The Clapping Book: Wind-Driven Oscillations in a Stack of Elastic Sheets

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We present a hybrid experimental and theoretical study on the oscillatory behavior exhibited by multiple thin sheets under aerodynamic loading. Our *clapping book* consists of a stack of paper, clamped at the downstream end and placed in a wind tunnel with steady flow. As pages lift off, they accumulate onto a bent stack held up by the wind. The book collapses shut once the elasticity and weight of the pages overcome the aerodynamic force; this process repeats periodically. We develop a theoretical model that predictively describes this periodic clapping process.

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There has been much recent interest in the coupling of a flexible solid structure and fluid flow. This arises in the classic contexts of vortex-induced vibrations (VIV) [1] and wind loading of bridges and buildings [2]. Instances in biology are also ubiquitous, including the dynamics of swimming [3,4] and flying [5], snoring [6], and flow in flexible tubes [7]. Model systems have been paramount for understanding the governing physics, where the flapping flag [8] has become a canonical problem. Arrays of multiple flexible objects in fluid flow have particular biological significance, such as in locomotion of microorganisms [9] and flow through terrestrial [10] and aquatic [11] vegetation. In the context of model systems, a few recent studies have focused on arrays of multiple flaglike objects [12]. However, the predictive understanding of systems of multiple sheets under flow in other geometries remains challenging.

Here, we introduce a novel model system, the *clapping book*. In our experiments, the book is a stack of 150 pages of paper (Hammermill), each of length $L = 17.0$ cm, width $W = 2.8$ cm, and thickness $h = 110\ \mu$m. Each page has mass per unit length $\rho_p hW = 0.021$ g/cm and bending stiffness $BW = 8.2 \times 10^{-6}$ N m^2, where ρ_p is the paper's density, $B = Eh^3/12(1-\nu^2)$ the bending modulus, E the Young's modulus, and ν the Poisson ratio. We clamp the pages at one end and place the book in the 30 cm $\times$ 30 cm test section of a wind tunnel, with the clamped end downstream, and image it from the side at 120 frames per second. A sequence of representative frames of the clapping process is presented in Figs. 1(a)–1(e). The pages lift off [Figs. 1(a)–1(d)] and form a bent stack of paper held up by the wind. As pages accumulate, this elevated stack thickens and moves progressively farther upstream. Eventually the book claps shut and this process restarts [Fig. 1(e)], resulting in continuous oscillations with a well-defined period. These oscillations involve the interaction of multiple sheets and occur on a time scale much slower than that associated with an individual sheet. In what follows, we develop a theoretical model to explain this behavior quantitatively and perform a series of detailed comparisons with our experiments. We consider the large elastic deformations, the interaction between the pages, and their coupling with the flow.

We model each elevated page as an inextensible elastic beam of dimensions $h \ll W \ll L$ whose motion is confined to the xy plane [Fig. 1(f)]. Its shape can therefore be represented by $\mathbf{x}(s,t)$, with unit tangent $\mathbf{x}_s = \mathbf{t} = (\cos\theta, \sin\theta)$ and normal $\mathbf{n}$. Its evolution is governed by the relation between curvature and torque and the angular and linear momentum equations, which can respectively be written in dimensionless form as [13],

$$\theta_s = \Gamma, \tag{1}$$

$$I\theta_{tt} = \Gamma_s + \mathbf{n}\cdot\mathbf{F}, \tag{2}$$

$$\mathbf{x}_{tt} = \mathbf{F}_s + K\mathbf{n} - \Delta\mathbf{e}_y, \tag{3}$$

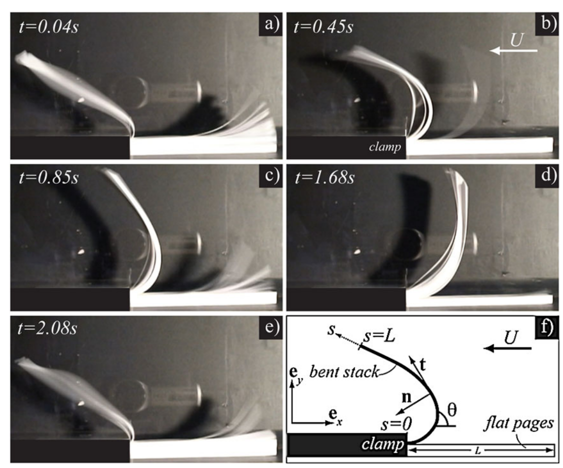

FIG. 1 (color online). (a)–(e) Representative photographs of the clapping book at selected times during a single period. (f) Schematic diagram of the system.

