Optimal Quantum Estimation of the Unruh-Hawking Effect

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We address on general quantum-statistical grounds the problem of optimal detection of the Unruh-Hawking effect. We show that the effect signatures are magnified up to potentially observable levels if the scalar field to be probed has high mean energy from an inertial perspective: The Unruh-Hawking effect acts like an amplification channel. We prove that a field in a Fock inertial state, probed via photon counting by a noninertial detector, realizes the optimal strategy attaining the ultimate sensitivity allowed by quantum mechanics for the observation of the effect. We define the parameter regime in which the effect can be reliably revealed in laboratory experiments, regardless of the specific implementation.

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Introduction.—The Unruh effect [1] is one of the most fundamental manifestations of the fact that the particle content of a field theory is observer-dependent [2]. A quantum (scalar) field in the Minkowski-Unruh vacuum from an inertial perspective is detected as thermal by a Rindler observer in uniform acceleration. This is deeply connected with the phenomenon of Hawking radiation [3]. In the presence of an eternal black hole, if a scalar field is in the Hartle-Hawking vacuum as observed by a Kruskal observer (freely falling into the black hole), a Schwarzschild observer outside the event horizon would detect, again, a thermal state [4]. The temperature T of the Unruh (Hawking) thermal bath depends on the observer's acceleration a (the black hole mass M [5]):

$$T_{\rm Unruh} = \frac{\hbar a}{2\pi c k_B}, \qquad T_{\rm Hawking} = \frac{\hbar c^3}{8\pi G M k_B}, \tag{1}$$

where c is the speed of light, k_B is the Boltzmann constant, and G is the gravitational constant. Despite their crucial role in modern theoretical physics, no experimental verification of the Unruh-Hawking (UH) effects has been accomplished so far, as conventionally the associated temperatures lie far below any observable threshold. There have been different proposals to detect the Unruh effect via accelerated detectors [2], involving electrons in Penning traps, atoms in microwave cavities, backreaction in ultraintense lasers, or Bose-Einstein condensates [7]. Another path pursued in recent research is the detection of Hawking-like effects in "artificial" black holes realized in liquids, optical fibers, or condensed matter systems [8], where a horizon analog (e.g., acoustic or optical) may be created under controllable conditions.

In this Letter, borrowing rigorous methods from quantum statistics and estimation [9], we investigate the ultimate precision limits for the estimation of the UH temperature, and we determine on fully general grounds the optimal conditions for the revelation of UH effects. We are guided by an interesting link between field theory and quantum information: The change of coordinates between an inertial (or freely falling into a black hole) observer—hereby called Alice—and a noninertial (or escaping the fall outside the horizon) observer—hereby called Rob—in the description of the state of a scalar field is equivalent to the transformation that affects a light beam undergoing parametric down-conversion in an optical parametric oscillator (see [4,10]). Detection by an accelerated observer formally amounts to the action of a bosonic amplification channel. We consider the realistic possibility that, in a laboratory implementation, the field mode can be prepared in an arbitrary state from Alice's perspective, beyond the typical Minkowski-Unruh or Hartle-Hawking vacuum. We then devise the most suitable field states and the best detection schemes to be performed by Rob, in order to achieve optimal visibility and sensitivity in measuring the UH temperature. We show that having an increasingly large field energy from Alice's perspective results in a magnification of the UH signatures. We then prove that engineering the field in a Fock inertial state followed by Rob's photon counting allows for the optimal estimation of the UH effect. Alternative valid strategies involve coherent inertial states and Rob's heterodyne detections. Our findings are independent of the specialized implementation, setting a general goal for any experiment striving towards the unambiguous observation of the UH effects.

