Assessing the Polarization of a Quantum Field from Stokes Fluctuations

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We propose an operational degree of polarization in terms of the variance of the Stokes vector minimized over all the directions of the Poincaré sphere. We examine the properties of this second-order definition and carry out its experimental determination. Quantum states with the same standard (first-order) degree of polarization are correctly discriminated by this new measure. We argue that a comprehensive quantum characterization of polarization properties requires a whole hierarchy of higher-order degrees.

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Introduction.—Polarization is a fundamental property of light that has received a lot of attention over the years [1]. Nowadays, the topic is witnessing a revival in interest because of fast developments in both applications and fundamental physics aspects. As polarization is a robust characteristic, relatively simple to manipulate without inducing more than marginal losses, it is not surprising that many experiments at the forefront of quantum optics involve this observable [2].

In classical optics, polarization can be elegantly visualized by using the Poincaré sphere and is determined by the Stokes parameters. These are directly measurable quantities that can be straightforwardly extended to the quantum domain [3].

The classical degree of polarization is simply the length of the Stokes vector. While this provides a very intuitive picture, for complex fields it has serious drawbacks. Indeed, this classical quantity does not distinguish between states having remarkably different polarization properties [4]. In particular, it can be zero for light that cannot be regarded as unpolarized, giving rise to the so-called "hidden polarization" [5]. We stress that this is not a mere academic curiosity, since many quantum states used in the literature suffer from these inconveniences [6].

These flaws have prompted some novel generalizations of the degree of polarization [7–12]. A notion that has been gaining support is to apply a properly chosen distance [13] (entropy can be regarded as a special case [14]). This has the potential advantage of circumventing most of the aforementioned difficulties, while making close contact with other measures introduced to quantify quantum resources [15].

There is, however, a problem with this approach: these distances can be computed (not measured) only after a complete knowledge of the state, which in practice implies a full quantum tomography. In other words, while offering very good properties, they do not have a clear operational meaning.

We adhere to the view that the Stokes variables constitute a natural tool in appraising polarization properties, so they should be the basic building blocks for any practical degree of polarization. One can expect that the problems arising with the classical degree are due to its definition in terms exclusively of first-order moments of the Stokes variables. This may be sufficient for most classical situations, but for quantum fields higher-order correlations are crucial.

Our goal in this Letter is to provide a practical solution to this question. We learn from coherence theory that a full description of interference phenomena may involve a hierarchy of degrees. In this vein, we go beyond the first-order description and look for a second-order degree as the minimum Stokes variance over all directions of the Poincaré sphere. This simple proposal will prove very satisfactory when facing the complications known in this field. We also present a couple of experimental examples confirming the feasibility of our scheme.

As a final remark, let us mention that our new measure is operational in style and is based on the underlying SU(2) symmetry of light polarization. This makes possible a direct translation of our results to other fields where the same symmetry plays an important role, such as cold atoms [16]. It is also well suited for other unitary symmetries, such as $\mathrm{SU}(2)^{\otimes n}$ or SU(3). The former is connected with the polarization of spatial-multimode fields [17], while the latter has recently attracted a lot of attention in relation with near-field optics [18].

Polarization structure of quantum fields.—We begin by briefly recalling some background material. We assume a two-mode quantum field that is described by two complex amplitudes, $\hat{a}_H$ and $\hat{a}_V$, where the subscripts H and V indicate horizontally and vertically polarization modes, respectively. The commutation relations of these operators are standard: $[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk}$, with $j,\ k \in \{H, V\}$. The analysis is greatly simplified if we use the Stokes operators

$$\hat{S}_x = \hat{a}_H \hat{a}_V^\dagger + \hat{a}_H^\dagger \hat{a}_V,$$
$$\hat{S}_y = i(\hat{a}_H \hat{a}_V^\dagger - \hat{a}_H^\dagger \hat{a}_V),$$
$$\hat{S}_z = \hat{a}_H^\dagger \hat{a}_H - \hat{a}_V^\dagger \hat{a}_V, \qquad (1)$$

together with the total photon number $\hat{S}_0 = \hat{N} = \hat{a}_H^\dagger \hat{a}_H + \hat{a}_V^\dagger \hat{a}_V$. The average values of these operators are precisely the classical Stokes parameters. One immediately finds that the components of the Stokes vector $\hat{\mathbf{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)^t$ (where t denotes the transpose) satisfy the commutation relations distinctive of the su(2) algebra: $[\hat{S}_x, \hat{S}_y] = i2\hat{S}_z$ and cyclic permutations. This noncommutability precludes their simultaneous precise measurement, which is expressed by the uncertainty relation

