and δ_{24}, one can observe a spin-asymmetric Josephson effect in which the currents in the two tunneling channels are different in amplitude, but oscillate at the same frequency. Moreover, the results predict a tunable dc Josephson effect: by choosing the detunings so that $\delta_{13} + \delta_{24} = 0$, the phase factor in Eq. (1) is constant but the critical current can still be tuned. The conclusions hold for experimentally realistic parameters and also if cross interactions [see Fig. 1(a)] and finite temperature are included into our analysis, as shown in the supplementary material [26]. For typical parameters and taking, e.g., $\delta_{13}/E_F = 0.4$ and $\delta_{24}/E_F = 0.5$, with E_F the Fermi energy, one obtains a considerable asymmetry of $I_{13}^C/I_{24}^C = 1.14$; see [26] for details. By performing the self-consistent calculation we have removed the ambiguity of whether our previous suggestion of the critical current asymmetry [14] was due to a simple linear response approach following the Ambegaokar-Baratoff treatment [27].

Our results (1)–(3) are in obvious contradiction with the standard interpretation of the Josephson supercurrent in terms of coherent *pair* tunneling excluding any difference in the Josephson currents of different spin components. In what follows we provide an explanation for the spin-asymmetric Josephson effect which not only resolves this paradox but also opens up a new point of view to the conventional Josephson effect. Let us begin by writing down the initial state of the two superfluids as a product state of two BCS states: $|\Psi\rangle = |\mathrm{BCS}\rangle_{12} \otimes |\mathrm{BCS}\rangle_{34} = \prod_k(u_k|0\rangle_k + v_k|12\rangle_k)\prod_{k'}(u_{k'}|0\rangle_{k'} + v_{k'}|34\rangle_{k'})$, where $|ij\rangle_k = c_{k,i}^\dagger c_{-k,j}^\dagger|0\rangle_k$. Next, we single out one Cooper pair from each superfluid. Since the rf coupling between 1–3 and 2–4 conserves momentum we focus on states with $k = k'$ (the momentum conservation can be relaxed and our conclusions still hold):

$$\begin{aligned} &(u_k|0\rangle_k + v_k|12\rangle_k)(u_k|0\rangle_k + v_k|34\rangle_k) \\ &\quad = u_k^2|0\rangle_k|0\rangle_k + v_k^2|12\rangle_k|34\rangle_k + u_k v_k|12\rangle_k|0\rangle_k \\ &\qquad + u_k v_k|0\rangle_k|34\rangle_k. \end{aligned} \tag{4}$$

The empty state $|0\rangle|0\rangle$ cannot contribute to the current and neither can $|12\rangle|34\rangle$ since it is Pauli blocked. Therefore, the Josephson physics arises from the $u_k v_k|12\rangle|0\rangle + u_k v_k|0\rangle|34\rangle$ superposition.

We then ask whether the essential features of our results can be explained by considering the dynamics of a single Cooper pair, initially in the above superposition state characteristic for the BCS state with a macroscopic phase. In addition to the paired states $|12\rangle|0\rangle \equiv |12\rangle$ and $|0\rangle|34\rangle \equiv |34\rangle$, the states $|1\rangle|4\rangle \equiv |14\rangle$ and $|2\rangle|3\rangle \equiv |23\rangle$ are required to have a closed subsystem with respect to the

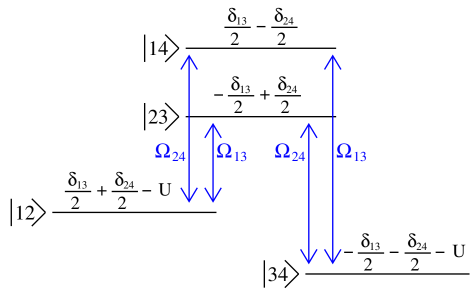

FIG. 2 (color online). Energy level diagram of the four-state model. The basis states $|12\rangle$, $|34\rangle$, $|14\rangle$, $|23\rangle$ form a closed subspace under the Rabi oscillation processes caused by the rf coupling. The paired states $|12\rangle$ and $|34\rangle$ have an energy lower by U than the states of broken pairs $|14\rangle$ and $|23\rangle$.

tunneling coupling; see Fig. 2. These broken pair states are analogous to single particle excitations within the BCS formalism.