The Unruh effect as a bosonic amplification channel.—Let us set our notation by focusing, for ease of clarity, on the framework of the Unruh effect in two-dimensional Minkowski spacetime. We consider a scalar field which is, from an inertial perspective, in a special superposition of Minkowski monochromatic modes (see [6,11] for details) such that, in the Unruh basis [4,6,12], Alice detects the field in the single-mode state $|\psi_0\rangle_\omega$. The annihilation operator of the mode satisfies the bosonic commutation relations: $[\hat{a}_\omega, \hat{a}^\dagger_{\omega'}] = \delta_{\omega,\omega'}$. In Rindler coordinates the field is described as an entangled state of two modes, truly monochromatic with frequency ω [6] (from now on we drop the frequency subscript), each living in one of the two

Rindler wedges I (right) and II (left). The Rindler field mode operators $\hat{b}_{\mathrm{I,II}}$ are connected to the Minkowski-Unruh ones via a Bogoliubov transformation [4], $\hat{a} = \cosh r \hat{b}_{\mathrm{I}} - \sinh r \hat{b}_{\mathrm{II}}$, where the "acceleration parameter" r is proportional to the Unruh temperature: $\cosh^{-2} r = 1 - \exp(-\hbar\omega/k_B T)$. A noninertial observer (Rob) in uniform acceleration a is confined to Rindler region I. Thus the equilibrium state from Rob's viewpoint, in the Schrödinger picture, is obtained by tracing over the modes in the causally disconnected region II: $\hat{\varrho}_r = \mathrm{Tr}_{\mathrm{II}}[\hat{U}(r)(\hat{\varrho}_0 \otimes |0\rangle\langle 0|)\hat{U}^\dagger(r)]$. Here $\hat{U}(r) = \exp[r(\hat{b}_{\mathrm{I}}^\dagger \hat{b}_{\mathrm{II}}^\dagger - \hat{b}_{\mathrm{I}}\hat{b}_{\mathrm{II}})]$ is the two-mode squeezing operator that encodes the particle pair production between the two Rindler wedges (or across an eternal black hole horizon), and $\hat{\varrho}_0 \equiv |\psi_0\rangle\langle\psi_0|$. Such a phenomenon has a well known analog in quantum optics [10], which plays a crucial role for continuous variable quantum information [13]. An input signal beam in the state $\hat{\varrho}_0$ interacts with an idler mode (environment) in the vacuum via a two-mode squeezing transformation (realized by parametric down-conversion) with squeezing r. Tracing over the output idler mode, the output signal is left precisely in the mixed state $\hat{\varrho}_r$. Overall, the nonunitary transformation from input to output, or from inertial to noninertial frame, corresponds to the action of a bosonic amplification channel (see also [14]) and can be described by the master equation $d\hat{\varrho}_r/dr = \tanh r \mathcal{L}[\hat{b}_{\mathrm{I}}^\dagger]\hat{\varrho}_r$, where $\mathcal{L}[\hat{b}_{\mathrm{I}}^\dagger]\hat{\varrho}_r = 2\hat{b}_{\mathrm{I}}^\dagger \hat{\varrho}_r \hat{b}_{\mathrm{I}} - \hat{b}_{\mathrm{I}}\hat{b}_{\mathrm{I}}^\dagger \hat{\varrho}_r - \hat{\varrho}_r \hat{b}_{\mathrm{I}}\hat{b}_{\mathrm{I}}^\dagger$. The solution to the master equation, using the disentanglement theorem [10], can be written as

$$\hat{\varrho}_r = N_r \sum_{k=0}^{\infty} C_r^k (\hat{b}_{\mathrm{I}}^\dagger)^k (\cosh r)^{-\hat{b}_{\mathrm{I}}^\dagger \hat{b}_{\mathrm{I}}} \hat{\varrho}_0 (\cosh r)^{-\hat{b}_{\mathrm{I}}\hat{b}_{\mathrm{I}}^\dagger} \hat{b}_{\mathrm{I}}^k, \quad (2)$$

with $N_r = \cosh^{-2} r$ and $C_r^k = (\tanh r)^{2k}/k!$. Equation (2) precisely denotes the state detected by Rob, who is noninertial with acceleration parameter r, corresponding to a field mode state $\hat{\varrho}_0$, with mean photon number (energy) $\bar{n}_0 = \mathrm{Tr}[\hat{\varrho}_0 b_{\mathrm{I}}^\dagger b_{\mathrm{I}}]$, from Alice's inertial perspective.

