Comment on "Suppression of Superconductivity in Mesoscopic Superconductors"

Sobnack and Kusmartsev [1] have proposed a theory of the two-dimensional superfluid transition in a finite-size superfluid film with hard-wall boundary conditions. They postulate that "single" vortices can be thermally excited, since the presence of the wall gives rise to an image vortex of opposite circulation in the wall. The main result of the theory is a prediction that decreasing the radius R_0 of the film decreases the transition temperature as a power law in R_0. A recent experiment [2] with confined helium films has claimed to at least partially confirm this behavior, though the power-law exponent was considerably larger than predicted in the theory. To check the correctness of the theory we have now numerically evaluated the recursion relations for the superfluid density given in Ref. [1], and find that in fact the transition temperature increases when R_0 is decreased, contradicting the claim of Sobnack and Kusmartsev.

Figure 1 shows the results of iterating the recursion relations [Eqs. 5(a) and 5(b)] of Ref. [1] for two different values of R_0/a_0, where a_0 is the vortex core radius. Since it describes a single vortex, the core energy is taken to be $1/2$ that of the Villain-model core energy of vortex pairs, $E_c/K_0 = \pi^2/4$, where K_0 is the starting dimensionless superfluid density at the smallest length scale a_0. The superfluid fraction is driven to zero at a critical temperature T_c (temperature is proportional to $1/K_0$), and it is readily seen that T_c increases as R_0 decreases, contradicting the claim of Ref. [1]. The source of this behavior can be easily traced to the factor of $R_l = R_0 e^{-l}$ in Eq. 5(a), which increases the loss of superfluid as R_0 increases. The inset of Fig. 1 shows that if the recursion relations are iterated to large length scales, very close to T_c the superfluid density displays a power-law dependence on $T_c - T$, and fits give a superfluid exponent of 0.7674, in agreement with the value $2/(\sqrt{13} - 1)$ calculated in Ref. [1]. Truncating the iteration at the scale R_0 leads instead to a finite-size broadening of the transition.

A problem with the single vortex theory is that it neglects the correlations between opposite-circulation vortices within the superfluid region, the correlations that give rise to the Kosterlitz-Thouless transition. This means that the theory will not correctly describe the transition even in the limit where R_0 becomes large. We have proposed an alternative theory [3] using the vortex-antivortex "pairs" of the Kosterlitz-Thouless theory in the finite superfluid film, but with these now modified by the corresponding vortex and antivortex images in the wall. The presence of the wall in this theory causes the superfluid density to become anisotropic with respect to the wall direction, and to depend spatially on the distance from the wall. Both effects are absent in the theory of Ref. [1].

The experimental results [2] in finite-width channels remain puzzling, and further measurements would be helpful in understanding the nature of the transition. We point out that these experiments only measure the temperature where the superfluid oscillation signal is lost, which is not necessarily the shifted transition temperature, but only the point where the attenuation becomes large. In fact, earlier similar experiments by the same group [4] using thicker films (with less attenuation) showed more than an order of magnitude smaller temperature shift for comparable or even smaller channel widths than in Ref. [2]. It should also be possible to better resolve the theoretical questions here if Monte Carlo simulations could be carried out on planar XY spins in the presence of free boundary conditions, which are a close approximation to the hard-wall boundaries of the experiments.

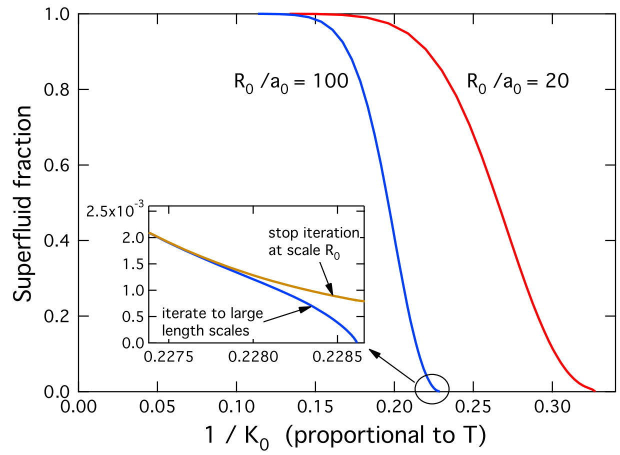

FIG. 1 (color online). 2D superfluid fraction of a film bounded by circular hard wall of radius R_0, computed from the recursion relations of Ref. [1] for two different radii. The inset shows a blowup of the region very close to T_c for the larger radius.

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