Simulating the Wess-Zumino Supersymmetry Model in Optical Lattices

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We study a cold atom-molecule mixture in two-dimensional optical lattices. We show that, by fine-tuning the atomic and molecular interactions, the Wess-Zumino supersymmetry (SUSY) model in 2 + 1 dimensions emerges in the low-energy limit and can be simulated in such mixtures. At zero temperature, SUSY is not spontaneously broken, which implies identical relativistic dispersions of the atom and its superpartner, a bosonic diatom molecule. This defining signature of SUSY can be probed by single-particle spectroscopies. Thermal breaking of SUSY at a finite temperature is accompanied by a thermal Goldstone fermion, i.e., phonino excitation. This and other signatures of broken SUSY can also be probed experimentally.

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Introduction.—Wess and Zumino proposed the first space-time supersymmetry (SUSY) model (WZ-SUSY model) 36 years ago [1]. Since then, SUSY has become a fundamental ingredient of theories beyond the standard model in high-energy physics [2]. However, none of the superpartners of the known elementary particles have been found thus far; it remains to be seen if they can be detected in the energy range of the Large Hadron Collider.

On a different front, *nonrelativistic* SUSY (a Bose-Fermi symmetry unrelated to space-time symmetry) has attracted considerable recent interest in the cold atom community, as it can be realized by using Bose-Fermi atom (molecule) mixtures which are loaded in optical lattices. Examples include attempts to simulate the nonrelativistic limit of a superstring by trapping fermionic atoms in the core of vortices in a Bose-Einstein condensate [3], the study of the SUSY effect in an exactly solvable one-dimensional Bose-Fermi mixture with the Bethe ansatz [4], and SUSY models for nonrelativistic particles in various dimensions [5–7]. In Refs. [6,7], we studied perhaps the simplest cold atom SUSY model and discussed detecting the Goldstino mode by measuring the single-fermion spectral function and the SUSY response. Although these studies are interesting, SUSY in these nonrelativistic systems is very different from the relativistic (or space-time) SUSY in high-energy physics.

In this Letter, we propose a way to simulate the simplest *relativistic* SUSY model, the WZ-SUSY model [1]. We show that it can emerge in the low-energy limit of a cold atom-molecule mixture in properly chosen two-dimensional lattices. The first requirement is the existence of Dirac points in the Brillouin zone. In most proposed models [8], two Dirac points K and K' are related to each other by $K' = -K$. This means that two fermionic atoms which form a usual BCS pair or a diatom molecule belong to two *different* Dirac points. This molecule cannot sever as a Klein-Gordon field which corresponds to a diatom molecule made by two Dirac fermions from the *same* Dirac point. Such molecules carry a $2K \neq 0$ momentum and are energetically unfavorable as a result. In a recent work, Lee attempted to avoid this difficulty by introducing frustrated hopping for the molecules, such that the boson dispersion has minima at $\pm 2K$ instead of zero [9]. It is found that the massless WZ-SUSY model emerges at the boson's superfluid-insulator critical point.

In this work, we show that the WZ-SUSY model can emerge not only at the critical point. We use a lattice model studied recently by Liu *et al.* [10] instead. This is a square lattice model in which the Dirac points at $K = (0, 0)$ and $K' = (0, \pi)$ are their own negatives, as $(0, -\pi) \equiv (0, \pi)$. This means a diatom molecule made of two atoms from the same Dirac points has zero momentum for $2K = (0, 0)$ and $2K' = (0, 2\pi) \equiv (0, 0)$. With this setup, we can simulate the WZ-SUSY model more straightforwardly, after appropriate interactions are introduced and fine-tuned.

The research interest in WZ-SUSY models has been renewed recently [11]. No spontaneous breaking of the SUSY implies that there are equal poles in the single-particle spectral functions of both the Dirac field and the Klein-Gordon field. A further calculation showed that these single-particle spectral functions are not renormalized from their free particle ones [12]. This is the identifier of the SUSY and may be detected by the established techniques of the single-particle spectroscopies [13]. It is known that a thermal bath always breaks the SUSY [14], and this thermal breaking of the SUSY is accompanied by a thermal Goldstone fermion, the phonino [12]; thus, studying this model at a finite temperature sheds light on the physics of SUSY breaking.

