Anyons from Strings

Luca Mezincescu[1] and Paul K. Townsend[2]
[1]Department of Physics, University of Miami, Coral Gables, Florida 33124, USA
[2]Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
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The Nambu-Goto string in a three-dimensional (3D) Minkowski spacetime is quantized preserving Lorentz invariance and parity. The spectrum of massive states contains anyons. An ambiguity in the ground state energy is resolved by the 3D $\mathcal{N} = 1$ Green-Schwarz superstring, which has massless ground states describing a dilaton and dilatino, and first-excited states of spin $1/4$.

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A standard claim of string theory texts is that a free string cannot be consistently quantized below its critical dimension, preserving Lorentz invariance, without the introduction of an additional "Liouville" variable that is absent classically. Here we show that strings in a three-dimensional (3D) spacetime are an exception to this rule. Specifically, we show that both the 3D Nambu-Goto string and the $\mathcal{N} = 1$ 3D Green-Schwarz (GS) superstring [1] may be quantized, preserving both Lorentz invariance and parity, without the introduction of any additional variables. It turns out that the spectrum of these strings includes anyons, i.e., particles of spin s such that $2s$ is not an integer [2,3]. Specifically, we find anyons in the bosonic string spectrum at level 2 or 3, depending on the choice of ground state energy. The superstring is massless in its ground state, has spin $1/4$ at level 1 and other "semions" (particles of spin $1/4 + n/2$ for integer n) at level 2.

One may ask why the existence of these new 3D quantum strings has not previously been noticed (we do not say "string theories" because we do not address here issues of modular invariance or interactions). Part of the answer to this question is surely that a manifestly covariant description of anyons requires fields in representations of some multiple cover of $Sl(2;\mathbb{R})$. The universal cover is required for irrational spin (and potentially for an infinite number of rational spins) implying infinite-component fields [4,5]. However, such fields do not arise in any of the standard approaches to covariant quantization. Although this limitation may be circumvented in the future, at present it is only in the light-cone gauge that one can easily see all possibilities for a consistent quantum theory, and that is the method used here.

We begin with the Hamiltonian form of the Nambu-Goto action for a closed relativistic 3D string of tension T in terms of the canonical three-vector variables $(\mathbb{X}, \mathbb{P})$, which are functions of the world sheet time τ and the string coordinate $\sigma \sim \sigma + 2\pi$:

$$S = \int d\tau \oint \frac{d\sigma}{2\pi}\{\dot{\mathbb{X}}^\mu \mathbb{P}_\mu - \frac{1}{2}\ell[\mathbb{P}^2 + (T\mathbb{X}')^2] - u\mathbb{X}'^\mu \mathbb{P}_\mu\}. \tag{1}$$

The overdot and prime indicate derivatives with respect to τ and σ, respectively, and ℓ and u are Lagrange multipliers for the Hamiltonian and string-reparametrization constraints, respectively. This action involves the Minkowski spacetime metric (with "mostly plus" signature) via the scalars $\mathbb{P}^2$ and $(\mathbb{X}')^2$. The standard Nambu-Goto action is recovered by elimination of the three-momentum $\mathbb{P}$ followed by elimination of ℓ and then u. In addition to the gauge invariances associated to the constraints, the action is invariant under the Poincaré transformations generated by the Noether charges

$$\mathcal{P}_\mu = \frac{1}{2\pi}\oint d\sigma \mathbb{P}_\mu, \quad \mathcal{J}^\mu = \frac{1}{2\pi}\oint d\sigma [\mathbb{X}\wedge\mathbb{P}]^\mu, \tag{2}$$

where $[\mathbb{U}\wedge\mathbb{V}]^\mu = \varepsilon^{\mu\nu\rho}\mathbb{U}_\nu\mathbb{V}_\rho$ for any two three-vectors $\mathbb{U}$ and $\mathbb{V}$, and the invariant antisymmetric tensor ε is defined such that $\varepsilon^{012} = 1$.

