Direct Observation of Dynamical Bifurcation in a Superfluid Josephson Junction

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We report a direct observation of dynamical bifurcation between two plasma oscillation states of a superfluid Josephson junction. We excite the superfluid plasma resonance into a nonlinear regime by driving below the natural plasma frequency and observe a clear transition between two dynamical states. We also demonstrate bifurcation by changing the potential well with temperature variations.

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A superfluid Josephson junction is formed by separating two superfluid reservoirs with an array of nanoscale apertures [1–3]. It exhibits many fascinating properties that are also found in superconducting Josephson tunnel junctions including Josephson oscillation [4], plasma oscillation [5], the Fiske effect [6], and the Shapiro effect [7]. These phenomena are nonlinear dynamical effects, some of which have found applications in both superconducting and superfluid fields as novel sensors [8–12] and amplifiers [13,14] that remain efficient in the quantum limit. Unique Josephson phenomena have also been reported in Bose-Einstein condensates [15,16]. In this Letter, we report the first observation of dynamical bifurcation between two driven superfluid plasma oscillation states. The observed switching behavior at the bifurcation point resembles that of a Josephson bifurcation currently utilized in superconducting qubit research [17]. The results presented here not only advance the close analogy between the macroscopic quantum physics of superconductivity and superfluidity but also open the possibility for the development of sensitive nonclassical amplifiers for superfluid quantum interference devices [11,12].

Our apparatus is schematically represented in Fig. 1. A cylindrical inner reservoir is bounded on the top by a metallized flexible diaphragm, and an array of nanoscale apertures is mounted at the bottom opening to the outer reservoir. Both inner and outer reservoirs are filled with superfluid ^{4}He, and the aperture array forms a superfluid Josephson junction that couples two macroscopic wave functions describing the two superfluid reservoirs [18].

Near the superfluid transition temperature T_λ, the aperture array junction behaves as an ideal, nondissipative, nonlinear oscillator [3,19]. The mass current $I(t)$ across the junction driven by a chemical potential difference $\Delta\mu$ is governed by the Josephson current-phase relation [5]: $I(t) = I_0 \sin\phi(t)$, where I_0 is the junction critical current and $\phi(t)$ is the quantum phase difference across the junction. The phase difference $\phi(t)$ evolves in time according to the Anderson phase evolution equation [18,20]: $\partial\phi(t)/\partial t = -\Delta\mu/\hbar$. In this ideal weak-coupling limit, one can parameterize the junction with a nonlinear hydrodynamic inductance $L_J = (\kappa_4/2\pi)(dI/d\phi)^{-1} = \kappa_4/(2\pi I_0 \cos\phi)$, where $\kappa_4 \equiv h/m_4$ is the circulation quantum for superfluid ^{4}He [3,19,21]. Note that this takes the same exact form as the nonlinear superconducting Josephson inductance $L_J = (\Phi_0/2\pi)(dI/d\phi)^{-1} = \Phi_0/(2\pi I_0 \cos\phi)$, where $\Phi_0 \equiv h/(2e)$ is the magnetic flux quantum and I_0 in this case is the superconducting Josephson critical current.

The nonlinear hydrodynamic inductance of a superfluid Josephson junction is shunted in parallel by a hydrodynamic capacitance C associated with the presence of a diaphragm and heat capacity and compressibility of the fluid that it displaces [21,22]. The combined system is an LC oscillator with the dynamics described by a phase particle with coordinate ϕ in a so-called washboard potential $U(\phi) = (\kappa_4 I_0/2\pi)(1 - \cos\phi)$. The oscillation within this potential well is called the plasma mode (also called the Helmholtz mode in a superfluid system), and its natural frequency $\omega_p = 1/\sqrt{L_J C}$ is called the plasma frequency. One can also view the system as a rigid pendulum with the phase difference ϕ playing the role of the pendulum's displacement angle. For $\phi \ll 1$, L_J reduces to $\kappa_4/(2\pi I_0)$, which suggests $\omega_p^2 \propto 1/L_J \propto I_0$ in this small oscillation amplitude limit.