$$(\Delta \mathbf{S})^2 = (\Delta S_x)^2 + (\Delta S_y)^2 + (\Delta S_z)^2 \geq 2\langle \hat{S}_0 \rangle, \qquad (2)$$

with $(\Delta S_k)^2$ $(k = x, y, z)$ being the corresponding variances. In addition, $[\hat{S}_0, \hat{\mathbf{S}}] = 0$, so we can treat each subspace with a fixed number of photons N separately. This can be emphasized if instead of the Fock basis for both polarization modes, $|n\rangle_H |m\rangle_V$ $(n, m = 0, \ldots, \infty)$, we employ the relabeling $|N, k\rangle = |k\rangle_H |N-k\rangle_V$ $(k = 0, 1, \ldots, N)$. In this way, for each fixed N, these states span an SU(2) invariant subspace of dimension $N + 1$.

The standard definition of the degree of polarization for a quantum state $\hat{\varrho}$ is

$$\mathbb{P}_1(\hat{\varrho}) = \frac{|\langle \hat{\mathbf{S}} \rangle|}{\langle \hat{S}_0 \rangle} = \frac{\sqrt{\langle \hat{S}_x \rangle^2 + \langle \hat{S}_y \rangle^2 + \langle \hat{S}_z \rangle^2}}{\langle \hat{S}_0 \rangle}, \qquad (3)$$

where the subscript 1 stresses here that it involves first-order moments of the Stokes variables. We note that for any single-mode state of the form $|\Psi\rangle_H |0\rangle_V$, we get $\mathbb{P}_1 = 1$, which seems unphysical for a variety of reasons. In particular, when $|\Psi\rangle_H \to |0\rangle_H$, we have $\mathbb{P}_1 = 1$ for field states arbitrarily close to the quantum two-mode vacuum. Moreover, unpolarized states according to (3) are determined by $\langle \hat{\mathbf{S}} \rangle = \mathbf{0}$. Nonetheless, there are states fulfilling this latter condition (as, e.g., $|n\rangle_H |n\rangle_V$) that cannot be regarded as unpolarized, as revealed by a number of features. These unwanted consequences indicate the need to go beyond $\mathbb{P}_1$.

Second-order quantum degree of polarization.—From the previous discussion it seems clear that higher-order moments must be taken into account, as advocated by Klyshko [19]. For the time being, we concentrate on the second order: our task is thus to link the resulting fluctuations with our notion of a polarization degree. To this end, we observe that a sensible modification of (3) is easily obtained by replacing $\langle \hat{S}_0 \rangle$ with $[\langle \hat{S}_0 (\hat{S}_0 + 2) \rangle]^{1/2} = \langle \hat{\mathbf{S}}^2 \rangle^{1/2}$ in the denominator. The resulting degree [20]

$$\mathbb{P}'_2(\hat{\varrho}) = \sqrt{1 - \frac{(\Delta \mathbf{S})^2}{\langle \hat{\mathbf{S}}^2 \rangle}}, \qquad (4)$$

contains the desired second-order information (hence the subscript 2) and fixes some of the above-mentioned problems. For example, $\mathbb{P}'_2 < 1$ for every state $|\Psi\rangle_H |0\rangle_V$, and $\mathbb{P}'_2 \to 0$ when $|\Psi\rangle_H \to |0\rangle_H$. However, other bugs still persist. The reason is that (4) does not properly represent the behavior of the fluctuations in phase space. To catch these aspects we propose to use

$$\mathbb{P}_2(\hat{\varrho}) = \sqrt{1 - \inf_{\mathbf{n}} \frac{(\Delta S_{\mathbf{n}})^2}{\frac{1}{3}\langle \hat{\mathbf{S}}^2 \rangle}}, \qquad (5)$$

where $\hat{S}_{\mathbf{n}} = \hat{\mathbf{S}} \cdot \mathbf{n}$, with $\mathbf{n}$ being a unit vector in an arbitrary direction of spherical angles (θ, ϕ). The factor $1/3$ has been introduced for normalization.

To further appreciate this idea, we define the real symmetric 3×3 covariance matrix for the Stokes variables as $\Gamma_{k\ell} = \frac{1}{2}\langle \{\hat{S}_k, \hat{S}_\ell\} \rangle - \langle \hat{S}_k \rangle \langle \hat{S}_\ell \rangle$, where $\{,\}$ is the anticommutator [21]. In terms of this matrix $\mathbf{\Gamma}$, we have $(\Delta S_{\mathbf{n}})^2 = \mathbf{n}^t \mathbf{\Gamma} \mathbf{n}$ and, since $\mathbf{\Gamma}$ is positive definite, the minimum of $(\Delta S_{\mathbf{n}})^2$ exists and it is unique. If we incorporate the constraint $\mathbf{n}^t \mathbf{n} = 1$ as a Lagrange multiplier γ, this minimum is given by $\mathbf{\Gamma} \mathbf{n} = \gamma \mathbf{n}$: the admissible values of γ are thus the eigenvalues of $\mathbf{\Gamma}$ and the directions minimizing $(\Delta S_{\mathbf{n}})^2$ are the corresponding eigenvectors, which are known as principal components of $\mathbf{\Gamma}$.