We now introduce light-cone coordinates

$$X^\pm = \frac{1}{\sqrt{2}}(\mathbb{X}^1 \pm \mathbb{X}^0), \qquad P_\pm = \frac{1}{\sqrt{2}}(\mathbb{P}_1 \pm \mathbb{P}_0), \tag{3}$$

and set $\mathbb{X}^2 = X$ and $\mathbb{P}_2 = P$. It is convenient to define

$$\begin{aligned} x(\tau) &= \frac{1}{2\pi}\oint d\sigma X, & x^-(\tau) &= \frac{1}{2\pi}\oint d\sigma X^-, \\ p(\tau) &= \frac{1}{2\pi}\oint d\sigma P, & p_+(\tau) &= \frac{1}{2\pi}\oint d\sigma P_+ \end{aligned} \tag{4}$$

and

$$\begin{aligned} \bar{X} &= X - x, & \bar{X}^- &= X - x^-, \\ \bar{P} &= P - p, & \bar{P}_+ &= P_+ - p_+. \end{aligned} \tag{5}$$

The light-cone gauge is defined by the choice

$$X^+ = \tau, \qquad P_- = p_-(\tau), \tag{6}$$

where $p_-(\tau)$ is a nonzero function of τ only. This choice leaves only the residual global gauge invariance that shifts the origin of the angular string coordinate σ. In this gauge, the Hamiltonian constraint imposed by ℓ may be solved for P_+:

$$P_+ = -\frac{1}{2p_-}[P^2 + (TX')^2]. \quad (7)$$

One also finds that $\bar{X}^-$ is a Lagrange multiplier imposing the constraint $u' = 0$, which has the solution $u = u_0(\tau)$. The final result is the Lagrangian

$$L = \left\{\dot{x}p + \dot{x}^- p_- + \oint \frac{d\sigma}{2\pi}\dot{\bar{X}}\bar{P}\right\} - H - u_0 \oint \frac{d\sigma}{2\pi}\bar{X}'\bar{P}, \quad (8)$$

where the Hamiltonian is

$$H = -p_+ = \frac{1}{2p_-}\left[p^2 + \oint \frac{d\sigma}{2\pi}\{\bar{P}^2 + (T\bar{X}')^2\}\right]. \quad (9)$$

As expected, there is a residual global constraint imposed by u_0. In the light-cone gauge, the Poincaré Noether charges of (29) are

$$\mathcal{P} = p, \qquad \mathcal{P}_- = p_- \qquad \mathcal{P}_+ = -H,$$
$$\mathcal{J} = x^- p_- + \tau H, \qquad \mathcal{J}^+ = \tau p - x p_-$$
$$\mathcal{J}^- = -x^- p - xH + \oint \frac{d\sigma}{2\pi}[\bar{X}\bar{P}_+ - \bar{X}^-\bar{P}]. \quad (10)$$

The two Poincaré invariants are

$$-\mathcal{P}^2 \equiv \mathcal{M}^2 = \oint \frac{d\sigma}{2\pi}[\bar{P}^2 + (T\bar{X}')^2],$$
$$\mathcal{P}\cdot\mathcal{J} \equiv \Lambda = p_- \oint \frac{d\sigma}{2\pi}[\bar{X}\bar{P}_+ - \bar{X}^-\bar{P}]. \quad (11)$$

We now Fourier expand the canonical pair $(\bar{X}, \bar{P})$ by writing

$$\bar{P} - T\bar{X}' = \sqrt{2T}\sum_{n=1}^{\infty}[e^{in\sigma}\alpha_n + e^{-in\sigma}\alpha_n^*],$$
$$\bar{P} + T\bar{X}' = \sqrt{2T}\sum_{n=1}^{\infty}[e^{in\sigma}\tilde{\alpha}_n^* + e^{-in\sigma}\tilde{\alpha}_n]. \quad (12)$$

