Anisotropic Mass Density by Radially Periodic Fluid Structures

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This Letter reports physical realization of acoustic metamaterials with anisotropic mass density. These metamaterials consist of a superlattice of two fluidlike components radially periodic. Several structures are spectroscopically characterized at large wavelengths (homogenization limit) by studying the acoustic resonances existing in the circular cavity where they are embedded. This characterization method allows us to extract the diagonal components of the sound speed tensor. Analytical expressions describing the anisotropic behavior as a function of the corrugation parameter are also developed and their predictions are in agreement with measurements.

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Acoustic metamaterials with anisotropic mass density have increasing interest due to their necessity in the fabrication of novel acoustic devices, especially for those proposed within the framework of transformation acoustics [1]. More specifically, cloaking shells [2], acoustic hyperlenses [3], or radial sonic crystals [4], among others, require mass anisotropy for their realization. Though anisotropy is not a property of common fluids, it can be obtained by using the metamaterial concept. It has been demonstrated that fluid-fluid or fluid-solid periodic composites of subwavelength dimensions behave as acoustic materials with anisotropic mass density [5–8].

Most of the above mentioned devices have radial symmetry and, therefore, their mass density tensor is described in polar coordinates by a diagonal tensor. Based on classical composite methods [9], a way to obtain this anisotropy has been recently reported [10,11]. It was shown that a multilayered cylindrical shell of two alternating homogeneous and isotropic fluid materials behaves, in the low frequency limit, like a fluidlike material with a cylindrically anisotropic mass density.

The problem of engineering cylindrically multilayered fluid-fluid composite is still unsolved, due to the difficulty in avoiding the mixing of fluid materials with different physical parameters. By using the metamaterial concept, the authors in Ref. [11] proposed a design of fluidlike materials based on homogenization properties of solid structures made of cylindrical scatterers [12]. A similar approach was employed by Farhat *et al.* to realize a cloak for surface waves in water [13]. However, a huge number of cylinders was required for each layer. More recently, Li and co-workers [3] have employed a fluid-solid multilayered structure to get a metamaterial with strongly anisotropic mass density. But this approach is not suitable to get finite anisotropy ratios.

In this Letter we introduce physical realization and experimental characterization of three acoustic metamaterials with anisotropic mass density. They are based on multilayered structures made of two fluidlike materials with different mass densities. Two main achievements are here reported. The first one is to realize a stable 2D fluid-fluid multilayered cylinder. The second one is to experimentally demonstrate that, in the low frequency limit, it behaves like an anisotropic fluidlike medium. This demonstration has been performed by studying the cylinder's resonances.

The proposed 2D multilayered fluid-fluid structure is obtained inside a planar wave guide made of aluminum, where a circular cavity is drilled with an embedded corrugated structure, as shown in Fig. 1. In this cavity two alternating regions of heights h_1 and h_2 and widths d_1 and d_2 are defined. This height discontinuity in a waveguide can be described, in a first approximation, by stating that region 1 and 2 are two different fluids with the same sound speed $c_1 = c_2 = c_b$, where c_b is the sound speed in the background, and a mass density mismatch given by [14]

$$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}. \quad (1)$$

This expression is valid only for the fundamental mode. Although a more accurate description could be obtained by considering the coupling through evanescent modes. For the purposes of this work a simple approach based on Eq. (1) will be sufficient.

Note that the multilayered structure considered here is made of two alternating air regions of height h_1 and h_2. In other words, the aluminum is used here just as a container and, due to the huge impedance mismatch with air, no sound propagation inside Al is considered.

The corrugated structure shown in Fig. 1 can be described by a cylindrical periodic multilayer of two alternating materials, where material 1(2) is a homogeneous and isotropic fluid of width $d_1(d_2)$ and acoustic parameters ρ_1, $B_1(\rho_2, B_2)$, where ρ_i and B_i are the density and bulk modulus of material i $i = (1, 2)$, respectively. This

