All-Optical Optomechanics: An Optical Spring Mirror

S. Singh, G. A. Phelps, D. S. Goldbaum, E. M. Wright, and P. Meystre

B2 Institute, Department of Physics and College of Optical Sciences The University of Arizona, Tucson, Arizona 85721, USA

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The dominant hurdle to the operation of optomechanical systems in the quantum regime is the coupling of the vibrating element to a thermal reservoir via mechanical supports. Here we propose a scheme that uses an *optical spring* to replace the mechanical support. We show that the resolved-sideband regime of cooling can be reached in a configuration using a high-reflectivity disk mirror held by an optical tweezer as one of the end mirrors of a Fabry-Perot cavity. We find a final phonon occupation number of the trapped mirror $\bar{n} = 0.56$ for reasonable parameters, the limit being set by our approximations, and not any fundamental physics. This demonstrates the promise of dielectric disks attached to optical springs for the observation of quantum effects in macroscopic objects.

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Operating macroscopic objects in the quantum regime is a challenge whose successful completion will have profound implications, ranging from an improved fundamental understanding of the quantum-classical interface and of the quantum measurement process to the development of quantum detectors of unsurpassed sensitivity [1]. Cooling a nanomechanical system to its ground state of center-of-mass motion is an important step toward that goal, and spectacular progress has recently occurred via an interdisciplinary approach combining tools from nanoscience, quantum optics, and condensed matter physics. A recent benchmark experiment has demonstrated the operation of a micromechanical resonator down to a phonon number $\bar{n} < 0.07$, as well as quantum control at the single-phonon level [2]. Such developments pave the way to the detection of exceedingly small forces and displacements, with applications ranging from the quantum control of molecular processes to gravitational wave detection [1].

One of the simplest systems being considered in this quest consists of a small vibrating element that forms one of the end mirrors of a Fabry-Perot cavity [3]. So far the biggest hurdle in achieving the ground-state cooling of such a mirror has been the coupling to a thermal reservoir by way of a mechanical support. This support acts as the dominant source of dissipation and decoherence. This note theoretically discusses an alternative configuration where the mechanical clamping of the system is replaced by an optical spring realized by an optical tweezer.

There is a large volume of work on the trapping of dielectric particles—from atoms to bacteria, in the focus of laser beams far detuned from any electronic resonance [4]. Over the last two decades optical tweezers have matured into a well established tool, providing elegant and relatively simple ways to control the motion and to measure the weak forces acting on particles suspended in a fluid or in vacuum. Exploiting this idea, several recent theoretical proposals have considered levitating macroscopic objects (spheres or even living organisms) in a cavity and cooling them to their ground state of center-of-mass motion [5–7]. A key observation in the present context is that macroscopic objects optically levitated in vacuum are remarkably isolated from most environmental noise sources [8]. This leads us to introduce the new paradigm of an optical spring mirror, in which an optomechanical cavity mirror is suspended by light rather than by mechanical clamps. This approach provides an elegant route toward the elimination of the mirror clamping losses already mentioned, and has the potential to relatively easily reach the quantum regime. One can also envision simple schemes to couple it to a two-state atom to fully characterize and control the quantum state of the mechanical motion [9]. We also remark that trapping and cooling a dielectric end mirror of a resonator, rather than an object inside a resonator, results in scattering losses significantly reduced compared to the case of spheres.

The optical spring mirror that we propose is a dual-disk structure comprised of a silica disc that is connected via a silica pillar or pedestal to a disk mirror, the geometry of which is illustrated in the inset in Fig. 1. The idea is that the structure is held in vacuum by the optical gradient force due to two linearly polarized elliptical Gaussian beams of equal wavelength λ that are applied solely to the silica disk to avoid laser heating of the disk mirror. The disk mirror is a Bragg mirror composed of alternating layers of two

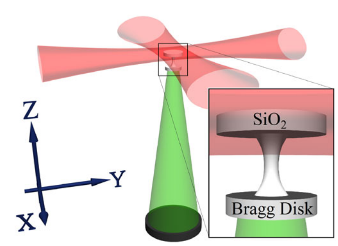

FIG. 1 (color online). Arrangement for an optomechanical cavity without clamping losses. The disk mirror is trapped in the optical tweezer by the crossed elliptical Gaussian beams shown in red, and provides the moving mirror for the Fabry-Perot aligned along the z axis shown in green.