The Lagrangian (8) becomes

$$L = \dot{x}p + \dot{x}^- p_- + i\sum_{n=1}^{\infty} n^{-1}(\dot{\alpha}_n\alpha_n^* + \dot{\tilde{\alpha}}_n\tilde{\alpha}_n^*)$$
$$- \frac{1}{2p_-}(p^2 + \mathcal{M}^2) + u_0\sum_{n=1}^{\infty}(\alpha_n^*\alpha_n - \tilde{\alpha}_n^*\tilde{\alpha}_n), \quad (13)$$

where

$$\mathcal{M}^2 = 2T\sum_{n=1}^{\infty}(\alpha_n^*\alpha_n + \tilde{\alpha}_n^*\tilde{\alpha}_n). \quad (14)$$

Observe that the other Poincaré invariant Λ of (11) depends on $\bar{X}^-$ as well as the canonical variables of the final action, but the equation of motion of u in the original action reduces in the light-cone gauge to

$$p_-(\bar{X}^-)' = -\bar{X}'P, \quad (15)$$

which allows us to express $\bar{X}^-$ in terms of the Fourier coefficients of $(\bar{X}, \bar{P})$. We pass over the details, which are similar to those for the critical string [6]; the final result is

$$\Lambda = \sqrt{2T}(\lambda + \tilde{\lambda}), \quad (16)$$

where λ depends only on the α_n and $\tilde{\lambda}$ is the same expression but in terms of the $\tilde{\alpha}_n$. Explicitly,

$$\lambda = \sum_{n=1}^{\infty}\frac{i}{n}(\alpha_n^*\beta_n - \alpha_n\beta_n^*), \quad (17)$$

where

$$\beta_n = \frac{1}{2}\sum_{m=1}^{n-1}\alpha_m\alpha_{n-m} + \sum_{n>m}\alpha_m\alpha_{n-m}^*, \quad (18)$$

and similarly for $\tilde{\lambda}$.

To quantize we promote the canonical variables to operators satisfying canonical commutation relations. The nonzero commutators are

$$[x^-, p_-] = [x, p] = i \qquad [\alpha_n, \alpha_n^\dagger] = [\tilde{\alpha}_n, \tilde{\alpha}_n^\dagger] = n. \quad (19)$$

The constraint imposed by u_0 becomes the level-matching condition in the quantum theory:

$$N = \tilde{N}, \qquad N = \sum_{n=1}^{\infty}\alpha_n^\dagger\alpha_n, \qquad \tilde{N} = \sum_{n=1}^{\infty}\tilde{\alpha}_n^\dagger\tilde{\alpha}_n. \quad (20)$$

Taking this into account, the mass-squared operator is

$$\mathcal{M}^2 = 2T(2N - a), \quad (21)$$

where a is an arbitrary constant arising from operator ordering ambiguities. Similarly, the operator Λ is as in (16) but now with $\alpha_n^* \to \alpha_n^\dagger$ and hence $\beta_n^* \to \beta_n^\dagger$, and similarly for $\tilde{\lambda}$.

The quantum Lorentz generators are

$$\mathcal{J} = \frac{1}{2}\{x^-, p_-\} + \tau H, \qquad \mathcal{J}^+ = \tau p - x p_-,$$
$$\mathcal{J}^- = -x^- p - \frac{1}{2}\{x, H\} + \Lambda/p_-. \quad (22)$$

In principle, there are operator ordering ambiguities in these expressions but they are fixed by the requirements of hermiticity and closure of the Lorentz algebra. It is straightforward to verify that the charges as given satisfy the required commutation relations:

$$[\mathcal{J}, \mathcal{J}^\pm] = \pm i\mathcal{J}^\pm, \qquad [\mathcal{J}^+, \mathcal{J}^-] = i\mathcal{J}. \quad (23)$$

This should not be a surprise because the "dangerous" commutators vanish "by default" in three dimensions.