There are many studies of SUSY in space-time lattice models [15]. The significant difference between the present work and those lattice SUSY models is that, while the latter are supersymmetric on the lattices, we study the *emergence* of SUSY from a microscopic space lattice (but continuous time) model with no SUSY to begin with.

Free fermion lattice model and continuum limit.—We briefly recall the lattice model proposed in Ref. [10]. Consider a single-component fermionic atom gas loaded in a square lattice. The potential minimum in the sublattice A is higher than that in the sublattice B. Two states with the energy difference $2M$, the s orbital at the A sites and the p orbital at the B sites, form a pseudospin-1/2 subspace. The sublattices are anisotropic with 1, 2, 3, and 4, the next nearest neighbor sites [Fig. 1(b)]. The hoppings between the nearest and next nearest sites are taken into account. The corresponding hopping amplitudes are $t_{A,A+\delta_{x(y)}} = -t_{A,A-\delta_{x(y)}} = t_{AB}$, $t_{A1} = t_{A3}$, $t_{A2} = t_{A4}$, $t_{B1} = t_{B3}$, and $t_{B2} = t_{B4}$ with δ_x (δ_y) being the unit vector in the x (y) direction. In addition, a periodic gauge field generated by two opposite-traveling standing wave laser beams coupling with atoms [16] is introduced. This gives rise to a tunable staggered Peierls phase $\pm\theta_0$ along the vertical links and vanishing in the horizontal and 1, 2, 3, 4 links. With these the single-fermion Hamiltonian is given by

$$H(\mathbf{k}) = p_x(\mathbf{k})\sigma_x + p_y(\mathbf{k})\sigma_y + h_z(\mathbf{k})\sigma_z, \qquad (1)$$

where $p_x = 2t_{AB}\sin\theta_0 \sin(k_y a)$, $p_y = 2t_{AB}[\sin(k_x a) + \cos\theta_0 \sin(k_y a)]$, and $h_z = -M - t_0 \cos(k_x a)\cos(k_y a) - 2\tilde{t}\sin(k_x a)\sin(k_y a)$ with $t_0 = t_{A1} - t_{B1} + t_{A2} - t_{B2}$ and $\tilde{t} = (t_{A1} - t_{B1} + t_{B2} - t_{A2})/2$. When $M = \pm t_0 \neq 0$, there is a unique gapless Dirac point: either $K = (0, 0)$ or $K' = (0, \pi)$. We choose $\theta_0 = \pi/2$ and define the "speed of light" $v_s = 2t_{AB}a$. In the continuum limit and near the Dirac points, $p_x(K + \delta k) \sim 2t_{AB}a\delta k_y \equiv v_s q_x$ and $p_x(K' + \delta k) \sim -2t_{AB}a\delta k_y = -v_s q_x$; $p_y(K + \delta k) \sim 2t_{AB}a\delta k_x \equiv v_s q_y$ and $p_y(K' + \delta k) \sim 2t_{AB}a\delta k_x \equiv v_s q_y$; and $h_z(K + \delta k) = -M - t_0 = m_0$ and $h_z(K' + \delta k) = -M + t_0 = m_\pi$. Thus, for the Dirac fields $\xi(x)$ near K and $\zeta(x)$ near K', the effective Hamiltonian reads

$$H_c^{(0)} = v_s \int d^2x \xi^\dagger(-i\alpha_+^a \partial_a + m_0\sigma_z)\xi + v_s \int d^2x \zeta^\dagger(-i\alpha_-^a \partial_a + m_\pi\sigma_z)\zeta, \qquad (2)$$

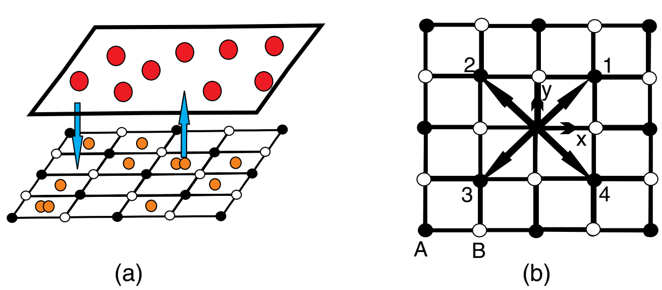

FIG. 1 (color online). (a) Josephson tunneling between the atom-molecule mixture (lower lattice plane) and the dimolecule Bose-Einstein condensate nearby (upper plane). The orange dots are molecules in the mixture, and red dots are dimolecules. Fermionic atoms are in the lattice sites. (b) The square lattice where 1, 2, 3, and 4 denote the next nearest neighbor sites.