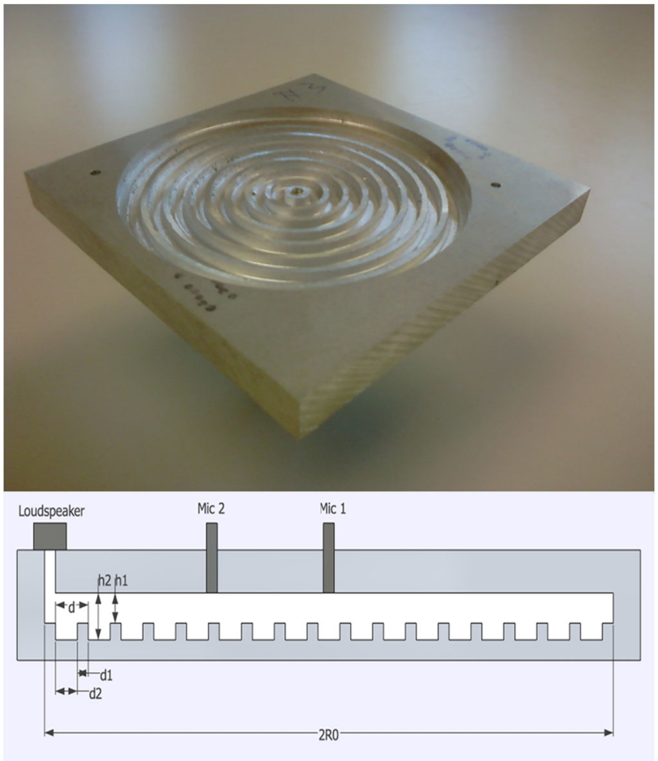

FIG. 1 (color online). Upper panel: One of the four samples built and characterized in the present work. The system of grooves creates a periodic fluid-fluid composite. Lower panel: Schematic view of the experimental set up. The loud speaker excites a sound field inside the cavity formed by the aluminum boundaries. The frequencies of the excited resonances are used to determine de effective parameters of the metamaterial.

structure behaves, in the low frequency limit, as an anisotropic fluidlike material whose effective acoustic parameters are given as follows [10,11]:

$$\rho_r = \frac{1}{d}[d_1\rho_1 + d_2\rho_2] = \frac{\rho_1}{d}\left[d_1 + d_2\frac{h_1}{h_2}\right], \qquad (2)$$

$$\rho_\theta^{-1} = \frac{1}{d}[d_1\rho_1^{-1} + d_2\rho_2^{-1}] = \frac{\rho_1^{-1}}{d}\left[d_1 + d_2\frac{h_2}{h_1}\right], \qquad (3)$$

$$B^{-1} = \frac{1}{d}[d_1B_1^{-1} + d_2B_2^{-1}] = \frac{\rho_\theta^{-1}}{c_b^2}, \qquad (4)$$

where the last expression has been obtained by using $B_i = \rho_i c_i^2$ and $c_1 = c_2 = c_b$.

Sound propagation inside this medium is determined by the radial component of the sound speed c_r and the anisotropy factor γ obtained as follows

$$c_r^2 \equiv B/\rho_r = c_b^2\frac{d^2}{[d_1 + d_2\frac{h_1}{h_2}][d_1 + d_2\frac{h_2}{h_1}]} \qquad (5)$$

$$\gamma^2 \equiv \rho_r\rho_\theta^{-1} = \frac{1}{d^2}\left[d_1 + d_2\frac{h_1}{h_2}\right]\left[d_1 + d_2\frac{h_2}{h_1}\right]. \qquad (6)$$

From these relations the angular component of the sound speed is trivially obtained,

$$c_\theta = \gamma c_r = c_b. \qquad (7)$$

This result can also be obtained from Eq. (4) and the definition of c_θ in terms of the bulk modulus; i.e., $c_\theta^2 \equiv B/\rho_\theta = c_b^2$. Note that this result is exact; that is, it does not depend on the accuracy in the determination of the equivalent density of regions 1 or 2.

The characterization of these structures is performed by measuring the frequencies of the cavity's resonances. By assuming harmonic time dependency the pressure field, P, inside a homogeneous and anisotropic cylinder, like in the isotropic case [15], can be expressed in terms of Bessel functions [16]

$$P(r, \theta; \nu) = \sum_{q=0}^{\infty} A_q J_{\gamma q}(2\pi\nu r/c_r)\cos(q\theta + \phi_q), \qquad (8)$$

where ν is the linear frequency of the sound wave, A_q and ϕ_q are integration constants.

This expression is used to obtain the cavity modes, which are determined by the condition of rigid walls at $r = R_0$ [15]

$$\frac{\partial P}{\partial r}\Big|_{r=R_0} = 0, \qquad (9)$$

which in this case leads to

$$J'_{q\gamma}(2\pi\nu_{qi}R_0/c_r) = 0, \qquad i = 0, 1, 2, \cdots \qquad (10)$$

where $J'_x(\cdot)$ stands for the derivative of the Bessel function of real order x. The i subindex refers to the ith zero of the corresponding Bessel function. In the present work only fundamental resonances $i = 0$ have been analyzed, so that hereafter the subindex in ν will give reference to the q value.

For the case of mode $q = 0$ Eq. (10) does not contain any dependence on γ, so that the only unknown of that equation is c_r, which is obtained from

$$c_r = 2\pi\nu_0 R_0/3.832, \qquad (11)$$

where 3.832 being the first zero of $J'_0(\cdot)$ [17] and ν_0 is the resonant frequency assigned to the $q = 0$ mode.