Optimal estimation of the UH effect.—Suppose the following experiment is repeated N times: The field is prepared in the state $\hat{\varrho}_0$ in the inertial frame, and then a positive-operator-valued measurement (POVM) $\{\hat{O}_\chi\}$ is performed by Rob on the modes in Rindler region I. Here $\langle\phi|\hat{O}_\chi|\phi\rangle \geq 0 \ \forall \ |\phi\rangle$ and $\sum_\chi \hat{O}_\chi = \mathbb{1}$. For each strategy $\mathcal{S} = (\hat{\varrho}_0, \hat{O})$, one can construct an unbiased estimator $\check{r}$ for r, of minimum variance [15] given by $N\mathrm{Var}[\check{r}] = I_r^{-1}(\mathcal{S})$. Here the Fisher information $I_r(\mathcal{S}) = \int d\chi p(\chi|r)(\frac{\partial \ln p(\chi|r)}{\partial r})^2$, with $p(\chi|\theta) = \mathrm{Tr}[\hat{O}_\chi \hat{\varrho}_r]$, is a figure of merit characterizing the performance of the strategy: The higher the Fisher information (FI), the more precise the estimation. At a fixed $\hat{\varrho}_0$, the quantum Cramér-Rao bound [16] states that for any strategy it is $I_r(\mathcal{S}) \leq I(\hat{\varrho}_0, \hat{O}_{\hat{\Lambda}}) \equiv H_r(\hat{\varrho}_0)$; i.e., there exists an optimal POVM yielding maximum sensitivity that consists of projections on the eigenstates of the so-called "symmetric logarithmic derivative" $\hat{\Lambda}_{\hat{\varrho}_r}$, an observable which depends on $\hat{\varrho}_r$ and is defined implicitly as follows: $2d\hat{\varrho}_r/dr = \hat{\Lambda}_{\hat{\varrho}_r}\hat{\varrho}_r + \hat{\varrho}_r\hat{\Lambda}_{\hat{\varrho}_r}$. The FI associated to such optimal measurement is known as quantum FI (QFI), $H_r(\hat{\varrho}_0) = \mathrm{Tr}[\hat{\varrho}_r \hat{\Lambda}_{\hat{\varrho}_r}^2]$. Alternatively, the QFI can be computed from the Bures metric [17], which in turn is related to the quantum fidelity [18] $\mathcal{F}(\hat{\varrho}_1, \hat{\varrho}_2) = (\mathrm{Tr}[\sqrt{\sqrt{\hat{\varrho}_1}\hat{\varrho}_2\sqrt{\hat{\varrho}_1}}])^2$ between two infinitesimally close states (in our case, evolved from the same $\hat{\varrho}_0$): $H_r(\hat{\varrho}_0) = 4[1 - \mathcal{F}(\hat{\varrho}_r, \hat{\varrho}_{r+dr})]/dr^2$. We will now investigate strategies for the estimation of r, involving Gaussian (coherent, squeezed) or non-Gaussian (Fock) field states from Alice's perspective beyond the typical Minkowski-Unruh vacuum, and we will aim for those with the highest possible (quantum) Fisher information [19–22].

Gaussian field states.—Gaussian states, e.g., ground and thermal states of harmonic oscillators, play an important role in quantum optics and many-body physics since they are easy to manipulate mathematically and provide a good description of states commonly produced in experiments. We begin by considering a displaced squeezed pure Gaussian field state $\hat{\varrho}_0^G$ in the inertial frame. It can be completely specified by its first moments $\xi_0 = (q_0, p_0)^T$ and its covariance matrix [13]

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} e^{2s}\cos^2\theta + e^{-2s}\sin^2\theta & \sin 2\theta \sinh 2s \\ \sin 2\theta \sinh 2s & e^{-2s}\cos^2\theta + e^{2s}\sin^2\theta \end{pmatrix}.$$