where the arclength s and position $\mathbf{x}$ are scaled by L and time t by $\sqrt{\rho_p h L^4/B}$. The z component of the internal torque (Γ), the internal force ($\mathbf{F}$), and the aerodynamic force per unit length (K) are scaled by BW/L, BW/L^2 and BW/L^3, respectively. This nondimensionalization results in two dimensionless parameters, $I = h^2/12L^2$ and the elastogravity number $\Delta = \rho_p g h L^3/B$, which accounts for the relative weight of the pages with respect to their bending rigidity. The pages are thin relative to their length, $I \ll 1$, and the corresponding term in Eq. (2) is therefore neglected, yielding a model equivalent to the flag in Ref. [8]. At the clamped end of the beam, the position is fixed, $\mathbf{x}|_{s=0} = \mathbf{0}$, and its tangent is horizontal, $\theta|_{s=0} = 0$. At the free end, the internal torque and force must vanish, i.e., $\Gamma|_{s=1} = 0$ and $\mathbf{F}|_{s=1} = \mathbf{0}$, respectively.

To describe the flow around the pages, we focus on the slender body limit [14] with a uniform far-field wind velocity $-U\mathbf{e}_x$. Viscous drag can be neglected since the Reynolds number (based on page length) ranges from 4×10^4 to 2×10^5. In the wind's frame of reference, the beam's shape is given by $\mathbf{X}(s, t) = \mathbf{x}(s, t) + Ut\mathbf{e}_x$ and the flow is determined entirely by the normal component of the beam's velocity, $V = \mathbf{n} \cdot \mathbf{X}_t = \mathbf{n} \cdot \mathbf{x}_t + U\mathbf{n} \cdot \mathbf{e}_x$. For our regime of interest, the speed of the pages is much slower than the speed of the flow itself, so the time derivative term is neglected for simplicity.

We consider two components of the external aerodynamic force on the beam, the *resistive* and the *reactive* forces. The resistive force on an element of the beam is the drag experienced by a flat plate moving in the direction perpendicular to its surface, with magnitude per unit length $F_1 = \frac{1}{2} C_d \rho_a V^2 W$, where ρ_a is the density of air and C_d is a drag coefficient that must be determined experimentally. Assuming the flow reaches a steady state faster than the beam moves, any contribution from the unsteadiness of the flow can be neglected. The reactive force arises as the fluid accelerates to follow the shape of the beam. Following Ref. [4], we take the flow locally to be the potential flow induced by a flat plate moving perpendicular to its surface. This flow has momentum $\mathbf{P} = \rho_a A V \mathbf{n}$, where $\rho_a A$ is the beam's added mass, with $A = \pi (W/2)^2$ being the area of its circumscribing circle. The change in the fluid momentum at a fixed point in the moving frame is $\mathbf{P}_t - (U\mathbf{e}_x \cdot \mathbf{t})\mathbf{P}_s$, the $-\mathbf{n}$ component of which gives the magnitude of the reactive force (per unit length) on the beam, $F_2 = -\rho_a A[V_t - (U\cos\theta)V_s]$. Again, the relatively low speed of the pages allows the time derivative term to be neglected.

The total aerodynamic force on the beam is the sum $F_1 + F_2$. This can be expressed in dimensionless form as

$$K = \frac{1}{2} C_d C_y \sin^2\theta - \frac{\pi}{4} \lambda C_y \cos^2\theta \frac{d\theta}{ds}, \tag{4}$$

where $C_y = \rho_a U^2 L^3/B$ is the Cauchy number, the dimensionless ratio of aerodynamic force $\rho_a U^2 W L$ to bending force BW/L^2, and $\lambda = W/L$ is the aspect ratio of the beam.

Our model has four parameters, Δ, λ, C_y, and C_d. The drag coefficient, C_d, is the only parameter that must be determined by fitting (once and for all) to experiments (described below). The ranges of the other parameters in our experiments are $0.1 < \Delta < 20$, $0.1 < \lambda < 0.5$, and $C_y < 60$. Given values for these parameters, the static shape of a beam at equilibrium in the wind can be computed by solving Eqs. (1)–(4) with the time derivative terms dropped. In Figs. 2(a)–2(c), we present photographs of the experimental shapes of a single elastic sheet taken at three wind speeds and superimpose the predicted shapes, showing excellent agreement.