The covariance matrix $\mathbf{\Gamma}$ can be made diagonal by an orthogonal matrix $\mathbf{R}$. In this rotated reference frame we have that $\hat{\boldsymbol{\mathcal{S}}} = \mathbf{R}\hat{\mathbf{S}}$ satisfies $\hat{\boldsymbol{\mathcal{S}}}^2 = \hat{\mathbf{S}}^2$, so that

$$(\Delta \mathcal{S}_1)^2 + (\Delta \mathcal{S}_2)^2 + (\Delta \mathcal{S}_3)^2 \leq \langle \hat{\boldsymbol{\mathcal{S}}}^2 \rangle = \langle \hat{\mathbf{S}}^2 \rangle, \qquad (6)$$

where the subscripts 1, 2, and 3 indicate the directions of the orthogonal eigenvectors of $\mathbf{\Gamma}$. The contour surface of these variances defines an ellipsoid that provides an accurate representation of the noise distribution of the state.

Properties and examples.—Let us explore some properties of the degree $\mathbb{P}_2$. Unpolarized states according to $\mathbb{P}_2$ are those whose fluctuations are isotropic and saturate the bound in Eq. (6). This means that the ellipsoid reduces to a sphere of a radius $(\frac{1}{3}\langle \hat{\mathbf{S}}^2 \rangle)^{1/2}$. We note, in passing, that the unpolarized states introduced in Ref. [22] as those invariant under SU(2) transformations are also unpolarized for $\mathbb{P}_2$. However, the converse is not true, in general.

It follows directly from its definition that any SU(2) polarization transformation $\hat{U}$ leaves $\mathbb{P}_2$ invariant: $\mathbb{P}_2(\hat{\varrho}) = \mathbb{P}_2(\hat{U}\,\hat{\varrho}\,\hat{U}^\dagger)$.

It is clear that the moments of any energy-preserving observable (such as $\hat{\mathbf{S}}$) do not depend on the coherences between different subspaces. The only accessible information from any state $\hat{\varrho}$ is thus its polarization sector, which is defined by the block-diagonal form $\hat{\varrho}_{\mathrm{pol}} = \sum_{N=0}^{\infty} \hat{\mathbb{1}}_N \hat{\varrho} \hat{\mathbb{1}}_N$, where $\hat{\mathbb{1}}_N$ is the projector onto the N-photon subspace. Therefore, any $\hat{\varrho}$ and its associated block-diagonal form $\hat{\varrho}_{\mathrm{pol}}$ have the same value of $\mathbb{P}_2$. This is consistent with the fact that polarization and intensity are, in principle, independent concepts: in classical optics the form of the ellipse described by the electric field (polarization) does not depend on its size (intensity). All this confirms that our proposal fulfills all the requirements for a bona fide second-order degree of polarization.

We further develop these ideas by presenting a few relevant examples. First, for any two-mode number state $|n\rangle_H|m\rangle_V$, $\mathbb{P}_2(|n\rangle_H|m\rangle_V) = 1$. In particular, this means that $\mathbb{P}_2$ identifies the hidden polarization of, e.g., the state $|n\rangle_H|n\rangle_V$.

For two-mode quadrature coherent states $|\alpha\rangle_H|\beta\rangle_V$, with an average number of photons $\bar{N} = |\alpha|^2 + |\beta|^2$, simple calculations give $\mathbb{P}_2(|\alpha\rangle_H|\beta\rangle_V) = [\bar{N}/(\bar{N}+3)]^{1/2}$, so when $\bar{N} \to \infty$, $\mathbb{P}_2$ tends to unity, and when $\bar{N} \to 0$, $\mathbb{P}_2$ tends to zero, showing a good classical limit. Interestingly, the covariance for these states is isotropic and the corresponding ellipsoid reduces to a sphere of radius $\bar{N}^{1/2}$.