dielectrics that acts as an end mirror for the cavity. The silica disk axis is along the z axis of the Fabry-Perot interferometer, and perpendicular to the trap beams, see Fig. 1. The tweezer beam traveling in the x direction is polarized along the y direction, and the beam traveling in the y direction is polarized in the x direction; the orthogonal polarizations being chosen to avoid the onset of interferences in the overlap region of the beams. Both beams have an elliptical transverse profile with the smallest beam waist along z, so as to provide a tight confinement along that axis and to avoid overlap with the disk mirror. The total intensity of the trapping beams has the form

$$I(\mathbf{r}) = I_{0x}\frac{\exp[\frac{-2y^2}{w_{0y}^2(1+x^2/y_r^2)} + \frac{-2z^2}{w_{0z}^2(1+x^2/z_r^2)}]}{\sqrt{(1+x^2/y_r^2)(1+x^2/z_r^2)}} + I_{0y}\frac{\exp[\frac{-2x^2}{w_{0x}^2(1+y^2/x_r^2)} + \frac{-2z^2}{w_{0z}^2(1+y^2/z_r^2)}]}{\sqrt{(1+y^2/x_r^2)(1+y^2/z_r^2)}}, \tag{1}$$

where I_{0x} and I_{0y} are the on-axis intensities of the laser beams traveling in the x and y directions, $w_{0\mu}$ is the focussed beam waists with $\mu = x, y, z$, and $\mu_r = \pi w_{0\mu}^2/\lambda$ the Rayleigh ranges along the respective directions.

For concreteness we consider the case of a Nd:YAG trapping laser ($\lambda = 1.064\ \mu$m) that is far-detuned from any material resonance in the silica disk. In this far-detuned limit we may assume that the field induces a dipole moment $\mathbf{p} = \alpha\mathbf{E}$ in the material, where α is the polarizability tensor and $\mathbf{E}$ the electric field envelope. Further assuming that the field envelope varies little over the dimensions of the disk, the components of the polarizability tensor can be approximated by those induced by a uniform electric field, and the trapping potential can be approximated by that of a static field, with a factor of 2 reduction due to time averaging. The static polarizability of a dielectric cylinder in a uniform static field has previously been calculated numerically [10]. Instead, we use the analytical expression for the polarizability of a spheroid [11], which is close to that of a cylinder of the same permittivity ϵ and aspect ratio. For our parameters, that approximation results in an error of about 3% in the value of the components of the polarizability tensor. The transverse and longitudinal polarizabilities of a spheroid of diameter d, height h, length l, eccentricity $e = \sqrt{(d/h)^2 - 1}$ and volume V are then given by

$$\alpha_{\perp,z} = \epsilon_0 V\left[\frac{\epsilon_r - 1}{1 + N_{\perp,z}(\epsilon_r - 1)}\right], \tag{2}$$

where $\epsilon_r = \epsilon/\epsilon_0$ is its relative permittivity, $N_z = (1+e^2)(e - \arctan e)/e^3$, and $N_\perp = 0.5(1 - N_z)$.

We next estimate the optical trap frequencies for our optical spring mirror. For a silica disk of dimensions $d = 60\ \mu$m, $h = 2.5\ \mu$m, and a net mass $m = 3.87 \times 10^{-11}$ kg for the dual-disk structure (the mirror diameter and height being 50 μm and 3.04 μm respectively), this gives $\alpha_\perp = 1.66 \times 10^{-25}$ C m^2 V^{-1} and $\alpha_z = 4.88 \times 10^{-26}$ C m^2 V^{-1}. The optical potential due to the gradient force is then $V(\mathbf{r}) = -\alpha_\perp I(\mathbf{r})/(2\epsilon_0 c)$, $\mathbf{r}$ being small displacements about the origin. For small deviations along the z axis this yields a harmonic potential of frequency

$$\omega_z = \left[\frac{2\alpha_\perp}{mc\epsilon_0 w_{0z}^2}(I_{0x} + I_{0y})\right]^{1/2}. \tag{3}$$

For Nd:YAG laser beams of intensity 0.1 W/μm^2 and beam waists $w_{0x} = w_{0y} = 100\ \mu$m and $w_{0z} = 4\ \mu$m, we then find $\omega_z = 2.01 \times 10^5$ rad/s, and in a similar manner we find $\omega_{x,y} = 8.29 \times 10^3$ rad/s for the transverse trapping frequencies.

