Sobnack and Kusmartsev Reply: The idea of the boundary-driven phase transition [1] is based on the fact that single (anti)vortices (SV) may arise near the boundary and this may change the vorticity of the system; however, their penetration inside the sample may be limited. Their lifetime and locations are defined by different physical parameters, such as vicinity to phase transition and size of the superconductor. In mesoscopic superconductors one has to take into account both contributions: (1) the conventional vortex-antivortex (V-A) pairs, which of course will be polarized by the boundary due to the mirror "electric" forces and (2) the contribution due to single (anti)vortices (whose creation energy is smaller than that of a V-A pair: the core energy is twice as small), which arise only from the boundary. Such vortices are preferably located near the boundary, in contrast to V-A pairs whose origin practically does not depend on the sample size. Although they (V-A pairs) are also attracted and polarized by the boundary, the attraction has a dipole-dipole character and is significantly weaker; that is why the boundary cannot significantly change the Berezinskii-Kosterlitz-Thouless (BKT) critical temperature.

In response we have performed calculations including both contributions and we can clearly identify two different fugacities associated with the creation of V-A pairs in bulk and single vortices and antivortices near the boundary. In the framework of the new renormalization group equations, we have studied the critical fixed point and clearly noticed that Williams [2] addressed only V-A pairs. In contrast, in [1] we focused only on the SV generation mechanism associated with the change in the effective boundary position. V-A pairs will not be able to change the effective boundary position due to their weaker interaction with the boundary.

Thus, single vortices could be thermally excited from the boundary and create effectively a new boundary that separates the central still superconducting part of the disk from the peripheral normal region located very close to the original bare boundary. SVs located near the boundary are energetically favorable to V-A pairs since they require half of the core energy compared to V-A pairs. This is the main reason why in the vicinity of the sample boundary there will be created many such S(A)Vs that may destroy the superfluidity or superconductivity in a thin layer close to the boundary. Obviously, such fluctuative vortices have no effect on the bulk superfluid or superconductor separated by a new effective boundary from the thin normal vortex fluctuative layer, whose relative thickness increases as the size of the sample becomes smaller or the temperature rises. Obviously, the temperature and the inverse system size are acting in a similar way. This is actually the main

physical observation that is the basis of the boundary-driven phase transition described in Ref. [1].

In the preceding Comment [2], the idea has been misinterpreted. In [1], we have not estimated finite size corrections to the BKT transition, while Williams [3] talks precisely about that. Such corrections have a minor effect as noticed in [3]. Obviously, these two different mechanisms are associated with two different classes of phase transitions that may arise in mesoscopics: (1) the boundary-driven transformation of the superfluid or superconductor into the normal state due to fluctuative SVs and (2) the transformation due to bulk formation of V-A pairs slightly polarized by the boundaries. Obviously, for bulk macroscopic samples the first mechanism does not work, while the second does.

The second paragraph of the Comment is wrong: the critical point (K^*, y^*) [1] follows directly from Eqs. (5) and (6). This is a highly degenerate critical point, since the final position of effective boundary defined by $R(l)$, or where the scaling should be stopped, is not defined by Eq. (6). Equation (6) defines the critical point but not the length scale; i.e., for any value of superconducting region, $R(l)$, we have a new critical single vortex fugacity, $y^*(l)$. This is the degeneracy, that simply reflects the fact that the effective boundary, $R(l)$, is moving with the scaling, i.e., with l. To stop such a scaling, or find where the effective boundary of the central superconducting region must be stopped, we have to compare the effective size of the central superfluid area, $R(l)$, with the effective correlation length, ζ, i.e., $R(l_c) = \zeta$. Then using Eqs. (7)–(10) we immediately obtain that the critical fugacity $y^*(l_c)$ depends on the disk radius as $y^*(l_c) = 6\pi^2[(R_0/r_c)^{1/2} - 1]^{-2}$, while $K^*(l_c)$ is defined by Eq. (6). When the radius R_0 increases, $y^*(l_c)$ vanishes and becomes the same as in the BKT phase transition $(K^*, y^*) = (3/2\pi, 0)$. These are analytical calculations and do not require numerical simulations.

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