In our neutral matter Josephson system, a plasma oscillation can be directly observed, for example, by applying a step voltage between the diaphragm and a nearby electrode

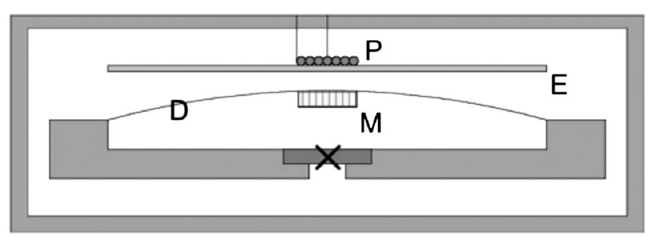

FIG. 1. A flexible diaphragm (D) and a rigid electrode (E) form an electrostatic pressure pump. The diaphragm with a small magnet (M) attached to it also forms the input element of a sensitive displacement sensor through a nearby pickup coil (P). A junction (X) consists of 75×75 60-nm apertures spaced on a 2-μm square lattice in a 60-nm-thick silicon nitride window.

to create a pressure difference across the junction. The applied $\Delta\mu$ sets the phase particle in motion within the potential well, and the fluid in the aperture array exhibits characteristic oscillations in time. This results in the oscillations of the diaphragm hydraulically coupled to the superfluid within the aperture array, which we detect with our displacement sensor. Plasma frequency for a superfluid Josephson system is typically below 100 Hz, whereas the superconducting plasma frequency is typically in 10–100 GHz range. See Refs. [3,23] for typical time traces of superfluid plasma oscillations.

Figure 2 shows the square of measured plasma frequency plotted against the critical current for several different temperatures close to T_λ. The temperatures shown are $T_\lambda - T$ values with $T_\lambda \approx 2.176$ K. The expected $\omega_p^2 \propto I_0$ dependence in an ideal Josephson regime can be clearly seen near T_λ (indicated by a solid line in the plot). However, as the temperature is lowered, the system leaves the ideal Josephson regime to become more dissipative, and the current-phase relation starts to gain more linearity [3]. The simple relation $\omega_p^2 \propto I_0$ clearly does not hold in the crossover regime.

The nonlinearity of the Josephson junction manifests itself in the phase particle picture in analogy with a rigid pendulum as described earlier. One of the simplest models for such a system is a damped, forced oscillator with a cubic nonlinearity: $\ddot{x} + \gamma\dot{x} + \alpha x + \beta x^3 = B\cos(\omega_d t)$, where B is the driving parameter, ω_d is the drive frequency, and γ, α, and β represent the strengths of damping, stiffness, and nonlinearity, respectively. This is the so-called Duffing oscillator model with a "soft spring" condition ($\beta < 0$) [24–26]. The expected oscillator response is plotted as a function of detuning parameter $2Q(\omega_d/\omega_0 - 1)$ in Fig. 3 for increasing values of the dimensionless driving amplitude $f \equiv B\sqrt{\beta/\alpha^3}$ [27]. We have set $Q = 300$ to match the observed quality factor of superfluid plasma mode at $T_\lambda - T = 10$ mK. For small driving strength, the sinusoidal potential is well approximated by a parabola, and therefore the particle behavior is harmonic with its response Lorentzian in shape. However, for large amplitude oscillations, nonlinear terms act to reduce the oscillation frequency, causing the peak to bend towards the left. For even stronger drives, the bending of the peak becomes so much that the oscillator bifurcates from a single-valued to a bistable regime. At locations such as the one indicated by a vertical line in Fig. 3, the system can have two possible oscillation states with different amplitudes and phases. Increasing the oscillation amplitude at such bias points eventually causes the system to switch between the two dynamical states (from L to H, for example), giving rise to a sharp step on the lower frequency side of the distorted peak. Since the superfluid Josephson junction is intrinsically nonlinear, it suggests a possibility for such bifurcation. If we ramp the plasma oscillation with an ac drive at a frequency slightly below ω_p, at some critical drive level, the system should exhibit a sharp transition between two oscillation states. In the experiment reported here, we probe this by applying an ac voltage (V_{ac}) between the diaphragm and the electrode at a drive frequency ω_d. Since the force on the diaphragm scales as the applied voltage squared, we add a dc offset (V_{dc}) to the ac excitation to linearize the drive. The voltage from our SQUID-based displacement sensor is fed into a lock-in amplifier with a reference signal synchronized with the ac drive. We then monitor the amplitude R of superfluid plasma oscillations at the drive frequency ω_d as well as the oscillation phase θ relative to the drive.