In the experiments, as U is decreased, we observe that there is a critical wind speed below which a given sheet can no longer be supported. In Fig. 2(d), we show measurements of this speed for a strip of acetate and a strip of vinylpolysiloxane (VPS), as L is varied. Similarly, for given values of Δ, λ, and C_d, Eqs. (1)–(4) fail to have a valid equilibrium solution when C_y is below a critical value C_y^*. Hence, from the definition of C_y, a critical wind speed $U = U^*$ is predicted. We compare the experimentally measured critical wind speeds with those predicted by our model [Fig. 2(d)], where the drag coefficient C_d is treated as the single fitting parameter. In the subsequent

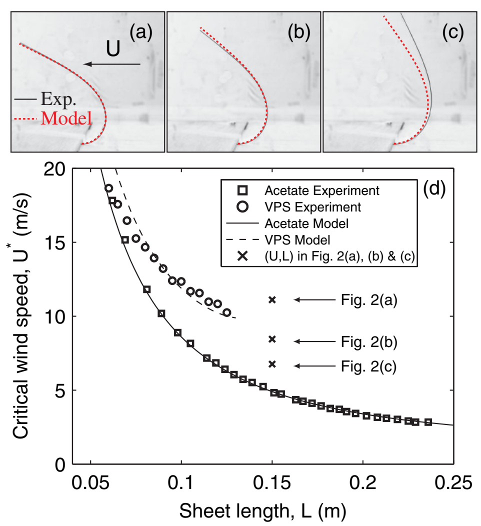

FIG. 2 (color online). (a),(b),(c) Photographs of a single acetate sheet (solid lines), with $L = 15.0$ cm, $W = 2.9$ cm, $\rho_p h W = 0.114$ g/cm, and $BW = 2.4 \times 10^{-4}$ N m^2, superimposed with predicted shape (dashed lines). Flow speeds were (a) 11.1, (b) 8.5, and (c) 6.8 m/s. For all static shape calculations, we use a spatial resolution of 1000. (d) Experimental and predicted (with $C_d = 1.76$) critical wind speed versus length for two materials: acetate strip with $W = 3.0$ cm, $\rho_p h W = 0.118$ g/cm, and $BW = 2.6 \times 10^{-4}$ N m^2; vinylpolysiloxane (VPS) strip with $W = 2.2$ cm, $\rho_p h W = 2.04$ g/cm, and $BW = 2.3 \times 10^{-4}$ N m^2, with paper on its front surface. Also shown ($\times$) are parameter values for the photographs in (a), (b), and (c).

calculations, we use the average value that best fits each material, $C_d = 1.76 \pm 0.03$, which is consistent with previously reported experimental measurements of the drag coefficient for a flat plate [15].

Having presented a model for a single wind-loaded sheet, we return to the periodic behavior of the clapping book. At any time, we consider some of the book's pages to be lifted up, forming a stack bent by the wind, while the rest remain flat. When the elevated stack contains more pages than the wind can support, the book claps shut. We proceed by separately considering three aspects of the process: (i) page accumulation, (ii) loss of support of the bent pages, and (iii) the subsequent collapse.

To help in describing page accumulation, a schematic diagram of the flow profile over the flat stack of pages is sketched in Fig. 3(a), with recirculation zones shown ahead of the step and on the surface of the top page. This geometry is known as the forward-facing step [16]. A detailed analysis of the flow would involve determining the locations of flow separation and reattachment, a challenging endeavor that goes beyond the scope of our study. Instead, we proceed by deducing an empirical relation for the page liftoff rate. We focus on the regime in which the wind speed is high enough that the pages lift off continuously. In Fig. 3(a), we present measurements of the dependence of the liftoff rate, f, on wind speed for our book. We find that f follows the linear relationship,

$$f = \beta(U - U_0), \tag{5}$$

with $\beta = 12.5 \pm 0.8\ \mathrm{m}^{-1}$ and $U_0 = 4.9 \pm 0.2$ m/s [Fig. 3(a)].