For single-mode states $|\Psi\rangle_H|0\rangle_V$, we find

$$\mathbb{P}_2(|\Psi\rangle_H|0\rangle_V) = \sqrt{1 - \frac{3\min[(\Delta N)^2, \bar{N}]}{(\Delta N)^2 + \bar{N}(\bar{N}+2)}}, \qquad (7)$$

where $\bar{N}$ is the average number of photons. The problems arising with these states when using $\mathbb{P}_1$ are thus avoided.

We finally consider SU(2) coherent states $|\theta, \phi\rangle = \hat{D}(\theta, \phi)|N, k = 0\rangle$, where $\hat{D}(\theta, \phi) = \exp[\theta/2(\hat{S}_+ e^{-i\phi} - \hat{S}_- e^{i\phi})]$, with $\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y$, is the standard displacement operator on the sphere. These are the only ones that satisfy relation (2) as an equality, so $\mathbb{P}_2(|\theta, \phi\rangle) = 1$ and they are completely polarized for our approach. Incidentally, we have also $\mathbb{P}_1(|\theta, \phi\rangle) = 1$ and one could expect that they would fulfill similar properties for all orders.

Experiment.—We demonstrate our proposal with two different quantum states: a very bright polarization squeezed state and a quadrature squeezed vacuum generated in an optical fiber and in an optical parametric oscillator (OPO), respectively.

To create the bright squeezed light, we employ ultrashort laser pulses in the soliton regime of an optical fiber to achieve a large effective nonlinear Kerr response and avoid dispersive pulse broadening (see Fig. 1) [23]. Our experiment uses a Cr^4+:YAG laser emitting near Fourier-limited 140 fs FWHM pulses at 1497 nm with a repetition rate of 163 MHz. We utilize the two polarization axes of a 13.2 m birefringent fiber (3M FS-PM-7811, 5.6 μm mode-field diameter) to simultaneously generate two independent quadrature squeezed states in the H and V modes, with a relative phase of $\pi/2$. The average output power from the fiber was 13 mW, which, with the bandwidth definition of our quantum state, corresponds to an average number of photons of 10^{11} per 1 μs.

The Stokes measurement is also shown in Fig. 1 and consists of a half-wave plate ($\lambda/2$, θ) followed by a quarter-wave plate ($\lambda/4$, ϕ) and a polarizing beam splitter (PBS). The transformation performed by the wave plates is represented by $\hat{D}(\theta, \phi)$, while the PBS projects on the basis $|N, k\rangle$. The outputs of the PBS are measured using high efficiency photodiodes (98%), the photocurrent difference

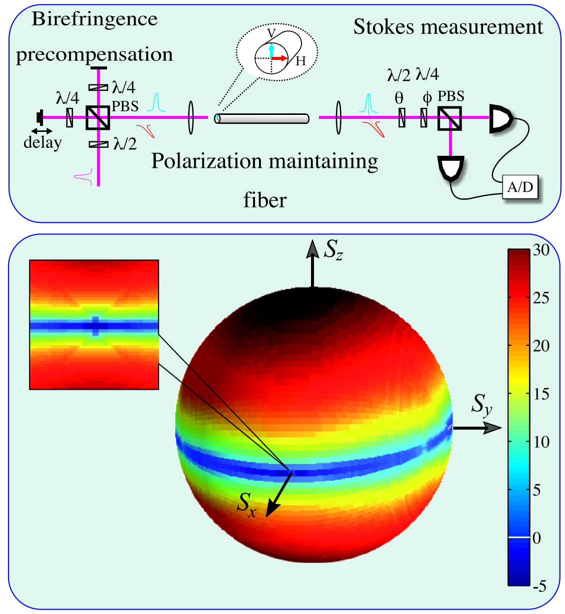

FIG. 1 (color online). (Top) Setup for efficient generation of a polarization squeezed state and the corresponding Stokes measurement apparatus. (Bottom) Measured variances for that state with the indicated scale (in dB noise power relative to the shot noise, marked by a white line). The minimum measured variance is -5.0 ± 0.3 dB. The white point in the S_y axis is the tip of $\langle\hat{\mathbf{S}}\rangle$. We include also a zoom around the S_x axis, near the minimum variance.

is produced, and the resulting fluctuations are evaluated at a sideband of 17.5 MHz (and a bandwidth of 1 MHz). In this way, the setup enables the measurement of $\hat{S}_\mathbf{n}$ [24]. For each pair of angles (θ, ϕ) the Stokes variances $(\Delta S_\mathbf{n})^2$ are obtained and the results are plotted in Fig. 1 as a color map on the sphere.