Next we assess the angular motion of the disk with respect to the x and y axes, see Fig. 1. In particular, we calculate the wobble frequency $\omega_{\rm wob}$ of the disk when it is misaligned by an angle θ with respect to the x-axis. Such motion of asymmetric isotropic objects in linearly polarized optical traps has previously been studied in detail, for example, in Ref. [12]. We estimate $\omega_{\rm wob}$ by considering a light beam propagating in the y direction and polarized along x. For a disk misaligned by an angle θ with respect to the x axis the induced dipole moment is $\mathbf{p} = [\alpha_\perp E_0(\cos\theta)\hat{\mathbf{x}} + \alpha_z E_0(\sin\theta)\hat{\mathbf{z}}]$. An analysis of small angle harmonic rotational motion along y shows that it has the frequency

$$\omega_{\rm wob} = \left[\frac{12 I_{0y}(\alpha_\perp - \alpha_z)}{\epsilon_0 c \mathcal{I}_x}\right]^{1/2}, \tag{4}$$

where $\mathcal{I}_x = m(3d^2/4 + h^2)$ is the moment of inertia of the disk along x. For our parameters we find $\omega_{\rm wob} = 2.3 \times 10^4$ rad/s. We note that $\omega_z \gg \omega_{\rm wob}$ thereby ruling out any parametric coupling between the wobble mode and the longitudinal mirror motion. This means that the wobble mode should not be detrimental to cooling the longitudinal mirror motion.

Although the silica disk is nominally transparent to the trapping lasers, it will absorb some light, and with no heat sinking the only way to dissipate this energy is through blackbody radiation [5]. Taking the absorption coefficient $\alpha = 10^{-5}$/m due to UV absorption, we find that the temperature of the mirror increases by a modest 0.4 K, thereby causing no material damage [13].

Having established the mechanical properties of the trapped Bragg disk, we now turn to a discussion of the Fabry-Perot cavity in which the Bragg disk serves as a vibrating end mirror [1]. The fixed mirror of the Fabry-Perot interferometer, assumed to have a reflectivity $R_f = 0.999\,998$, is placed at a distance $L = 3.999$ cm from the movable mirror of lower reflectivity $R_m = 0.9998$. We note that small mirrors of comparable or smaller sizes with reflectivity exceeding 0.9998 are already being used in experiments [14,15]. For $\lambda = 852$ nm, the cavity damping rate is $\kappa = \pi c/\mathcal{F}L \approx 7.5 \times 10^5$ rad/s ($\mathcal{F}$ is the finesse), a value comparable to the optical trap frequency, so that the system is only marginally approaching the resolved sideband limit of radiation pressure cooling. Ignoring all sources of noise, these parameters result in a minimum thermal phonon occupation number of [16]

$$\langle n\rangle_{\min} = -\frac{4(\Delta + \omega_z)^2 + \kappa^2}{16\omega_z\Delta}. \quad (5)$$

For our parameters and a detuning $\Delta = (\omega_{\text{laser}} - \omega_c) = -4.25 \times 10^5$ rad/s from the cavity resonance (ω_c), we get $\langle n\rangle_{\min} \simeq 0.56$, well into the quantum regime. We remark that we may reduce this value by using tighter trapping, though this would violate our approximation that the field varies little over the dimensions of the disk. The quoted value is thus an upper-bound consistent with our approximations, but by no means a fundamental limit.

In calculating $\langle n\rangle_{\min}$ we have ignored all effects of noise. The major sources of noise are the fluctuations of the trapping and the Fabry-Perot cavity lasers, and background gas collisions. We next evaluate their impact on $\langle n\rangle_{\min}$.

Trapping laser fluctuations.—There are three noise sources due to the optical tweezer laser beams: intensity fluctuations, beam-pointing fluctuations, and photon scattering losses. The first two noise sources have been studied extensively in the context of trapping alkali atoms in optical traps [17]. The intensity fluctuations lead to a change in trap frequency, see Eq. (3), resulting in transitions $n \rightarrow n \pm 2$ between states of vibration of the trapped mirror. This produces a rate of parametric heating due to intensity fluctuations given by $\gamma_I = \frac{1}{4}\omega_z^2 S_I(2\omega_z)$, where $S_I(2\omega_z)$ is the noise power spectrum of the laser. For example, using $S_I = 10^{-10}$ Hz^{-1} results in an exponential energy growth rate of 1.01 rad/s. We note that Nd:YAG lasers with a lower noise spectrum are available and would further reduce this source of heating.