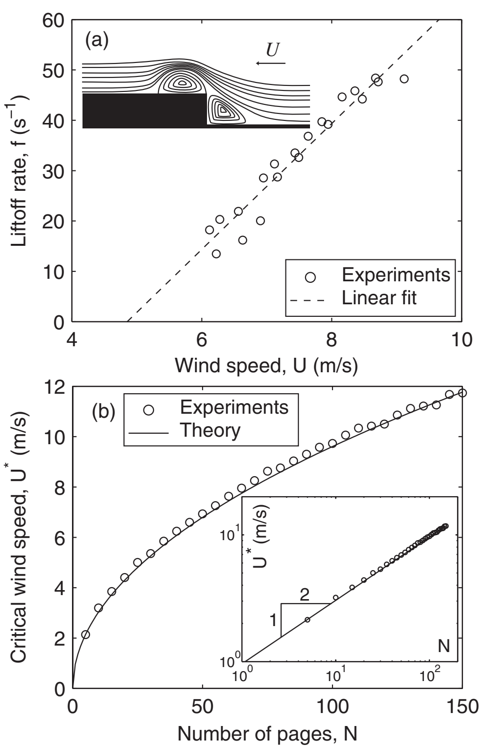

FIG. 3. (a) Experimental page liftoff rate for the paper book. The dashed line is the liftoff rate given by Eq. (5). Inset: Sketch of expected flow profile. (b) Critical wind speed measured for an elevated stack of N pages. The solid line is calculated using Eq. (6). Inset: log-log version of the plot.

We proceed by determining the maximum number of pages that the wind can support, using the equilibrium model for a beam introduced above, Eqs. (1)–(4), again after dropping the time derivative terms. The elevated pages are treated as a single beam, and we assume that the pages slide against each other without friction. As such, the bending energy of the stack of elevated pages is the sum of the bending energies of the individual pages. Thus, if $\rho_p hW$ and B characterize a single page, the elastic beam representing the entire elevated stack of N pages has mass per unit length $N\rho_p hW$ and bending modulus NB [17]. We note that both parameters Δ and λ are then independent of N. Hence, since the model's prediction for C_y^* depends only on Δ and λ, this prediction should be unique for a particular book, regardless of how many pages have been lifted up. Using this C_y^*, the definition of C_y yields a relation between the number of pages in a stack, N, and the critical wind speed, U^*:

$$C_y^* = \frac{\rho_a U^{*2} L^3}{NB}. \tag{6}$$

For our book, the model gives $C_y^* = 18.5$, from which the critical wind speed can be calculated, $U^* = \sqrt{NBC_y^*/\rho_a L^3}$. In Fig. 3(b) (and inset), we show that this prediction is in excellent agreement with experimentally measured critical wind speeds and validates our description of the behavior of multiple pages. Conversely, for a given wind speed U, we use Eq. (6) to obtain the number of pages, N^*, for which U is the critical speed. This is the maximum number of pages this wind speed can support.

Once this maximum number of pages is exceeded, stability is lost and the bent stack collapses. We simulate the collapsing stack of pages as a beam using the full time-dependent form of Eqs. (1)–(4), supplemented with the inextensibility condition $\mathbf{x}_{tts} = \mathbf{t}_{tt}$. To account for the momentum of the pages when collapse begins, we start the simulation with the equilibrium shape $P/4$ time before the maximum number of pages has accumulated, with no initial velocity. Here $P = 2\pi/m^2$ is the dimensionless period of the first mode of an unforced beam, with the constant $m = 1.88$ given by Rayleigh [18]. In the simulations, as in the experiments, we observe the entire page to become horizontal nearly simultaneously, and denote the time at which this occurs T_c. For simplicity, we use the collapse simulation run at the middle wind speed, finding that the collapse time differs for the minimum and maximum wind speeds by less than 0.1 s.

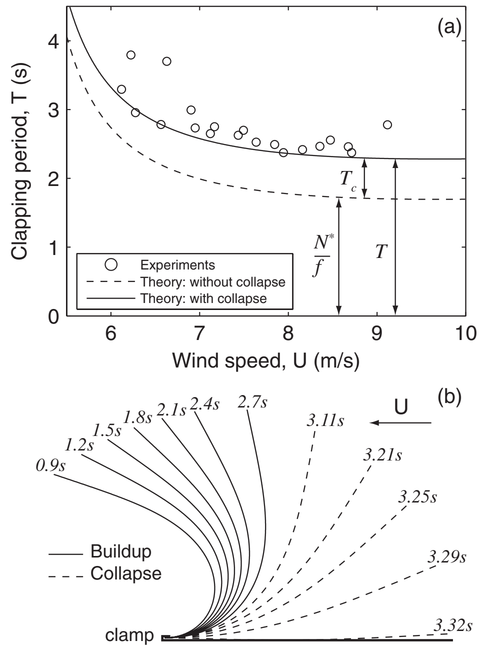

FIG. 4. (a) Experimentally measured clapping period as a function of wind speed compared with the prediction from Eq. (7). The dashed line is the first term of Eq. (7); the solid line incorporates the collapse time (using a spatial resolution of 50). (b) Simulated sequence of the clapping process ($U = 6$ m/s, $t = 0$ s at the start of accumulation). The solid and dashed lines correspond to accumulation and collapse, respectively.

