Graphene-Based Electronic Spin Lenses

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We theoretically demonstrate the capability of a ferromagnetic-normal interface in graphene to focus an electron wave with a certain spin direction. The essential feature is the negative refraction Klein tunneling, which is spin resolved when the exchange energy of ferromagnetic graphene exceeds its Fermi energy. Exploiting this property, we propose a graphene normal-ferromagnetic-normal electronic spin lens through which an unpolarized electronic beam can be collimated with a finite spin polarization. Our study reveals that magnetic graphene has the potential to be the electronic counterpart of the recently discovered photonic chiral metamaterials that exhibit a negative refractive index for only one direction of the circular polarization of the photon wave.

There exists a close analogy between the propagation of photons inside a photonic crystal and that of electrons in a solid state system as a result of the wave-particle duality in quantum physics. This analogy has been revealed by several counterpart effects in the two progressing fields of photonics and solid state electron optics [1]. Of particular interest in both fields has been the focusing of a beam. In photonics, recent advances in the fabrication of artificial metamaterials has provided the ability to control the electromagnetic wave flow inside matter almost completely. This is not possible in natural materials. In particular, the realization of left-handed metamaterials, which can have a negative refractive index [2,3], has shown exciting technological promises such as perfect lenses [4] and electromagnetic cloaking [5]. On the other hand, significant developments have been made in electron optics through the fabrication of metallic and semiconducting nanostructures in which the ballistic and phase-coherent transport of electrons makes it possible to observe electronic effects with photonic analogues [6]. The idea of using quantum point contacts to focus the electron wave in a two-dimensional electron gas subjected to a magnetic field has already been experimentally achieved [7]. The capability of graphene, a single atomic layer of graphite, to become an electronic metamaterial was predicted recently [8]. It was shown that an interface between electron (n)-doped and hole (p)-doped regions in graphene can focus an electron beam, which may lead to the realization of an electronic Veselago's lens in analogy with the photonic left-handed metamaterials.

Despite the promising achievements in focusing the electron and photon waves, until now little attention has been paid [9,10] to the polarization degree of freedom of the focused beam. The most recent development in photonics is the realization of the so-called chiral metamaterials [11,12] in which the degeneracy between the two circularly polarized waves is broken. A strongly chiral metamaterial may exhibit negative refraction for one circularly polarized beam, while retaining positive refraction for the other. Thus, the interface of such a metamaterial with an ordinary medium will focus only the waves with a certain direction of the circular polarization, which results in a circularly polarized focusing of a linearly polarized incident wave.

In electron optics, however, the question of the possibility of *spin-polarized* focusing of an electron wave has remained unanswered. The aim of this Letter is to address this question by introducing a model based on magnetic graphene. We show that a weakly doped ferromagnetic (FM) graphene can be the electronic counterpart of photonic chiral metamaterials, in the sense that it can be used for focusing electrons with a certain spin direction. Based on this finding, we propose an electronic spin lens, shown schematically in Fig. 1, through which an unpolarized incident electron wave can be focused into an image point with a finite spin polarization. The spin polarization of the
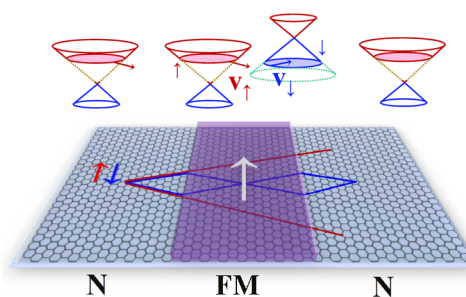

FIG. 1 (color online). Schematic drawing of the ferromagnetic (FM) graphene spin lens in a normal (N) sheet. The configuration of the two spin subbands (being n type or p type) is also shown, when the electrostatic potential is set to $U_F = 0$, $U_N = h$ in FM and N regions, respectively. The electrons are injected from a nonmagnetic point source inside the left N region. A spin-up electron beam (outer straight lines) diverges, but a spin-down electron beam (inner broken lines) undergoes negative refractions at the FN interfaces and focuses in the right N region at the image point of the source.

image is directed antiparallel to the magnetization vector of FM. Such a possibility for the realization of a focused spin accumulation with a tunable direction can also be of great interest in the field of spintronics [13].