Figure 4 is an example of the amplitude and the phase of superfluid plasma oscillations for different excitations, while sweeping the frequency ω_d. When the drive is kept minimal, both R and θ plots show behaviors expected for a resonant system in a linear regime. However, a dramatic change takes place when we increase the excitation to push the system into a nonlinear regime. The resonant peak bends towards lower frequency as predicted, and an abrupt step appears on the left side of the peak as the system bifurcates and transitions from one state to another. This effect is pronounced and more apparent in the θ plot where a smooth zero crossing evolves to an abrupt switching

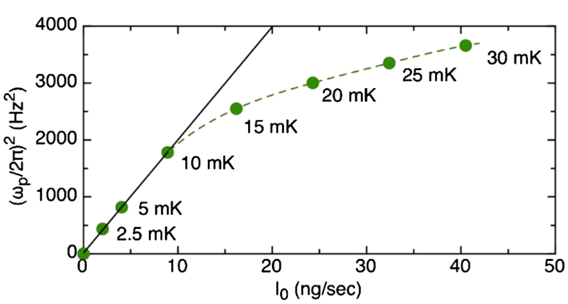

FIG. 2 (color online). Measured plasma frequency squared vs critical current. The solid line is a linear fit to the first four points closest to the superfluid transition temperature. The dotted line is a guide to the eye. The point at the bottom left corner is for $T = T_\lambda$, where both ω_p and I_0 are zero.

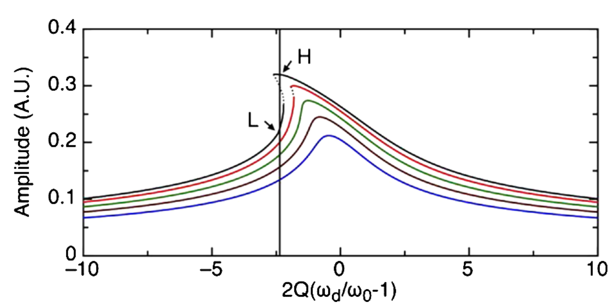

FIG. 3 (color online). Predicted oscillator responses. Dashed lines indicate unstable solutions. The top two curves exhibit dynamical bifurcation as the system is driven into a highly nonlinear regime. The drive amplitude parameter f is increased from 1.5 to 3.5×10^{-4} at 5×10^{-5} increments.

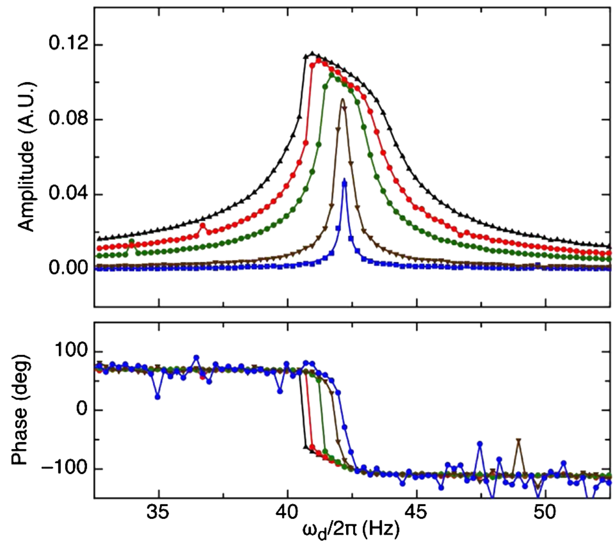

FIG. 4 (color online). Amplitude of plasma oscillation and oscillation phase relative to the drive as a function of drive frequency ω_d. These data are taken at $T_\lambda - T = 10$ mK. Excitations ($V_{\rm ac}V_{\rm dc}$) are 0.003, 0.01, 0.04, 0.06, and 0.09 [V^2].