Beam-pointing fluctuations cause fluctuations of the trap center and lead to a constant heating rate given by $\gamma_x = \frac{1}{4}\omega_z^4 m S_x(\omega_z)$. For a spectrum of position fluctuations $S_x(\omega_z)$ of 10^{-10} μm^2 Hz^{-1} this yields a negligible constant heating rate of the order of 10^{-12} J/s.

We next turn to scattering losses. An object with a diameter much bigger than the wavelength of light, such as our "floating" mirror, can be thought of as being comprised of a collection of optically driven induced dipoles. Scattering from these dipoles is the mechanism behind Rayleigh scattering [18]. The momentum kicks due to the scattering average to zero, but their fluctuations in the z direction result in heating. Details of this scattering loss can be found, e.g., in Ref. [18]. Here we summarize the main results.

For a light intensity I_0 incident on a scatterer of volume V, the scattered power per solid angle is given by $\frac{dP}{d\Omega} = I_0 V R$, where the scattering coefficient can be obtained by thermodynamic arguments and is given by $R = \frac{\omega^4}{16\pi^2 c^4}\gamma_e C_T k_B T \sin^2\phi$. Here, ω is the frequency of the (trapping) laser, $\gamma_e = (n^2 - 1)(n^2 + 2)/3$ where n is the index of refraction, C_T is the isothermal compressibility, T_e is the effective temperature of the density fluctuations and ϕ the angle from the direction of propagation (say the x axis). All other constants have their usual meaning. The power is the rate of optical energy scattered, E_{scatt}, and $\langle E_{\text{scatt}}\rangle = \langle N_{\text{scatt}}\rangle\hbar\omega$, where N_{scatt} is the number of photons scattered.

The component of the trapping photon momentum along z is $p_z = \hbar k\cos\phi$, resulting in an increase $\langle E_{\text{kin}}\rangle$ in kinetic energy of the trapped mirror. The fraction of scattered optical energy per photon that contributes to that increase is $\eta = \frac{1}{\hbar\omega}\frac{(\hbar k\cos\phi)^2}{2m}$. Integrating over the solid angle $d\Omega$, we find the scattered power to be

$$\frac{d}{dt}\langle E_{\text{kin}}\rangle = \frac{8\hbar\pi^4 I_0 V\gamma_e C_T k_B T_e}{15mc\lambda^5}. \quad (6)$$

For our parameter we find $\eta = 2.68 \times 10^{-26}\cos^2\phi$—which confirms the intuitive argument that most of the scattered light does not contribute to the heating of the center-of-mass mode—which leads to a negligible constant heating rate of 1.93×10^{-32} J/s. This is in contrast to the situation with nanospheres, where dipole scattering is the dominant source of noise [5,6].

Fabry-Perot laser fluctuations.—Another source of noise that places a fundamental limit on the occupation number of the center-of-mass motion of the moving mirror is the linewidth of the Fabry-Perot laser [19]. Here we model the laser linewidth in terms of a phase diffusion process that drives the laser field $E_{\text{in}}e^{i\phi(t)}$. The phase $\phi(t)$ is given by $\phi(t) = \sqrt{2\Gamma_L}\int_0^t \eta(s)ds$, where Γ_L is the laser linewidth and $\eta(s)$ is a gaussian white noise process with mean $\langle\eta(s)\rangle = 0$ and correlation $\langle\eta(s)\eta(v)\rangle = \delta(s - v)$. For $|(\omega_c z)/(\omega_z L)| \ll 1$ this results in the linewidth-modified cooling rate

$$\gamma_{rp} = -\left(\frac{\omega_c\kappa}{m\omega_z L^2}\right)\frac{8P_{\text{in}}[A_- - A_+]}{[(2\Gamma_L + \kappa)^2 + 4\Delta^2](\kappa^2 + \omega_z^2)} \quad (7)$$

where P_{in} is the input power and $A_\pm$ is given by

$$A_\pm = \frac{(\Gamma_L + \kappa)(2\Gamma_L + \kappa)^2 + 2\Gamma_L((\Delta \pm \omega_z)^2 + \Delta^2) + \kappa\omega_z^2}{(2\Gamma_L + \kappa)^2 + 4(\Delta \pm \omega_z)^2}. \quad (8)$$

The laser linewidth generally results in less efficient backaction cooling, but there is a range of detunings Δ for which the cooling rate is essentially unchanged from the ideal case, a result of the excitation of the anti-Stokes sideband from higher frequencies in the laser spectrum. For our parameters, a 1 μW laser of linewidth 10×10^3 rad/s, detuned -4.25×10^5 rad/s from the cavity resonance results in a cooling rate γ_{rp} of 5.2×10^4 rad/s.