where $\alpha_\pm^1 = \pm\sigma^x$ and $\alpha_\pm^2 = \sigma^y$; the two-component Dirac field $\xi(\mathbf{r})$ is given by

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \int_0^\Lambda d^2q e^{i\mathbf{q}\cdot\mathbf{x}} \left[\xi_{1\mathbf{q}} \begin{pmatrix} \theta_\mathbf{q} \\ 1 \end{pmatrix} + \xi_{2,-\mathbf{q}}^\dagger \begin{pmatrix} \theta_\mathbf{q} \\ -1 \end{pmatrix} \right],$$

with $\theta_\mathbf{q} = \frac{q_x + iq_y}{|\mathbf{q}|} = -\theta_{-\mathbf{q}}$ and similarly for ζ. The momentum cutoff Λ corresponds to that when the lattice fermion dispersion deviates severely from the linear one. The mass terms here can be fine-tuned. When $M = \pm t_0$, which is not zero in this lattice setup [10], one of the Dirac field is massless and another is massive. The latter can be integrated out in the low-energy limit. We note that the zero matter density (necessary for Lorentz invariance) in the relativistic quantum field theory corresponds to fermionic atoms being at half filling in this lattice realization. After an external source is introduced, the fermion number (including those forming molecules) will fluctuate but average at half filling. To facilitate pairing or molecule formation, we introduce attraction between fermionic atoms, which is modeled in a two-channel fashion below. For such spinless fermions, the two-atom attraction and p-wave-type bound state have already been achieved experimentally [17].

Two-channel model.—We take $m_0 = 0$ and $m_\pi \neq 0$ and integrate out ζ in the low-energy limit where only the states with their energy lower than $\min\{m_\pi, E_\Lambda\}$ are relevant. We now extend our Hamiltonian to a two-channel model, i.e., the lowest two hyperfine atom states with two-atom scattering states in an open channel and the two-atom bound state (Feshbach molecule) in a closed channel. We denote as $\xi^{(o)}(x)$ the Dirac fermions in the open channel and $\xi^{(c)}(x)$ the Dirac fermion in the closed channel. Analogous to the many-body theory of the atom-molecule coherence in Ref. [18], the effective Lagrangian describing this two-channel Dirac fermion model is given by

$$\mathcal{L} = -\xi^{(o)\dagger}\sigma^\mu\partial_\mu\xi^{(o)} - \xi^{(c)\dagger}\sigma^\mu\partial_\mu\xi^{(c)} + U^{(c)}\xi_2^{(c)\dagger}\xi_1^{(c)\dagger}\xi_1^{(c)}\xi_2^{(c)} + U^{(co)}\xi_2^{(c)\dagger}\xi_1^{(c)\dagger}\xi_1^{(o)}\xi_2^{(o)} + \text{H.c.},$$

where $\sigma_\mu = (I, \sigma_x, \sigma_y)$ and $\partial_\mu = (\partial_t, v_s\nabla)$. $U^{(c)}$ and $U^{(co)}$ are the interaction between closed channel fermions and the interchannel interaction, respectively. We have neglected the background interaction in the open channel. By introducing the pairing field $\Delta(\mathbf{r}, t)$ for $\xi^{(c)}$ via a Hubbard-Stratonovich transformation and integrating out $\xi^{(c)}$, the resulting Lagrangian is given by

$$\mathcal{L}(\xi^{(o)}, \Delta) = -\frac{1}{2}\xi^{(o)\dagger}\sigma^y\sigma^\mu\partial_\mu\xi^{(o)} - \frac{|\Delta|^2}{U^{(c)}} + \mathrm{Tr}\ln G^{(c)-1}. \qquad (3)$$