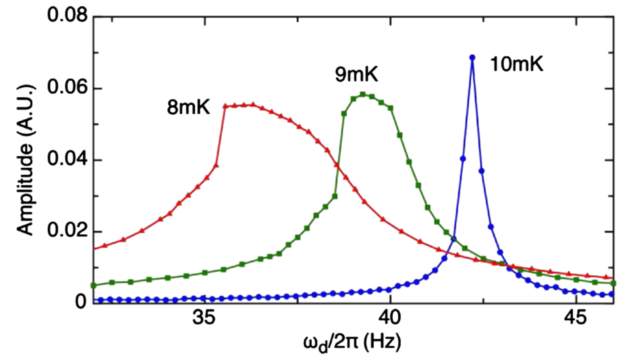

FIG. 5 (color online). Probing the nonlinearity with temperature changes with a constant drive. The overall shift in frequency is due to the temperature dependence of ω_p. Temperatures shown are $T_\lambda - T$ values. Excitation $V_{\rm ac}V_{\rm dc} = 0.006$ [V^2].

between two oscillation states as the drive is increased. This sudden transition from a low-amplitude and phase-lagging state to a high-amplitude and phase-leading state is a manifestation of nonlinear dynamical bifurcation, and such a phenomenon often seen in electrical and optical systems [24] is observed for the first time here in a superfluid system.

The predicted nonlinear behaviors in Fig. 3 have been generated for different values of $f = B\sqrt{\beta/\alpha^3}$ defined for the Duffing model. This implies that, even for a constant drive B, one should, in principle, be able to observe the predicted evolution by varying $\sqrt{\beta/\alpha^3}$, which corresponds to changing the shape of the potential well. However, for most physical systems, all the parameters are fixed by experimental configurations, and increasing the drive B and thus sweeping different parts of the potential well is often the only way to harness different degrees of anharmonicity. An unique feature of the reported phenomenon in superfluid helium is that one can change various parameters by varying the temperature. We demonstrate this in Fig. 5. Here, we have kept the drive level constant but increased the temperature, which is equivalent to holding B constant but increasing β and decreasing α in the Duffing model. The Lorentzian shape becomes distorted and a bifurcation step appears again, verifying the general picture for the driven nonlinear dynamical system.

Although the main nonlinear features such as decreasing oscillation frequency with increasing amplitude and the appearance of bifurcation are consistent with the predictions shown in Fig. 3, we find that the exact functional forms describing the data such as those shown in Figs. 4 and 5 are more complicated than what a simple Duffing model predicts. A more sophisticated model on the dynamics of superfluid Josephson junctions and plasma oscillations is needed to better parametrize the system response and further investigate different parameter regimes.

We note that the height of the potential well that binds the phase particle decreases with increasing temperature. This effect combined with a decreasing quality factor near T_λ makes it difficult to slowly ramp up the plasma oscillation without kicking the particle out of the potential well once the temperature reaches a few millidegrees Kelvin away from T_λ. The phase particle escaping from one potential well to another corresponds to the so-called Josephson oscillation, which we need to avoid in achieving the dynamical bifurcation with plasma mode oscillations.

One interesting aspect of the results reported here is the possibility for exploiting the anharmonicity of the junction oscillator as a novel amplifier, in close analogy with rf-driven Josephson bifurcation amplifiers currently utilized in quantum computing research [13]. As long as the system remains in the Josephson regime, critical current can be deduced from the measured plasma frequency, and a signal from small change in ω_p will be enhanced greatly if we tune the system to sit at the bifurcation point. The smallest variation in the critical current that can be resolved should be limited by vibrational noise and ultimately thermal fluctuations [28,29]. In this digital mode of operation, the bifurcation phenomenon itself becomes a sensitive threshold detector for the critical current, and this could be a useful tool for superfluid quantum interference devices. A superfluid interference device typically consists of a loop of superfluid interrupted by two junctions as in the case of a dc-SQUID. It has been utilized for rotation sensing through the Sagnac effect [10–12], direct measurement of a phase gradient associated with superfluid flow [30], detection of quantized vortex motion [31], and construction of an absolute gauge for quantum phase shifts [32]. In these experiments, quantum phase shifts to be measured are typically stable, unlike the qubit case, where short-lived signals benefit greatly from a fast latching readout [33]. However, to detect a small phase shift with a superfluid device, one is often required to scale up the sense-loop size, leading to higher kinetic inductance and shallower modulation depth. A detection scheme based on bifurcation phenomena could provide a sensitivity enhancement without the increase in device size. A longer measuring time required for resetting the system after each switching event could, however, be a trade-off and should be considered.

In conclusion, we have performed a new experiment where we excite the superfluid plasma mode into a nonlinear regime. We have shown that the system transitions between two dynamical states. We have also demonstrated the bifurcation phenomena by temperature variations. The findings reported here also advance the deep analogy between the dynamics of superconducting Josephson junctions and superfluid Josephson systems.

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