Here $s > 0$ is the squeezing degree, with phase θ. Under the action of the bosonic amplification channel associated with the UH effect [Eq. (2)], the Gaussian state is transformed into another Gaussian state $\hat{\varrho}_r$ characterized by the following first and second moments [23]: $\xi_r = X_r\xi_0$, $\boldsymbol{\sigma}_r = X_r\sigma^{\mathrm{in}}X_r^T + Y_r$, where $X_r = \cosh r \mathbb{1}_2$ and $Y_r = \sinh^2 r \mathbb{1}_2$. In order to compute the QFI via the Bures metric [24] (see also [22]), we recall that the fidelity between two arbitrary one-mode Gaussian states $\hat{\varrho}_1^G$ and $\hat{\varrho}_2^G$, with respective covariance matrix $\boldsymbol{\sigma}_{1,2}$ and first moments $\xi_{1,2}$, is $\mathcal{F}(\hat{\varrho}_1^G, \rho_2^G) = 2(\sqrt{\Sigma + \Gamma} - \sqrt{\Gamma})^{-1}\exp[-(\xi_2 - \xi_1)^T \times (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)^{-1}(\xi_2 - \xi_1)]$ [25], where $\Sigma = \det(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$ and $\Gamma = (\det\boldsymbol{\sigma}_1 - 1)(\det\boldsymbol{\sigma}_2 - 1)$. In our case, we note that the fidelity $\mathcal{F}(\hat{\varrho}_r^G, \hat{\varrho}_{r+dr}^G)$ is minimized for $\theta = 0$ and $q_0 = 0$. The inertial energy of the Gaussian field becomes $\bar{n}_0 = \sinh^2 s + p_0^2/2$. By setting $\sinh^2 s = x\bar{n}_0$ and $p_0^2 = 2(1 - x)\bar{n}_0$, we introduce an energy ratio x, ranging from 0 (purely coherent inertial state) to 1 (squeezed inertial state) [20]. We finally obtain the QFI: $H_r[\hat{\varrho}_0^G(\bar{n}_0, x)] = \{[2(x - 1)\bar{n}_0]/[\cosh(2r)(x\bar{n}_0 + 1) - \sqrt{x\bar{n}_0}\sqrt{x\bar{n}_0 + 1}]\} + \{16x\bar{n}_0/[\cosh(4r)(x\bar{n}_0 + 1) - x\bar{n}_0 + 3]\} - 2(x - 1)\bar{n}_0 - 2(x - 1)\bar{n}_0\sqrt{x\bar{n}_0/(x\bar{n}_0 + 1)} + 4$. The maximal QFI is obtained by numerical optimization over x for any given $\bar{n}_0$ and r. For very small r, squeezed states from Alice's perspective are optimal ($x = 1$), while for higher r a nonzero displacement improves the estimation. The maximal $H_r(\hat{\varrho}_0^G)$ is a monotonically increasing function of the energy $\bar{n}_0$ measured in the inertial frame (see Fig. 1, middle surface). Therefore, by using suitable engineered Gaussian field modes one can significantly improve the sensitivity in the detection of the UH effect compared to the case of a Minkowski-Unruh vacuum, whose variance $H_r^{-1}[\hat{\varrho}_0^G(0,0)] = 1/4$ is actually the largest. However, this strategy may be hard to implement in practice, as the optimal Rindler measurement depends on r, thus requiring an adaptive estimation scheme [9,20].

We can then consider the following simpler strategy. Let $\hat{\varrho}_0^G \equiv |\alpha_0\rangle\langle\alpha_0|$ be just a coherent state [with $\alpha_0 = (q_0 + ip_0)/\sqrt{2}$]. Rob detects the field in a displaced thermal state with energy $|\alpha_r|^2 = \cosh^2 r|\alpha_0|^2$. Let the POVM $\{\hat{O}_\chi\}$ be a heterodyne detection, i.e., projection onto a set of coherent states, $\{1/\pi|\chi\rangle\langle\chi|\}$. The FI associated to this strategy is $I_r(|\alpha_0\rangle, |\chi\rangle\langle\chi|) = 4(1 + \bar{n}_0/2)\tanh^2 r < H_r[\hat{\varrho}_0^G(\bar{n}_0, 0)]$. Although suboptimal, this strategy still encodes an increase in sensitivity with increasing inertial energy (displacement), as shown in Fig. 1 (bottommost surface). Namely, if $\bar{n}_0 \equiv |\alpha_0|^2 \gg 2/\sinh^2 r$, then the coherent strategy combines practical feasibility with an arbitrarily good improvement over the conventional vacuum case.