The inverse of the propagator $G^{(c)}$ of $\xi^{(c)}$ is given by

$$G^{(c)-1} = \begin{pmatrix} 0 & i\sigma^\mu\partial_\mu \\ -i\sigma^\mu\partial_\mu & 0 \end{pmatrix} - \begin{pmatrix} \Xi & 0 \\ 0 & \Xi^\dagger \end{pmatrix},$$

where $\Xi = \Delta + U^{(co)}\xi_1^{(o)}\xi_2^{(o)}$. Expanding the Lagrangian in powers of Δ and its gradients yields

$$\mathcal{L}[\varphi] = -\frac{1}{2}\partial_\mu\varphi^\dagger\partial^\mu\varphi - \frac{1}{2}\varepsilon_m|\varphi|^2 - \frac{\lambda}{8}|\varphi|^4 + O(|\varphi|^6), \tag{4}$$

where $\varphi \propto \Delta/U^{(c)}$ is the Feshbach molecular field with the detuning energy ε_m and the interacting strength $\lambda \propto (U^{(c)})^2$. We have $v_b = v_s$ in the weak coupling limit (i.e., $U^{(c,co)}$ much smaller than all other energy scales in the system including m_π and E_Λ) due to (emergent) Lorentz invariance. Lattice effects (which break Lorentz invariance) give rise to nonuniversal corrections to v_s and v_b; thus, tuning of one parameter (e.g., molecule dispersion through an additional lattice potential seen by the molecule only) is needed to ensure $v_b = v_s$ to maintain Lorentz invariance in the low-energy limit. Also included in (3) is the Yukawa coupling between φ and $\xi^{(o)}$, i.e., $\mathcal{L}_{\varphi\xi} = -\frac{g}{2}(\varphi\xi_2^{(o)\dagger}\xi_1^{(o)\dagger} + \varphi^\dagger\xi_1^{(o)}\xi_2^{(o)})$ with $g \propto -2U^{(co)}$.

WZ-SUSY model: Massless.—For simplicity, we drop the superscript of $\xi^{(o)}$ hereafter. By combining (3) and (4) and the Yukawa coupling together, the effective Lagrangian after neglecting $O(|\varphi|^6)$ is given by

$$\mathcal{L}(\xi,\varphi) = -\frac{1}{2}\partial_\mu\varphi^\dagger\partial^\mu\varphi - \frac{1}{2}\varepsilon_m|\varphi|^2 - i\xi^\dagger\sigma^\mu\partial_\mu\xi - \frac{\lambda}{8}|\varphi|^4 - \frac{g}{2}(\varphi\xi_2^\dagger\xi_1^\dagger + \varphi^\dagger\xi_1\xi_2). \tag{5}$$

By tuning $\varepsilon_m = 0$ by varying $U^{(c)}$, and further tuning pair-pair (or molecule-molecule) interaction by varying $U^{(co)}$ so that the coupling constant $\lambda = g^2$, the effective Lagrangian $\mathcal{L}(\xi,\varphi)$ is exactly the massless WZ-SUSY model with the SUSY under the SUSY transformations $\delta\varphi = \epsilon^\dagger\sigma^y\xi$ and $\delta\xi = \sigma^\mu\sigma^y\epsilon\partial_\mu\varphi^\dagger - \frac{g}{2}\varphi^{\dagger 2}\epsilon$, where ϵ is a constant two-component spinor parameter.

WZ-SUSY model: Massive.—To have a massive WZ-SUSY model, we need to introduce an external source. This can be realized by putting a Bose-Einstein condensate of dimolecules nearby, which is made of pairs of molecules (or 4-atom molecules) [see Fig. 1(a)]. Through Josephson tunneling with an amplitude κ, the dimolecule condensate exchanges pairs of molecules with the mixture. The effective Lagrangian reads

$$\mathcal{L}(\xi,\varphi,\Psi) = -\frac{1}{2}\partial_\mu\varphi^\dagger\partial^\mu\varphi - i\xi^\dagger\sigma^\mu\partial_\mu\xi - \frac{g^2}{8}|\varphi|^4 - \frac{g}{2}(\varphi\xi_2^\dagger\xi_1^\dagger + \varphi^\dagger\xi_1\xi_2) + \kappa(\Psi^\dagger\varphi^2 + \Psi\varphi^{\dagger 2}), \tag{6}$$