Background gas collisions.—The fluctuations in mirror motion due to background gas collisions can be described by the Langevin equation $\ddot{z} + \gamma_{\text{bg}}\dot{z} = \xi(t)$ where the fluctuating force $\xi(t)$ obeys the Markovian correlation relations $\langle\xi(t)\rangle = 0$, $\langle\xi(t)\xi(t')\rangle = q\delta(t - t')$ with q given by the fluctuation-dissipation theorem as $q = 2k_B T\gamma_{\text{bg}}/m$. To derive an expression for γ_{bg} we consider motion along the z axis only. A gas molecule of mass m_g and velocity v_g undergoing an elastic collision with the disk imparts a momentum change $\delta p = 2m_g v_g$. In the moving frame of the disk, this gives $\Delta p_{\text{disk}} = 2m_g(v_g - v_{\text{disk}}) - 2m_g(v_g + v_{\text{disk}})$, the two contributions corresponding to forward and backward collisions. The rate of momentum transfer is then obtained by multiplying this expression by the number of collisions per unit time ($nAv_g/2$), where n is the number density of gas molecules, A is the cross-section area of the disk, and v_g is the mean speed of the molecules, taken to be to be the average thermal velocity for an ideal gas of pressure P. This gives $\gamma_{\rm bg} = 4PA/(mv_g)$. For a pressure of 10^{-8} torr, $\gamma_{\rm bg} = 2.55 \times 10^{-6}$ rad/s. We note also that background gas collisions do not introduce any significant wobble.

Both intensity fluctuations and background collisions are mechanisms of damping for the disk mirror and provide the equivalent of a mechanical Q factor. The coupling to a thermal reservoir increases the attainable mean phonon number $\langle n\rangle_{\rm min}$ by $\gamma_{\rm bg}n_R/(\gamma_{\rm rp} - \gamma_I)$, n_R being the average occupation number of the relevant mode before cooling, $n_R \simeq k_BT/\hbar\omega_z$. For our parameters, at room temperature, the contribution of this mechanical damping is very small, $\simeq 0.01$, and can be reduced further via better stabilized lasers and an improved vacuum.

In conclusion, we have shown that the coupling to the thermal reservoir in standard optomechanical setups can be completely eliminated by optical levitation of the Fabry-Perot mirror, resulting in mean phonon occupation numbers significantly below unity. Following the argument of Ref. [6] it can also be shown that for the parameters considered here a levitated mirror cooled to its quantum mechanical ground state would undergo of the order of 10^4 oscillations before undergoing a shot-noise induced quantum jump.

As noted earlier the quoted value of $\langle n_{\rm min}\rangle$ can be reduced by stiffening the optical spring. The optical spring effect has been studied extensively in the gravitational wave detection community [20], where the moving mirror's mechanical resonance frequency has been greatly enhanced using a two-color laser configuration. A similar approach could also increase ω_z in our case without increase in the intensity of the trapping lasers.

An alternative cooling technique is cold damping quantum feedback. Using the theory of Ref. [21] we can evaluate the minimum mean phonon occupation number ignoring all sources of noise. For the parameters used in the sideband cooling calculation above, with a feedback bandwidth of $\omega_{\rm fb} = 3\omega_m$, cold damping results in $\langle n\rangle_{\rm min} \simeq 230$, far from the quantum limit. However, in contrast to sideband cooling, cold damping is more effective for larger κ/ω_m, and is more sensitive to the initial phonon occupation number. For instance, for an initial temperature of 100 mK with $\kappa = 10\omega_m$, then $\langle n\rangle_{\rm min} \simeq 0.86$. We can therefore see that for our setup, sideband cooling is advantageous.

Future work will include the extension of this proposal to a three-mirror geometry, as well as the coupling of the levitated mirror to ultracold atomic and molecular systems, either for the quantum control of the state of the mirror, or conversely for the manipulation of the atoms. In particular, the generation, detection and control of nonclassical motional states of the mirror will be considered. In addition, we will carry out a more detailed analysis of the optical coupling of the optical tweezers to the dual-disk structure.

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