The potential of graphene to be used for electron focusing is suggested by its unique zero-gap semiconducting electronic band structure [14–16]. Its conically shaped conduction and valence bands touch each other at the corners of the hexagonal first Brillouin zone, known as Dirac points. The carrier type and its density can be tuned by means of electrical gates or by doping the underlying substrates. At low energies, the quasiparticles are described by the massless Dirac Hamiltonian, $\hat{H}_D = v_F \mathbf{p} \cdot \hat{\boldsymbol{\sigma}}$, with Fermi velocity v_F, momentum $\mathbf{p}$, and Pauli matrices $\hat{\boldsymbol{\sigma}} = (\sigma_x, \sigma_y)$ defined in pseudospin space to characterize the two trigonal sublattices of the hexagonal structure of graphene. The linear dispersion, together with the pseudospin aspect, gives the carriers a pseudorelativistic chiral nature with electrons and holes having different chiralities $\mathbf{p} \cdot \hat{\boldsymbol{\sigma}}/p = \pm 1$. An important manifestation of the chirality is the so-called Klein tunneling, which is a negative refraction process through a p-n junction [17–19].

An interesting consequence of the specific band structure described above is that, in a ferromagnetic graphene with an exchange potential exceeding its Fermi energy, the Fermi level for the spin-up and the spin-down carriers lies in the conduction and the valence spin subbands, respectively [20,21]. This means that the opposite-spin carriers are of different types, electronlike and holelike, and, hence, have opposite chiralities. We show that the interface between such *spin-chiral* materials and nonmagnetic graphene (with the same type of carriers in the two spin subbands) exhibits negative refraction for electrons with a certain spin direction, while retaining positive refraction for electrons with an opposite spin direction. We demonstrate that spin resolving the sign of the electronic refractive index in this manner can lead to the realization of a graphene normal-ferromagnetic-normal (NFN) spin lens.

Our model consists of a spin-chiral FM stripe of width w inside an N graphene sheet as shown in Fig. 1. Such a FM region can be produced by using, in part, an insulating ferromagnetic substrate. Alternatively, FM metals or added magnetic impurities on top of a graphene sheet can induce an exchange potential [22,23]. Intrinsic ferromagnetic correlations are also predicted to exist in graphene sheets [20] and nanoribbons with zigzag edges [24] under certain conditions. To study the focusing effect, we use the single-electron Green's function method. The Hamiltonian for a spin-$s(=\pm)$ electron in one of the valleys is given by

$$\hat{H}_s^0 = \hat{H}_D - sh(\mathbf{r}) - U(\mathbf{r}), \qquad (1)$$

where $h(\mathbf{r})$ and $U(\mathbf{r})$ are the exchange and the electrostatic potential, respectively, and are functions of the 2D position vector $\mathbf{r}$. We model a nonmagnetic electronic point source

at the position $\mathbf{r}_0 \equiv (x_0, y_0)$ in the left N region as the perturbation potential $\hat{V}_s = \hat{V}_0 \delta(\mathbf{r} - \mathbf{r}_0)$, with strength $\hat{V}_0$. The total Hamiltonian then becomes $\hat{H}_s = \hat{H}_s^0 + \hat{V}_s$. The local density of states (LDOS) of spin-s electrons can be calculated using the relation $n_s(\varepsilon, \mathbf{r}) = -(1/\pi) \operatorname{Im} \operatorname{Tr} \hat{G}_s(\mathbf{r}|\mathbf{r})$ in which the retarded Green's function is defined as

$$\hat{G}_s(\mathbf{r}|\mathbf{r}') = \lim_{\eta \to 0^+} \langle \mathbf{r}|(\varepsilon + i\eta - \hat{H}_s)^{-1}|\mathbf{r}'\rangle, \qquad (2)$$

with Tr denoting the trace over the space of the pseudospin. Using the Dyson expansion, the change of the LDOS induced by the perturbation up to the first order in $\hat{V}_0$ can be calculated from the equation

$$\delta n_s(\mathbf{r}) = -\frac{1}{\pi} \operatorname{Im} \operatorname{Tr}[\hat{G}_s^0(\mathbf{r}|\mathbf{r}_0)\hat{V}_0\hat{G}_s^0(\mathbf{r}_0|\mathbf{r})], \qquad (3)$$

in which the unperturbed Green's function G_s^0 satisfies the relation $[\varepsilon - \hat{H}_s^0(\mathbf{r})]\hat{G}_s^0(\mathbf{r}|\mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$.

A voltage V applied to the source point can induce a current in the left N region, which we found to be spin polarized. The current density for spin-s electrons is obtained from

$$\mathbf{i}_s(\mathbf{r})/eV = ev_F \operatorname{Im} \operatorname{Tr}[\hat{\boldsymbol{\sigma}}\hat{G}_s^0(\mathbf{r}|\mathbf{r}_0)i\Gamma\hat{G}_s^{0\dagger}(\mathbf{r}_0|\mathbf{r})], \qquad (4)$$

where the level broadening function Γ is the measure of the tunneling rate between the source lead and the N graphene sheet.