We now solve the time evolution of this system perturbatively in the couplings Ω_{ij}. We consider an initial state in the general superposition form $\alpha_{\mathrm{I}}|12\rangle + \beta_{\mathrm{II}}|34\rangle$ as suggested by Eq. (4). The total current becomes

$$\begin{aligned}\langle\phi(t)|\dot{N}_1|\phi(t)\rangle = {}& |\alpha_{\mathrm{I}}|^2\langle\phi_{\mathrm{I}}(t)|\dot{N}_1|\phi_{\mathrm{I}}(t)\rangle \\ &+ |\beta_{\mathrm{II}}|^2\langle\phi_{\mathrm{II}}(t)|\dot{N}_1|\phi_{\mathrm{II}}(t)\rangle \\ &+ \alpha_{\mathrm{I}}\beta_{\mathrm{II}}^*\langle\phi_{\mathrm{II}}(t)|\dot{N}_1|\phi_{\mathrm{I}}(t)\rangle \\ &+ \alpha_{\mathrm{I}}^*\beta_{\mathrm{II}}\langle\phi_{\mathrm{I}}(t)|\dot{N}_1|\phi_{\mathrm{II}}(t)\rangle, \qquad (5)\end{aligned}$$

where $|\phi_{\mathrm{I}}(t)\rangle = \exp(-iHt)|12\rangle$ and $|\phi_{\mathrm{II}}(t)\rangle = \exp(-iHt)|34\rangle$ are calculated to second order in Ω_{ij}. For details and for a complementary discussion in terms of exact numerical eigenstates (dressed states) see [26]. Here the first two terms contain only single particle Rabi processes, which correspond to the standard single particle (quasiparticle) currents in a Josephson junction, I_{ij}^S in Eq. (1). Only the last two terms in Eq. (5), with $\alpha_{\mathrm{I}}\beta_{\mathrm{II}}^* \equiv |\alpha_{\mathrm{I}}\beta_{\mathrm{II}}|e^{i\varphi}$, contribute to the Josephson current. Thus, the Josephson effect originates from the interference part of the Rabi oscillations in the $|12\rangle, |34\rangle, |14\rangle, |23\rangle$ state space.

Isolating the terms oscillating at the Josephson frequency in $\langle\phi(t)|\dot{N}_1|\phi(t)\rangle$, we get the Josephson current

$$\begin{aligned} I_{13}^J &= I_{13}^C(\delta_{24})\sin[(\delta_{13}+\delta_{24})t+\varphi], \\ I_{13}^C(\delta_{24}) &= 2\Omega_{13}\Omega_{24}|\alpha_{\mathrm{I}}\beta_{\mathrm{II}}|\left[\frac{1}{U+\delta_{24}}+\frac{1}{U-\delta_{24}}\right]. \end{aligned} \qquad (6)$$

Hereby, we obtain the same qualitative result as our linear response calculation gave in Eq. (1): the amplitude of I_{13}^J depends only on the detuning δ_{24}. Now we can identify the source of the dependence of I_{13}^C on δ_{24}. The current I_{13}^J derives from the population of species $|1\rangle$, i.e., both the population of state $|12\rangle$ and $|14\rangle$. The result of Eq. (6) is the sum of these two contributions.

The contribution from state $|12\rangle$ is the result of tunneling between the paired states $|12\rangle$ and $|34\rangle$, via the intermediate state $|14\rangle$ (or $|23\rangle$) as shown in Fig. 3(a). The term is symmetric in δ_{13} and δ_{24} and proportional to $M_{\mathrm{pair}} = \frac{1}{U+\delta_{13}} + \frac{1}{U+\delta_{24}}$. The contribution is of second order because it originates from a zeroth order and a second order process: the population due to pair tunneling from $|34\rangle$ to $|12\rangle$ (second order) is indistinguishable from the initial (zeroth order) population in state $|12\rangle$. Pair tunneling contributions that do not originate from interference also exist but they are of fourth order in Ω.

The contribution to I_{13}^C from state $|14\rangle$ arises from interference of broken pairs (single particles) as depicted in Fig. 3(b). This contribution is responsible for the asymmetry as it is proportional to $M_{\mathrm{single}} = \frac{1}{U-\delta_{24}} - \frac{1}{U+\delta_{13}}$. Physically, this term arises because we cannot distinguish between broken pairs from the states $|12\rangle$ and $|34\rangle$. The two paths leading to the state $|14\rangle$ are not symmetric with respect to δ_{13} and δ_{24}; see Figs. 2 and 3. This gives important new insight also to the Josephson effect in general: also *single particle* interferences contribute to the Josephson current $I_{ij}^C\sin(\delta_{13}+\delta_{24})$ of Eq. (1).