Having described the accumulation and collapse of the pages, these results can be combined to calculate the total clapping period, a representative feature of the clapping process. The accumulation time can be taken to be the maximum number of pages that can be supported, N^* from Eq. (6), divided by the rate at which pages accumulate, f from Eq. (5). This is added to the collapse time, T_c, to give the complete clapping period,

$$T = \frac{N^*}{f} + T_c. \tag{7}$$

In Fig. 4(a) we show the measured clapping period for a book as a function of U, which is in excellent agreement with our theoretical prediction. Representative calculated page shapes over one period are shown in Fig. 4(b) and compare well with the photographs in Fig. 1.

In summary, we have introduced a novel system, the clapping book, that exhibits oscillations resulting from the interaction of multiple thin sheets and flow. This system was challenging to model due to the large-amplitude deformation of the sheets and their interaction with each other and with the wind. Using a model for an elastic beam in high Reynolds number flow, we combined descriptions of page accumulation, loss of stability, and dynamic collapse. We were thereby able to calculate the clapping period in good agreement with experiments. Rationalization of this model system should provide insight into other problems involving multiple flexible structures and flow, which are abundant in the natural world.

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[1] C. H. K. Williamson and R. Govardhan, Annu. Rev. Fluid Mech. **36**, 413 (2004).
[2] J. Holmes, *Wind Loading of Structures* (Taylor & Francis, London, 2007).
[3] G. Taylor, Proc. R. Soc. A **214**, 158 (1952).
[4] M. J. Lighthill, J. Fluid Mech. **9**, 305 (1960).
[5] D. Tam, J. W. M. Bush, M. Robitaille, and A. Kudrolli, Phys. Rev. Lett. **104**, 184504 (2010).
[6] L. Huang, J. Acoust. Soc. Am. **97**, 3642 (1995).
[7] T. Pedley and X. Luo, Theor. Comput. Fluid Dyn. **10**, 277 (1998).
[8] S. Alben and M. J. Shelley, Phys. Rev. Lett. **100**, 074301 (2008).
[9] C. Brennen and H. Winet, Annu. Rev. Fluid Mech. **9**, 339 (1977).
[10] C. Py, E. de Langre, and B. Moulia, J. Fluid Mech. **568**, 425 (2006).
[11] M. Ghisalberti and H. M. Nepf, J. Geophys. Res. **107**, 3011 (2002).
[12] L. B. Jia, F. Li, X. Z. Yin, and X. Y. Yin, J. Fluid Mech. **581**, 199 (2007); L. Schouveiler and C. Eloy, Phys. Fluids **21**, 081703 (2009); S. Michelin and S. G. Llewellyn Smith, J. Fluids Struct. **25**, 1136 (2009); L. Zhu, J. Fluid Mech. **635**, 455 (2009); L. Ristroph and J. Zhang, Phys. Rev. Lett. **101**, 194502 (2008).
[13] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon Press, London, 1959), section 19; B. Coleman, E. Dill, M. Lembo, Z. Lu, and I. Tobias, Arch. Ration. Mech. Anal. **121**, 339 (1993).
[14] L. Schouveiler, C. Eloy, and P. Le Gal, Phys. Fluids **17**, 047104 (2005).
[15] D. Lisoski, Ph.D. thesis, California Institute of Technology, 1993.
[16] M. Sherry, D. Jacono, J. Sheridan, R. Mathis, and I. Marusic, in *Sixth International Symposium on Turbulence and Shear Flow Phenomena* (Seoul National University, Seoul, Korea, 2009); D. Wilhelm and L. Kleiser, J. Sci. Comput. **17**, 619 (2002).
[17] If the pages were to stick together without sliding, the beam's thickness would scale with N, resulting in a bending modulus that scales with N^3.
[18] J. W. S. Rayleigh, *The Theory of Sound* (Dover, New York, 1945), Vol. 1, Chap. VIII.