Assuming that the potentials U and h vary only along the x direction, we can use the Fourier transformation $\hat{G}_s^0(\mathbf{r}|\mathbf{r}') = \int dp_y \exp[ip_y(y - y')]\hat{g}_{s,p_y}(x|x')$. The new Green's function $\hat{g}$ satisfies the one-dimensional evolution-like equation (as a function of the position x instead of the time),

$$[iv_F \partial_x - \hat{\mathcal{L}}_{s,p_y}(x)]\hat{g}_{s,p_y}(x|x') = \sigma_x \delta(x - x'), \qquad (5)$$

with a non-Hermitian Hamiltonian $\hat{\mathcal{L}}_{s,p_y}(x) = -[U(x) + sh(x) + \varepsilon]\hat{\sigma}_x + iv_F p_y \hat{\sigma}_z$. In principle, Eq. (5) together with Eqs. (3) and (4) can be solved numerically to obtain the spin-resolved variations of the LDOS and the current density for the given profiles of $h(\mathbf{r})$ and $U(\mathbf{r})$.

Before we proceed with the full quantum mechanical calculation, we may apply the adiabatic approximation to the non-Hermitian Hamiltonian $\hat{\mathcal{L}}$ in Eq. (5), which is valid when the variation of the potentials is slow on the scale of the Fermi wavelength in N and FM. In this way the semiclassical expression of $\hat{G}_s^0$ is obtained, from which we deduce that the semiclassical trajectory of a spin-s electron in N and FM regions consists of straight lines given by the relations

$$y - y_0 + x_0 \tan\theta_N$$

$$= \begin{cases} x\tan\theta_N & x < x_L \\ (x - x_L)\tan\theta_{Fs} + x_L\tan\theta_N & x_L < x < x_R \\ (x - w)\tan\theta_N + w\tan\theta_{Fs} & x > x_R. \end{cases} \quad (6)$$

Here $\theta_N = \arcsin(p_y/\mu_N)$ and $\theta_{Fs} = \arcsin(p_y/\mu_{Fs})$ are the propagation angles (measured from the normal to FN interfaces) and μ_{Fs} and μ_N are the electrochemical potentials for a spin-s electron inside FM and N regions, respectively; $x_{L(R)}$ indicates the locations of the left (right) interface.

From the relations (6) we find that the focusing can occur for spin-down $s = -$ ($h > 0$) electrons provided that μ_{F-} and μ_N have opposite signs. In this case, the angle θ_{F-} undergoes a change of sign at both FN interfaces, indicating that the NFN structure operates as a *spin n-p-n* structure for spin-down electrons. Equation (6) also gives the location of the two focuses inside FM and the right N region as $x_F - x_L = (x_0 - x_L)\tan\theta_N/\tan\theta_{F-}$ and $x_N - x_0 = w(1 - \tan\theta_{F-}/\tan\theta_N)$, respectively. We note that, in general, the location of the focal point depends on the transverse momentum p_y of the incident electron, which could lead to the appearance of many focusing points. This problem can be solved if we set a symmetric spin p-n potential profile at the interfaces by having $\mu_N = -\mu_{F-}$, which results in a unique, profound focus at $x_F - x_L = -(x_0 - x_L)$ and $x_N - x_0 = 2w$. We note that even with a symmetric profile at the interfaces, only the electrons close to the Fermi level are focused effectively, which shows the effectiveness of the focusing at low temperatures. From Eq. (6) we have estimated that at a finite temperature T the focal point will spread along the x direction over a length of the order $(k_B T/\mu_N)L$, where L is the distance of the source from the left FN interface (k_B is the Boltzmann constant). To have a profound spin lensing, this length should be much smaller than L, which gives a rough estimate of $T < \mu_N/k_B$. With $\mu_N \sim 10$–100 meV in graphene sheets, a temperature lower than 100 K is thus required.

On the other hand, to have a significant spin polarization at the focal points, the spin-up electrons have to remain unfocused. This is achieved by assigning the same sign to both μ_{F+} and μ_N, which means that the spin-up electrons remain at the same subband (valence or conduction) throughout the whole structure. Let us consider two special cases of $\mu_{F+} = \mu_N$ and $\mu_{F+} = 0$. In the first case, a spin-up electron does not feel any potential change and thus propagates divergently through the system. In the second case, the density of states of spin-up electrons vanishes in FM. This implies that a spin-up electron cannot propagate into the FM region, but rather tunnels through evanescent modes, which have a small contribution to the variation of LDOS. These two specific cases correspond to the potential sets of $U_F = 0$, $U_N = h$ and $U_F = -h$, $U_N = 2h$, respectively.