where Ψ is the external dimolecular field. There is a global $U(1)$ symmetry (called R symmetry) under $\xi \to e^{i\theta}\xi$, $\varphi \to e^{2i\theta}\varphi$, and $\Psi \to e^{4i\theta}\Psi$ [11]. If Ψ slowly varies in space-time, it is also SUSY invariant under $\delta\varphi = \epsilon^\dagger\sigma^y\xi$ and $\delta\xi = \sigma^\mu\sigma^y\epsilon\partial_\mu\varphi^\dagger - \frac{g}{2}\varphi^{\dagger 2}\epsilon + \frac{4\kappa\Psi^\dagger}{g}\epsilon$. By taking Ψ to be its condensed order parameter $\langle\Psi\rangle = \langle\Psi^\dagger\rangle = m^2/8\kappa$, the R symmetry is broken and reduced to a discrete $\mathbb{Z}_2$ symmetry with $\xi \to i\xi$ and $\varphi \to -\varphi$, and the on-shell WZ Lagrangian appears (up to an additive constant):

$$\mathcal{L}(\xi,\varphi,m) = -\frac{1}{2}\partial_\mu\varphi^\dagger\partial^\mu\varphi - i\xi^\dagger\sigma^\mu\partial_\mu\xi - \frac{g^2}{8}\left(\varphi^{\dagger 2} - \frac{m^2}{g^2}\right)\left(\varphi^2 - \frac{m^2}{g^2}\right) - \frac{g}{2}(\varphi\xi_2^\dagger\xi_1^\dagger + \varphi^\dagger\xi_1\xi_2). \tag{7}$$

The SUSY is exact by replacing $\Psi^\dagger$ with $\langle\Psi^\dagger\rangle$ in the SUSY transformations. The $\mathbb{Z}_2$ symmetry is always spontaneously broken in one of the degenerate ground states with $\varphi = \phi \pm m/g$. The SUSY Lagrangian with spontaneous breaking of $\mathbb{Z}_2$ becomes

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi^\dagger\partial^\mu\phi - i\xi^\dagger\sigma^\mu\partial_\mu\xi \mp \frac{1}{2}m(\xi_2^\dagger\xi_1^\dagger + \xi_1\xi_2) - \frac{g^2}{8}|\phi|^4 - \frac{1}{2}m^2|\phi|^2 \mp \frac{gm}{4}|\phi|^2(\phi+\phi^\dagger) - \frac{g}{2} \times (\phi\xi_2^\dagger\xi_1^\dagger + \phi^\dagger\xi_1\xi_2). \tag{8}$$

This is the (2 + 1)-dimensional reduction of the original WZ-SUSY model in 3 + 1 dimensions [1,2].

Supercurrent and supercharge.—SUSY leads to a conserved supercurrent, whose conserved supercharges are generators of SUSY. The supercurrent is defined by $\delta\int dtd^2x\mathcal{L} = \int dtd^2x\epsilon^\dagger\sigma^y\partial_\mu J_s^\mu$. The supercharges are then given by $Q = \int d^2xJ_s^0(x)$ and $Q^\dagger$. The SUSY transformation generated by Q for a field O reads $\delta O = -i\epsilon^\dagger\sigma^y[Q,O]_\pm$. We focus on the on-shell model with the $\mathbb{Z}_2$ symmetry spontaneously broken, where the on-shell supercurrent [19] is given by $J_s^\mu = i\sigma^\mu\sigma^\nu\xi\partial_\nu\phi + i\frac{g}{2}(\phi^2 \pm 2m\phi/g)\sigma^y\sigma^\mu\xi$. The SUSY spontaneous breaking is signaled by $\langle\{Q,O\}\rangle \neq 0$ for a fermionic operator O. However, for this simplest SUSY model, the SUSY is not spontaneously broken at zero temperature [2].