Because the Poincaré invariant operators $\mathcal{M}^2$ and Λ commute, they may be simultaneously diagonalized. It follows that the space spanned by the level-N states is an invariant subspace of Λ. At levels $N = 0, 1$ there is a single state that is annihilated by Λ, so nonzero eigenvalues of Λ can occur only for $N \geq 2$. The eigenvalues of Λ at each level divided by the mass of the level are the "relativistic helicities" at that level. The four states at level 2 have helicities

$$\left(0, 0, \pm\frac{3}{\sqrt{4-a}}\right), \tag{24}$$

which implies that $a < 4$. The 9 states at level 3 have helicities

$$\left(0, 0, 0, \pm\sqrt{\frac{179}{12(6-a)}}, \pm\sqrt{\frac{179}{12(6-a)}}, \pm\sqrt{\frac{179}{3(6-a)}}\right). \tag{25}$$

Observe that nonzero helicities appear in parity doublets of opposite helicity, so parity is preserved by the quantization. For any $a \leq 4$ there is an anyon in either level 2 or level 3. It would be natural to choose $a = 0$, so that the level-0 state is massless; in this case we have a massive scalar at level 1, spin $3/2$ at level 2 and irrational spin anyons at level 3. Another natural choice is $a = 2$ because this makes the level-0 state a tachyon and the level-1 state a massless scalar, which might be interpretable as a dilaton, as for the critical bosonic string; in this case there is a massive state with irrational spin at level 2. We expect that irrational spins are generic in higher levels.

The freedom in the choice of ground state energy might be considered a defect of the bosonic model. In any case, this freedom is absent from the 3D $\mathcal{N} = 1$ GS superstring, to which we now turn. The action may be constructed by first making the replacement

$$d\mathbb{X}^\mu \to \Pi^\mu \equiv d\mathbb{X}^\mu + i\bar{\Theta}\Gamma^\mu d\Theta \tag{26}$$

where Γ^μ are real 3D Dirac matrices and Θ is a real anticommuting two-component spinor field, with Majorana conjugate $\bar{\Theta} = \Theta^T\Gamma^0$. Then we add to the action a Wess-Zumino term for the supertranslation algebra associated to the closed super-Poincaré invariant superspace three-form $\Pi^\mu d\bar{\Theta}\Gamma_\mu d\Theta$. This gives rise to the following "quasi-Hamiltonian" form of the action

$$S[\mathbb{X}, \mathbb{P}; \ell, u] = \int d\tau \oint \frac{d\sigma}{2\pi}\{\Pi^\mu_\tau \mathbb{P}_\mu - \frac{1}{2}\ell[\mathbb{P}^2 + (T\Pi_\sigma)^2] - u\Pi^\mu_\sigma \mathbb{P}_\mu + iT(\dot{\mathbb{X}}^\mu\bar{\Theta}\Gamma_\mu\Theta' - \mathbb{X}'^\mu\bar{\Theta}\Gamma_\mu\dot{\Theta})\}, \tag{27}$$

where Π_τ and Π_σ are the $d\tau$ and $d\sigma$ components of the world sheet one-form induced by Π. The term linear in T is the Wess-Zumino term, and we have chosen its coefficient to ensure invariance of the action under the following fermionic gauge invariance ("κ symmetry") with anticommuting Majorana spinor parameter κ:

$$\begin{aligned}\delta_\kappa\Theta &= \Gamma_\mu(\mathbb{P}^\mu - T\Pi^\mu_\sigma)\kappa,\\ \delta_\kappa\mathbb{X}^\mu &= -i\bar{\Theta}\Gamma^\mu\delta_\kappa\Theta,\\ \delta_\kappa\mathbb{P}_\mu &= 2iT\bar{\Theta}'\Gamma_\mu\delta_\kappa\Theta,\ \delta_\kappa u = -T\delta_\kappa\ell,\\ \delta_\kappa\ell &= -4i\bar{\kappa}[\dot{\Theta} + (\ell T - u)\Theta'].\end{aligned} \tag{28}$$

Observe that $\Gamma_\mu(\mathbb{P}^\mu - T\Pi^\mu_\sigma)$ has zero determinant on the surface defined by the constraints. This means that only one of the two independent components of κ has any effect, so that only one real component of Θ can be "gauged away". The Poincaré Noether charges are now

$$\begin{aligned}\mathcal{P}_\mu &= \oint\frac{d\sigma}{2\pi}\{\mathbb{P}_\mu + iT\bar{\Theta}\Gamma_\mu\Theta'\},\\ \mathcal{J}^\mu &= \oint\frac{d\sigma}{2\pi}\Big\{[\mathbb{X}\wedge(\mathbb{P} + iT\bar{\Theta}\Gamma\Theta')] + \frac{i}{2}\bar{\Theta}\Theta(\mathbb{P} - T\mathbb{X}')\Big\}^\mu.\end{aligned} \tag{29}$$