Once c_r is known the value of γ is determined from the $q \neq 0$ resonances. Obviously, if γ is not known the zeros of $J_{\gamma q}(\cdot)$ cannot be either; however, they can be obtained by solving numerically the transcendental equation

$$f(\gamma) \equiv J'_{q\gamma}(2\pi\nu_q R_0/c_r) = 0, \qquad (12)$$

which allows us to determine a set of values γ_q from the measurement of the resonant frequencies ν_q.

Four samples were built and characterized by using the procedure explained above. All the samples have the same cavity radius $R_0 = 52$ mm, the same layer thicknesses $d_1 = 2$ mm and $d_2 = 4$ mm, and the same height of region 2, $h_2 = 7$ mm. The only parameter that changes from sample to sample is the height h_1 of region 1, which takes the values $h_1 = 2$, 3, 4, and 7 mm for the samples 1,2,3, and 4, respectively. Note that the sample 4 has $h_1 = h_2 = 7$ mm, which makes it an isotropic cylindrical cavity employed here to obtain the background speed of sound c_b.

The experimental set up is shown in the lower panel of Fig. 1: A loudspeaker excites a sound field inside the cavity through a hole drilled on the upper tap. Two microphones are placed to measure the excited field; Mic1 is located at $r = 0$ and Mic2 at $r \neq 0$.

The excited sound field is a band-limited white noise covering the range 1.5 kHz up to 5.5 kHz, because the fundamental resonances of the different samples are expected in this frequency range. The fact that the larger height is $h_2 = 7$ mm for all the samples grants no z oscillations for propagating modes up to a frequency of $\nu_c \approx 25$ kHz, which is larger than the maximum working frequency $\nu_{\max} = 5$ kHz. Therefore, the problem can be considered as two-dimensional.

Spectra are taken simultaneously in two points: One at $r = 0$ (Mic 1 in Fig. 1) and another at $r \neq 0$ (Mic 2). The sound field measured at position $r = 0$ is only due to the contribution of the zeroth-order Bessel function, because $J_{q\gamma}(0) = 0$ for $q \neq 0$. However the sound field recorded at position $r \neq 0$ contains all the multipolar components; their symmetry can be easily determined for low q as follows. Firstly, the symmetry is initially assumed and the anisotropy factor γ is computed with Eq. (12). Since only three resonances are studied it is easy to check experimentally the consistence of the initial guess by rotating the sample.

TABLE I. Experimental values of the radial component of the sound speed tensor c_r and of the anisotropy factor γ_q obtained for each sample. The $q = 1, 2, 3$ index refers to the multipolar q mode from which the anisotropy factor was obtained. The main source of error in the parameter is the determination of the peak frequency, for which a Lorenz shape was assumed for each resonance.

Sample	$c_r(m/s)$	γ_1	γ_2	γ_3
1	230 ± 17	1.58 ± 0.15	1.50 ± 0.15	1.55 ± 0.14
2	273 ± 13	1.35 ± 0.13	1.3 ± 0.1	1.25 ± 0.12
3	298 ± 15	1.2 ± 0.1	1.15 ± 0.08	1.16 ± 0.09
4	341 ± 9	1	1	1

Figure 2 shows the spectra taken at $r = 0$ for all the samples. The peak frequency in the spectra allows the determination of c_r by means of Eq. (11). Their values are given in the second column of Table I.

The spectra measured at $r \neq 0$ are shown in Fig. 3, where the polar symmetry of each resonance is also indicated. For a given frequency the trascendental Eq. (12) is solved and the values obtained for γ_q are summarized in Table I. Note that for a given sample, the γ_q values obtained for resonances $q = 1, 2, 3$ are almost the same.

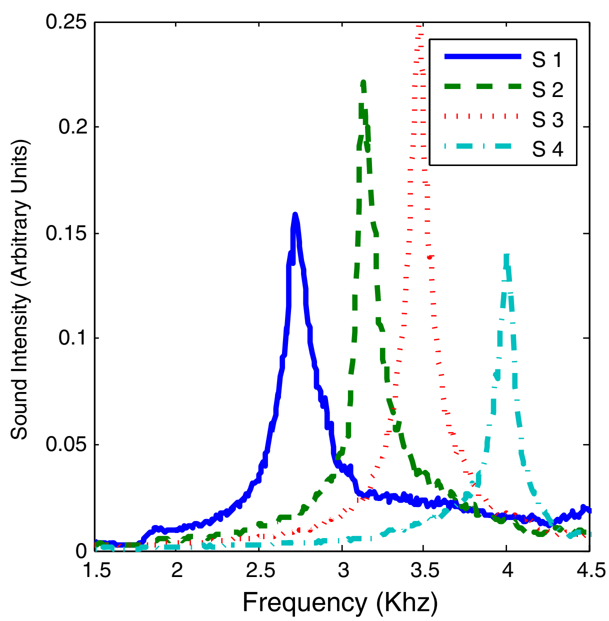

FIG. 2 (color online). Sound spectra measured by microphone located at the center of the cavity (Mic 1 in Fig. 1) for all the samples. The resonance only depend on the radial component of the sound speed tensor c_r, which is obtained from the frequency at the resonant peak [see Eq. (11)].