Fock field states.—We turn now to explore an estimation strategy involving non-Gaussian field states. Let the state be a Fock state $\hat{\varrho}_0^F = |n_0\rangle\langle n_0|$ from Alice's perspective, for which trivially $\bar{n}_0 = n_0$. From Eq. (2), the state as detected by Rob is $\hat{\varrho}_r^F = \sum_{k=0}^{\infty} c_{n,k}(r)|n_0 + k\rangle\langle n_0 + k|$, with $c_{n,k}(r) = \binom{n+k}{n}(\cosh r)^{-2(n+1)}\tanh^{2k} r$. In this case, to find the optimal measurement strategy, it is more convenient to use the definition of the QFI in terms of the symmetric logarithmic derivative. We find

$$H_r(|\bar{n}_0\rangle) = \sum_{k=0}^{\infty} \frac{[\partial c_{\bar{n}_0,k}(r)/\partial r]^2}{c_{\bar{n}_0,k}(r)} = 4(1 + \bar{n}_0). \tag{3}$$

As shown in Fig. 1 (topmost surface), the QFI for Fock inertial states beats the optimal Gaussian one in the whole parameter space (they coincide only in the limit $r \to 0$), allowing for a significantly reduced variance in the estimation of r at fixed $\bar{n}_0$. The optimal measurement strategy is in this case simply photon counting (independently of the value of r). Thus, by having a field which is prepared in a Fock state with high enough energy from an inertial perspective (in the Unruh basis), one can estimate the UH acceleration parameter—i.e., reveal the effect—with *arbitrarily high sensitivity* by simply letting Rob count photons in the field he detects.

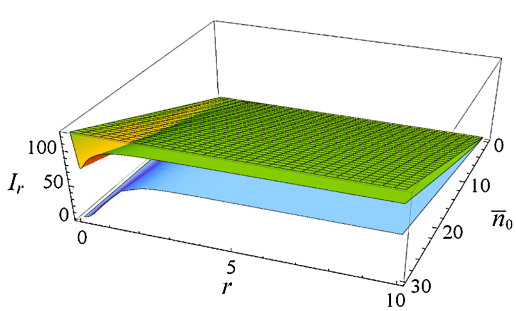

FIG. 1 (color online). Fisher information I_r corresponding, from bottom to top, to coherent states in Alice's frame and heterodyne detection by Rob (blue), general Gaussian states in Alice's frame and optimal detection by Rob (green), and the ultimate quantum bound, attained by Fock states in Alice's frame and photon counting by Rob (wire frame).

Ultimate quantum bound.—We will now prove that no improvement over the above Fock-based quantum estimation strategy can be achieved even by allowing causality violation. Suppose a hypothetical observer existed, able to measure jointly the field in the two spacelike separated Rindler regions, i.e., able to estimate globally the two-mode (unitary) squeezing transformation of coordinates $\hat{U}(r)$, so as to extract an ultimately precise estimator for r without any information loss. The QFI $H_r^{\max}$ in this unphysical setup can be written as $H_r^{\max} = 4(\langle\hat{G}^2\rangle - \langle\hat{G}\rangle^2)$ [9], where $\hat{G} = -i(\hat{b}_{\mathrm{I}}^\dagger\hat{b}_{\mathrm{II}}^\dagger - \hat{b}_{\mathrm{I}}\hat{b}_{\mathrm{II}})$ and the average is taken over the state of the system plus idler (from an inertial perspective), $\hat{\varrho}_0 \otimes |0\rangle\langle 0|$. We have $\langle\hat{G}\rangle = 0$ and $\langle\hat{G}^2\rangle = 1 + \bar{n}_0$. Hence the ultimate quantum bound on the estimation of the UH effect is given by $H_r^{\max} = 4(1 + \bar{n}_0)$, exactly equal to the QFI in Eq. (3). We conclude that, most notably, Fock field states in the inertial frame, in conjunction with photon counting performed by Rob (physically confined to Rindler region I), allow for the absolutely optimal estimation of r: *No advantage* could be achieved even if access to the degrees of freedom in Rindler region II was permitted.