The minimum-variance determination prescribed by $\mathbb{P}_2$ is also an optimal strategy for polarization-squeezing detection [25]. In this case, it suffices to consider a general Stokes parameter rotated by θ in the dark plane (orthogonal to the direction of $\langle\hat{\mathbf{S}}\rangle$), namely, $\hat{S}_\theta = \cos\theta \hat{S}_x + \sin\theta \hat{S}_z$, so that $\langle\hat{S}_\theta\rangle = 0$. Since for bright fields the fluctuations are small compared with the mean values, one has $(\Delta S_\theta)^2 \simeq \frac{1}{2}\bar{N}[(\Delta X_{H,\theta})^2 + (\Delta X_{V,\theta})^2]$, where $\hat{X}_\theta$ are the rotated quadratures for each polarization mode. The searched point is obtained by optimizing over θ, finding $2° \pm 0.3°$ [26].

From the data we get $\mathbb{P}_1 = 1$ and $\mathbb{P}_2 \simeq 1$ (within the experimental precision). This is simply due to the large excitation of the Stokes vector, which dominates in the definition (5).

In the next experiment, we use a state with a very small excitation, so that the second-order degree is governed by the Stokes fluctuations. We use an OPO operating below threshold and pumped with a 532 nm laser beam (see Fig. 2) [27]. The parametric down-conversion interaction is based on a type I phase-matched periodically poled KTP crystal, which produces a squeezed vacuum in the H mode while leaving the V mode in the vacuum. The resulting

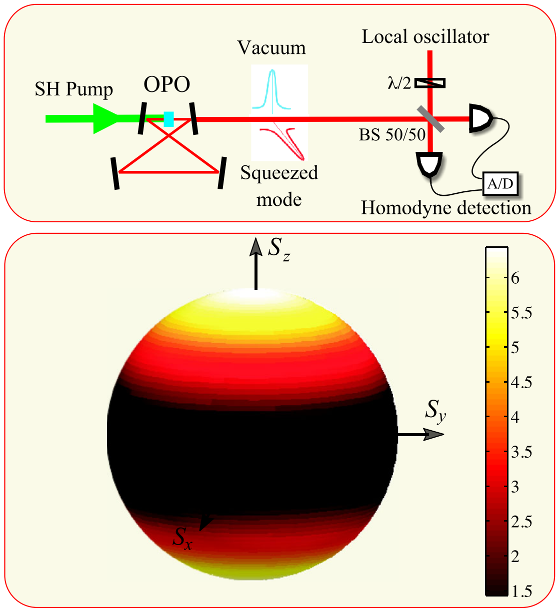

FIG. 2 (color online). (Top) Setup for the generation of a squeezed vacuum in the horizontal mode and vacuum in the vertical one. We measure −3.8 dB (8.6 dB) of quadrature squeezing (antisqueezing) in the horizontal mode. (Bottom) Measured variances $(\Delta S_{\mathbf{n}})^2$ for the state as a color map on the Poincaré sphere with the indicated (linear) scale.

state is then $\hat{\varrho}_H \otimes |0\rangle_{VV}\langle 0|$, where $\hat{\varrho}_H$ is the density operator of the state produced in the OPO.

In contrast to the previous experiment, now we characterize the polarization by using homodyne detection. As this provides complete knowledge about the measured state, the Stokes fluctuations will be contained in the homodyne data. Since the H and V modes are uncorrelated, a complete reconstruction can be obtained just by measuring the two orthogonal modes independently. To this end, we direct our state to a standard homodyne detector where the polarization of the local oscillator could be swapped between H and V polarizations. The measurements results are demodulated at a sideband frequency of 5 MHz with a bandwidth of 100 kHz. The total detection efficiency is about 87%.

Using the time-resolved data as well as the *a priori* state information, we fully reconstruct the density matrix in a 16-dimensional Fock space using a maximum likelihood algorithm. From this density matrix we calculate the moments of the Stokes parameters $\hat{S}_{\mathbf{n}}$ and plot the result as a color map on the Poincaré sphere, as shown in Fig. 2. Now $\bar{N} \simeq 1.5$ and the degree of polarization of the quadrature squeezed vacuum state is not only governed by the first moment (as with the bright squeezed state). This is nicely illustrated in the new definition of the degree of polarization: we get $\mathbb{P}_1 = 0.998 \pm 0.001$ and $\mathbb{P}_2 = 0.79 \pm 0.01$.

Concluding remarks.—The definition (5) has proved to be a satisfactory solution to deal with second-order polarization properties. Of course, a complete characterization must involve a whole hierarchy of degrees $\mathbb{P}_k$ containing all the orders, as it happens with field correlations in coherence theory. Although the second order considered here surely accounts for most of the interesting, and in many cases dominant effects, some subtleties may arise when taking into account higher orders. A full analysis of these questions exceeds the scope of this work and will be presented elsewhere.

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