Nonrenormalization.—In this simplest WZ-SUSY model, single-particle Green's functions are not renormalized. For example, the renormalization to the Klein-Gordon field's propagator in a one-loop self-energy calculation is given by $q^2 - m^2 \to q^2 - m_\phi^2(q)$ with [12] $m_\phi(q) = m + g_R\langle A\rangle_0 + O(g_R^2\langle A\rangle_0^2)$, where $A = \mathrm{Re}\phi$. For 2 + 1 dimensions, due to the nonzero anomalous critical exponents [20], the coupling constant may be renormalized to g_R. However, $\langle A\rangle_0 \propto \langle\{Q,\xi\}\rangle_0 = 0$ because the SUSY is not spontaneously broken. The mass of the Dirac field is also not renormalized as required by SUSY. Therefore, the Green's functions, both of the Dirac and Klein-Gordon fields, are not renormalized from their free version. The spectral functions of the Green's functions can be measured by the single-particle spectroscopic technique which has been developed recently [13]. The nonrenormalization of the Green's functions implies sharp peaks in their spectral functions, with identical *relativistic* dispersions for the atoms and molecules. Experimentally, this would be the hallmark of achieving SUSY.

Thermal breaking of SUSY.—By replacing t by $i\tau$, the imaginary time, the Euclidean version of Lagrangian (8) describes the WZ-SUSY model in finite temperature T. When $T \neq 0$, SUSY is always broken because $\langle\{Q, Q^\dagger \sigma^y\}\rangle_T = \langle \sigma^\mu P_\mu \rangle_T \neq 0$ with P_μ being the energy-momentum operator [14], due to the nonvanishing thermal energy. This SUSY thermal breaking is accompanied by a thermal Goldstone fermion (phonino) but not necessarily by a phonon because the Lorentz symmetry is also broken by $\langle P_0 \rangle_T \neq 0$ [12]. The phonino dispersion is given by [12] $q_0 = \pm v_{ss}|\mathbf{q}|$, where the SUSY sound velocity $v_{ss} = v_s/3$ for $T \gg m$ and $v_{ss} = T v_s/m$ for $T \ll m$.

To detect the phonino mode, one can consider the response to an external "fermionic" field coupled to the supercurrent. The phonino is a pole of the supercurrent-supercurrent correlation function. This external fermionic field can be a combination of an external photon with another hyperfine state of the fermionic atom which is decoupled to the mixture. We have studied this kind of SUSY response theory for a nonrelativistic SUSY mixture [7]. However, in the present case, the supercurrent is not simple, and thus the coupling between the external fermionic field and the supercurrent is not that easy to be experimentally handled.

By replacing $\langle A \rangle_0$ by $\langle A \rangle_T$, the masses are thermally renormalized. The masses of A, B ($\phi = A + iB$), and the spinor ξ have been calculated in low temperature and high temperature limits [12]. Namely, in $2 + 1$ dimensions up to one loop, for $T \ll m$, one has $m_B = m$, $m_A^2 - m_B^2 \propto g^2 m\alpha$, and $m_\xi - m_A \propto g^2 m \alpha$, where $\alpha = \frac{2T}{\pi m} e^{-m/T}$; for $T \gg m$, $m_B = m$, $m_A^2 = m^2 - 2g^2 T$, and $m_\xi^2 = m^2 - g^2 T$. These unequal masses of these fields signal SUSY breaking and can be probed quantitatively. We expect spectroscopy measurements to show double peaks in the molecule spectral function due to the unequal masses, while the atom spectral function has a single peak with a mass of the Dirac field equal to neither m_A nor m_B.

Experimental challenges.—Optical lattices that trap cold atoms can be routinely set up in the laboratory. The staggered Peierls phase originates from production of the artificial magnetic field [16]. As discussed earlier, one needs to tune three parameters to achieve SUSY: atom-atom interaction, molecule-molecule interaction, and molecule velocity. The former two may be done by adjusting the real magnetic field in a Feshbach resonance, while the latter may be related to the interaction between the laser field and the Feshbach molecule. These are all achievable within existing experimental capabilities. Perhaps the biggest challenge is finding the right fermionic atom, which needs to have a highly tunable interaction through a p-wave Feshbach resonance. It also needs to support a sufficiently stable dimolecule state, whose condensate provides the source term in Eq. (6), which gives rise to equal particle masses. Experimentally, one needs to overcome the atom loss due to the heating of the atom gas caused by three- and four-body collisions. Without the last ingredient, however, one can still realize the massless version of the WZ-SUSY model, Eq. (5), which already contains very rich SUSY physics. Despite these and other challenges, we believe simulating the WZ-SUSY model by using cold atom-molecule mixtures is a worthwhile endeavor, as the present model may be generalized to $(3 + 1)$ dimensions, which provides new opportunities to explore the real space-time SUSY physics.

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