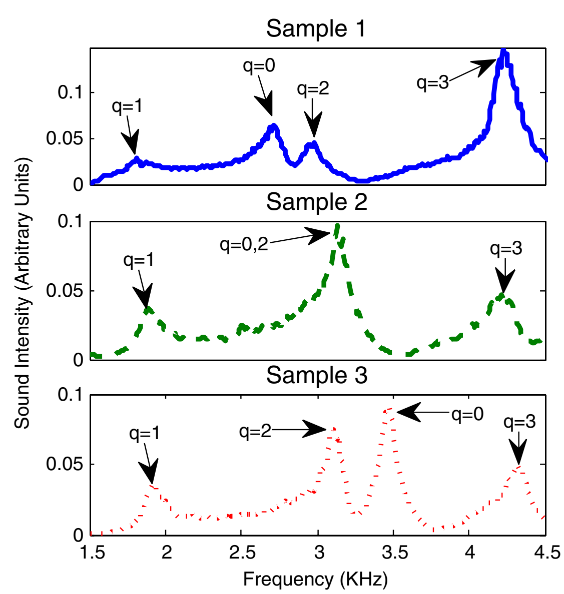

FIG. 3 (color online). Sound spectra measured by the microphone located at $r \neq 0$ (Mic 2 in Fig. 1) for the three anisotropic samples (samples 1 to 3 in Table I). The anisotropy factor γ is obtained from these spectra by solving Eq. (12). The multipolar symmetry of each mode is also indicated.

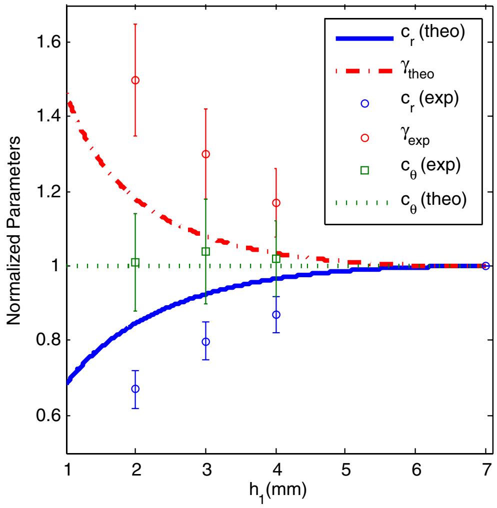

FIG. 4 (color online). Symbols with error bars represent experimental data for the components of the sound speed tensor relative to that of the background (blue circles and green squares) and the anisotropy factor (red circles). The lines describe the corresponding theoretical values given by the simple model in Eqs. (5)–(7) [see text].

This result supports the model employed in this work, showing that long wavelength sound propagation within the cavity is described by Eq. (8) and, consequently, that the effective medium is an anisotropic fluidlike medium.

Figure 4 summarizes the results obtained for the components of the sound speed tensor relative to that of the background and the anisotropy factor. This figure also depicts the theoretical values given by the simple analytical expressions in Eqs. (5)–(7). The experimental value of the angular component of the sound speed has been determined by $c_\theta = \gamma c_r$.

It should be noted that the disagreement obtained between theory and experimental data for c_r and γ is mainly due to the fact that the analytical model employed is very simple and, as already has been pointed out, a better agreement could be obtained by considering the interaction between evanescent modes. A similar effect was previously discussed for two-dimensional structures [18,19]. Note, however, that the experimental value obtained for c_θ strongly agrees with the theory, which establishes that it has to be equal to that of the background. This agreement has its origin in the fact that Eq. (7) is exact.

Results in Fig. 4 lead us to the conclusion that 2D anisotropic fluidlike materials can be easily achieved by using the method introduced here. Therefore, new advanced 2D acoustic devices based on these handmade materials can be envisaged.

In summary, a feasible method to build and characterize fluidlike cylinders with cylindrically anisotropic mass density has been presented. The fabrication method is based on the idea that a corrugated structure with radial symmetry can be described by a fluid-fluid multilayered structure that, in the low frequency limit, behaves like fluidlike cylinder with anisotropic mass density. The method of characterization is based on the frequency resonances of a 2D cylindrical acoustic cavity. With this setup two parameters of the media are directly obtained; the radial speed of sound, c_r, and the anisotropy factor γ. From these two parameters the angular speed of sound c_θ can also be determined, so that the propagation of sound in this medium is completely characterized.

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