Discussion.—To draw more practical conclusions, it is convenient to derive figures of merit associated with the direct estimation of the UH thermal (amplification) energy $n_T = \sinh^2 r$, which amounts to the mean photon number of a thermal mode with temperature T [Eq. (1)]. By using Eq. (3) and the transformation rule for the FIs [9] $I_{n_T} = I_r(\partial r/\partial n_T)^2$, the minimum variance corresponding to the optimal detection of n_T becomes $N\mathrm{Var}^{\min}[\check{n}_T] = (n_T + n_T^2)/(1 + \bar{n}_0)$. For a single experimental run ($N = 1$), the relative error on the estimation of n_T is defined as $\varepsilon_{n_T} = (\mathrm{Var}^{\min}[\check{n}_T])^{1/2}/n_T = [(1 + n_T^{-1})/(1 + \bar{n}_0)]^{1/2}$. For a precise estimation, it must be $\varepsilon_{n_T} \ll 1$, i.e.,

$$\bar{n}_0 \gg n_T^{-1}. \tag{4}$$

This result defines the regime for optimal sensitivity [26], and we will now link it with a simple assessment of visibility of the UH effect. From Eq. (2) we have, for a generic field state with mean energy $\bar{n}_0$ as detected by Alice, that the mean energy as detected by Rob is $\bar{n}_r = \mathrm{Tr}[\hat{\varrho}_r\hat{b}_{\mathrm{I}}^\dagger\hat{b}_{\mathrm{I}}] = \bar{n}_0 + n_T(\bar{n}_0 + 1)$. To make the UH effect observable, the energy difference (visibility) $\Delta\bar{n} = \bar{n}_r - \bar{n}_0$ must be at least of the order of one photon: $\Delta\bar{n} \gtrsim \max\{1, n_T\}$. Interestingly, the threshold in precision [Eq. (4)] translates in a visibility $\Delta\bar{n} \gg 1 + n_T$, i.e., precisely the regime that renders the effect amenable to detection. We conclude that Eq. (4) sets the ideal threshold on the inertial field energy for any reliable—both accessible and accurate—verification of the UH effect.

Additional fine-tuning is certainly needed to adapt our general prescription to the facilities of specific proposals to

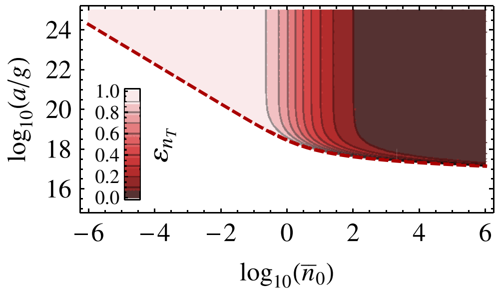

FIG. 2 (color online). Contour plot of the relative error ε_{n_T} on the estimation of the Unruh thermal energy versus the acceleration a of Rindler detectors and the energy $\bar{n}_0$ of a microwave field in an inertial frame (in log scales). The thick dashed line [Eq. (4)] sets the threshold for accessible and precise detection of the Unruh effect.

measure the Unruh effect or to mimic Hawking radiation [7,8]; this lies beyond the scope of this work. Just to have a flavor of the involved orders of magnitude, let us consider the setting of Rindler detectors (e.g., two-level atoms) accelerated in microwave fields ($\omega \approx 10^{10}$ Hz) [2]. For this example, we plot in Fig. 2 the relative error ε_{n_T} in the estimation of n_T versus the acceleration a of the detector and the microwave field energy $\bar{n}_0$ in the inertial frame. While for $\bar{n}_0 = 10^{-6}$ (a quasivacuum field) an acceleration of at least 10^{25} g is needed to detect the Unruh bath, already with $\bar{n}_0 = 1$ photon the threshold [determined by equality in Eq. (4)] drops to a more accessible 10^{18} g: a dramatic magnification of the Unruh effect. The error is further reduced by a statistical factor $\sqrt{N}$ by repeating the experiment N times. These are promising findings in view of the recent progress in the production of Fock states in microwave cavities and circuit QED [27] and in the degree of control of the photon counting technique [28].

Conclusions.—We have proven that the UH effects are magnified when a scalar field is in a state of nonminimal mean energy, e.g., coherent or Fock, from an inertial perspective. Accessible and precise measurement of the UH temperature is enabled by heterodyne detection or photon counting performed by a noninertial observer. Beyond a fundamental interest, our findings are of direct practical relevance, delivering clear-cut prescriptions for the optimal revelation of the UH effects, independent of the specific implementation.

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