The supersymmetry Noether charges are ($\alpha = 1, 2$)

$$\mathcal{Q}^\alpha = \sqrt{2}\oint\frac{d\sigma}{2\pi}\Big\{[\mathbb{P}_\mu - T\mathbb{X}'_\mu]\Gamma^\mu\Theta - \frac{i}{2}(\bar{\Theta}\Theta)\Theta'\Big\}^\alpha. \tag{30}$$

The κ symmetry variation of all these charges vanishes on the constraint surface.

To go to the light-cone gauge we proceed as before but now we also fix the κ symmetry by imposing the usual condition [1]

$$\Gamma^+\Theta = 0, \qquad \Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^1 \pm \Gamma^0). \tag{31}$$

For the representation $\Gamma^\mu = (i\sigma_2, \sigma_1, \sigma_3)$ of the 3D Dirac matrices, this condition implies that

$$\Theta = \left(1/\sqrt{2\sqrt{2}p_-}\right)\begin{pmatrix}\theta\\0\end{pmatrix} \tag{32}$$

for some anticommuting world sheet function $\theta(\tau, \sigma)$. As for the bosonic variables, it is convenient to define

$$\bar{\theta} = \theta - \vartheta, \qquad \vartheta(\tau) = \frac{1}{2\pi}\oint d\sigma\theta. \tag{33}$$

There should be no confusion with the notation for a conjugate spinor as θ is not a two-component spinor. Proceeding as before, but now with the additional Fourier expansion

$$\bar{\theta} = \sum_{n=1}^\infty[e^{in\sigma}\theta_n + e^{-in\sigma}\theta_n^*], \tag{34}$$

we end up with the Lagrangian

$$L = \dot{x}p + \dot{x}^-p_- + \frac{i}{2}\vartheta\dot{\vartheta} + \sum_{n=1}^\infty[n^{-1}(\dot{\alpha}_n\alpha_n^* + \dot{\tilde{\alpha}}_n\tilde{\alpha}_n^*) + \theta_n^*\dot{\theta}_n] - \frac{1}{2p_-}(p^2 + \mathcal{M}^2) + u_0\sum_{n=1}^\infty(\alpha_n^*\alpha_n - \tilde{\alpha}_n^*\tilde{\alpha}_n + n\theta_n^*\theta_n), \tag{35}$$

where

$$\mathcal{M}^2 = 2T\sum_{n=1}^\infty(\alpha_n^*\alpha_n + \tilde{\alpha}_n^*\tilde{\alpha}_n + n\theta_n^*\theta_n). \tag{36}$$

The supersymmetry charges in the light-cone gauge are

$$\begin{aligned} \mathcal{Q}^1 &= \sqrt{\frac{1}{\sqrt{2}p_-}}\left\{p\vartheta + \sqrt{2T}\sum_{n=1}^{\infty}(\alpha_n\theta_n^* + \alpha_n^*\theta_n)\right\} \\ \mathcal{Q}^2 &= \sqrt{\sqrt{2}p_-}\,\vartheta. \end{aligned} \tag{37}$$

The Lorentz Noether charges in the light-cone gauge are as in (10) but with a different expression for Λ. We will not give this expression here because the super-Poincaré invariant is not $\Lambda \equiv \mathcal{P}\cdot\mathcal{J}$ but rather

$$\Omega \equiv \mathcal{P}\cdot\mathcal{J} + \frac{i}{4}\bar{\mathcal{Q}}\mathcal{Q}. \tag{38}$$