In the standard symmetric case $\delta_{13} = \delta_{24} \equiv \delta$, we have $M_{\mathrm{pair}} = 2/(U+\delta)$ and $M_{\mathrm{single}} = 2\delta/(U^2-\delta^2)$. At the dc Josephson limit, $\delta \to 0$, the pair transfer M_{pair} dominates, which is intuitively appealing. Closer to the Riedel peak the excited state (single particle) interference M_{single} becomes equally important. To illustrate further, in a typical $\mathrm{Al/AlO}_x\mathrm{/Al}$ junction at the voltage of 0.015 mV the single particle interference accounts for 1.7% of the AC Josephson current. Note that our "single particle interference term" is not the cosine-term (also called "quasiparticle interference term" [28]) of the Josephson effect. The cosine-term involves real single particle transitions and exists only for voltages above 2Δ at zero temperature. In contrast, our single particle interference term is inherent

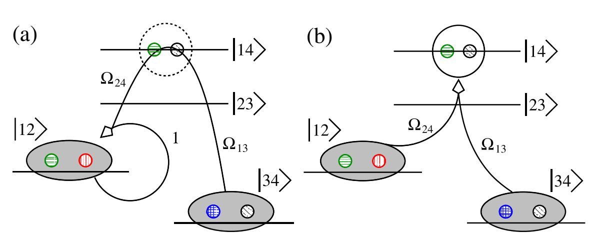

FIG. 3 (color online). Interference processes leading to the Josephson supercurrent. The initial state is $|\phi\rangle = (|12\rangle + |34\rangle)/\sqrt{2}$. (a) The second order transition from $|34\rangle$ through $|14\rangle$ to $|12\rangle$ creates an interference with the initial (zeroth order) population in state $|12\rangle$. The resulting contribution is of the order $\Omega_{13}\Omega_{24}$. Also the same process via the state $|23\rangle$ contributes. (b) The first order transitions to the excited state $|14\rangle$ from $|12\rangle$ and $|34\rangle$ create an interference proportional to $\Omega_{13}\Omega_{24}$.

in the supercurrent and corresponds to virtual transitions. Similarly, our findings are different from various combined effects of single particle currents and supercurrents in small Josephson junctions [29], where again the single particle transitions are real, not virtual. Note also that we do not consider any interactions between the Cooper pairs (analogue of charging effects) nor the effect of the environment.

We emphasize the interference nature of the Josephson effect: it requires lack of which-way information on the tunneling path of the particle. Above, the superposition $u_k v_k(|12\rangle|0\rangle + |0\rangle|34\rangle)$ was due to the uncertainty of the particle number in the BCS state. It is interesting to contrast this with the number-projected BCS state [30] or the Fock state [31], $|\Psi\rangle_{N\text{cons}} = (u_k v_{k'}|0\rangle_k|12\rangle_{k'} + u_{k'} v_k|12\rangle_k|0\rangle_{k'})(u_k v_{k'}|0\rangle_k|34\rangle_{k'} + u_{k'} v_k|0\rangle_{k'}|34\rangle_k)$ (for two k states.) Here the part relevant for Josephson physics would be $u_k v_{k'} u_{k'} v_k|12\rangle_{k'}|34\rangle_k + u_k v_{k'} u_{k'} v_k|12\rangle_k|34\rangle_{k'}$. Now, the entanglement of $|34\rangle_{k'}$ with $|12\rangle_k$ allows to determine, whether a particle in state $|1\rangle$ belongs to an initial 1–2 pair or to a tunnelled 3–4 pair. The indistinguishability is lost and there is no Josephson effect. Note that the system of Fig. 1(a) should be cooled with the rf couplings on to realize the necessary uncertainty in particle number. One can also ask whether, in the case of the separated 1–2 and 3–4 condensates (Fock states), the measurement process itself is sufficient to generate the relative particle number uncertainty, in analogy to the famous problem of interference between two BECs [32].

Importantly, based on the picture of interfering Rabi processes, one would anticipate corrections from the coupling strength Ω_{ij} to the Josephson frequency $\delta_{13} + \delta_{24}$ given by linear response. We have observed such a correction in the numerical simulations, see supplementary material [26]. We expect this correction to vanish in the thermodynamic limit, but it could be experimentally tested in ultracold gases and in superconducting grains.

In summary, we have predicted a spin-asymmetric Josephson effect which could be observed in ultracold Fermi gases and solid state systems and is likely to be relevant for spintronics. We provide a microscopic description of the Josephson effect as interference in Rabi oscillations of pairs and single particles, where the latter cause the predicted asymmetry. In particular, our finding that the interference part of single particle currents contributes to the Josephson supercurrent, in addition to the interference in pair tunneling, is fundamental. The single particle interference contribution is present already in the standard (symmetric) case, and becomes manifest and experimentally verifiable in the asymmetric one.

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