To compute Ω we need the analog of (15), which is

$$p_-(\bar{X}^-)' = -\bar{X}'P + \frac{i}{2}\theta\bar{\theta}'. \tag{39}$$

We again pass over the details; the final result is

$$\Omega = \sqrt{2T}\left[\lambda + \tilde{\lambda} + \sum_{n=1}^{\infty}\frac{i}{n}(\alpha_n^*\gamma_n - \alpha_n\gamma_n^*)\right], \tag{40}$$

where λ and $\tilde{\lambda}$ are as before, and

$$\gamma_n = \sum_{m=1}^{n-1}(n-m)\theta_m\theta_{n-m} + \sum_{m>n}\left(m - \frac{n}{2}\right)\theta_{m-n}^*\theta_m. \tag{41}$$

Note that although the Poincaré invariant Λ depends on the fermion zero mode ϑ, this mode cancels from the super-Poincaré invariant Ω.

We quantize as before, replacing the Grassmann odd variables by operators obeying canonical anticommutation relations. The nonzero anticommutators are

$$\{\vartheta, \vartheta\} \equiv 2\vartheta^2 = 1, \qquad \{\theta_n, \theta_n^\dagger\} = 1. \tag{42}$$

The level-matching constraint is now

$$\tilde{N} = N + \nu, \qquad \nu = \sum_{n=1}^{\infty} n\theta_n^\dagger\theta_n. \tag{43}$$

Taking this into account, one finds that

$$\{\mathcal{Q}^\alpha, \mathcal{Q}^\beta\} = \begin{pmatrix} \sqrt{2}H & p \\ p & \sqrt{2}p_- \end{pmatrix} = (\Gamma^\mu\Gamma^0)^{\alpha\beta}\mathcal{P}_\mu, \tag{44}$$

provided that the ground state energy is zero, which means that

$$\mathcal{M}^2 = 4T(N+\nu) = 4T\tilde{N}. \tag{45}$$

The Lorentz charges close under commutation exactly as for the 3D bosonic string, and it may be verified that they have the expected commutators with the supersymmetry charges: $[J^\mu, \mathcal{Q}] = -\frac{i}{2}\Gamma^\mu\mathcal{Q}$.

Because of the fermion zero mode, there is a double degeneracy at all levels, and because this zero mode cancels from Ω the degeneracy is that implied by the supersymmetric pairing of one bose with one Fermi state, corresponding to the ± 1 eigenspaces of $\sqrt{2}\vartheta$. In particular, the ground state is doubly degenerate, and we may interpret the corresponding massless particles in the spectrum as a dilaton and dilatino; note that although helicity is not defined for massless states, there is still a distinction between bosons and fermions [2,7]. This result is consistent with what one would expect from the low energy effective supergravity of a 3D $\mathcal{N} = 1$ superstring theory because neither the metric nor a two-form potential propagate modes in this context.

All higher levels are massive and the states at any given level span an invariant subspace of Ω, the eigenvalues of which are the superhelicities after division by the mass of the level. The superhelicity is just the average of the two helicities in a supermultiplet, which differ by $\frac{1}{2}$, and nonzero superhelicites appear in parity doublets of opposite sign. At level 1 there are four states, which combine into two massive supermultiplets of superhelicity zero, each of which contains helicities $\pm\frac{1}{4}$. At level 2 there are 16 states, and hence 8 eigenvalues of Ω. Four are zero, giving four spin-$\frac{1}{4}$ particles of helicities $\pm\frac{1}{4}$. The four nonzero eigenvalues of Ω are $(\frac{3}{2}, \frac{3}{2}, -\frac{3}{2}, -\frac{3}{2})$, corresponding to doubly degenerate supermultiplets of helicities $(\frac{7}{4}, \frac{5}{4})$ and $(-\frac{7}{4}, -\frac{5}{4})$. At levels 1 and 2 we thus find semion supermultiplets, which were first investigated in Ref. [8]. At higher levels we expect generic anyon supermultiplets, see, e.g., Ref. [9].

There is a classical 3D $\mathcal{N} = 2$ GS superstring and we believe that our quantum results will extend to this case. If so, there will be four massless states, interpretable as a dilaton and axion and their superpartners. Of course, it remains to be seen whether any of these free quantum 3D strings can interact to yield new 3D string theories.

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