Figure 2 shows the result of our quantum calculation for the spin LDOS, defined as $\delta n_+ - \delta n_-$, inside the right N region when a point nonmagnetic perturbation is located in the left N region and for the two sets of potentials (a) $U_F = 0$, $U_N = h$ and (b) $U_F = -h$, $U_N = 2h$ described above. The distribution of the amplitude of the spin-current density, $|\mathbf{i}_+| - |\mathbf{i}_-|$, is also shown when a voltage V is applied to the source point. We have assumed that the potential varies abruptly at the FN interfaces. The spin LDOS shows a peak at the image point of the perturbation with Friedel-like oscillations whose period is of the order of the Fermi wavelength $\lambda_N = \hbar/\mu_N$ in N. The difference between the two sets of potential is visible at points far from the image point. We note that focusing electrons by a NFN graphene creates a mirage that replicates LDOS oscillations which, unlike the original perturbation, are spin polarized and mimic the effect of a magnetic perturbation at the image point. Thus, the NFN structure produces a magnetic image from a nonmagnetic point source.

We have also investigated the effect of the smooth variation of the potential at the interfaces on spin lensing. The result is shown in Fig. 3, in which the spin LDOS [Fig. 3(a)] and the distribution of the amplitude of the spin current-density [Fig. 3(b)] are plotted for the potential set of Fig. 2(b), but for a finite thickness of the interfaces $\Delta x = \lambda_N$, over which $U(x)$ varies linearly from $U_F = -h$ to $U_N = 2h$. Compared to the case of sharp interfaces, the peaks of the spin LDOS and of the amplitude of the spin-current density are broadened. Therefore, introducing a smooth variation of the potential at the interfaces leads to the weakening of spin lensing. We note that the potential variation length is restricted in graphene because of the screening effect [25]. This, together with the low carrier densities and large Fermi wavelengths of graphene, make it
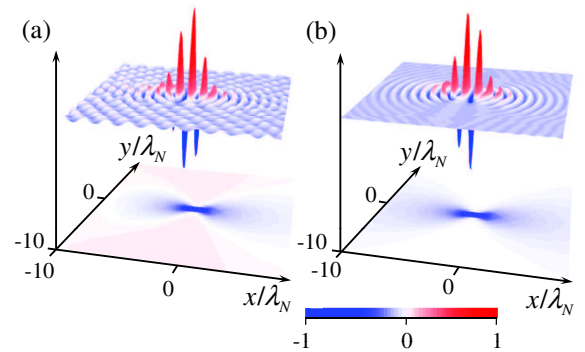

FIG. 2 (color online).   Spin local density of states (upper plots) and the amplitude of the spin-current density (lower plots) around the image point in the left N region (both in arbitrary units), when a nonmagnetic point source is located in the right N region. The potential is set to (a) $U_F = 0$, $U_N = h$ and (b) $U_F = -h$, $U_N = 2h$. Both quantities have peaks around the image point of the source, with spin LDOS showing Friedel-like oscillations around its peak.
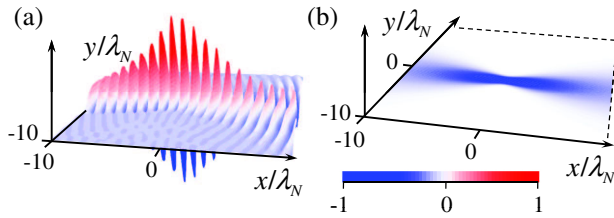
FIG. 3 (color online).   Effect of the smooth variation of the potential at FN interfaces on the spin image of Fig. 2(b). The potential is assumed to vary linearly across the interfaces from $U_F = -h$ to $U_N = 2h$ over a distance $\Delta x = \lambda_N$. Compared to the abrupt interfaces, the peaks of spin LDOS (a) and of the amplitude of the spin-current density (b) are broadened.

possible to envisage contacts smaller than the Fermi wavelength. Spin lensing can be observed experimentally by spin-polarized scanning tunneling microscopy [26] of the N graphene region around the focal point, which can image the variation of the spin LDOS with a resolution of the order of a few nm.

Regarding the validity of the independent valleys model [16] of the Hamiltonian (1), it is well known [27] that unlike a *p-n* contact in graphene nanoribbons with zigzag edges for which the intrinsic intervalley mixing is strong no matter how smooth the potential variation might be, in the wide contact geometry of our model, the intervalley scattering becomes effective only for very abrupt contacts of length on the order of an atomic lattice constant $a \sim 1$ Å. In graphene, this length scale is much shorter than λ_N, which is typically a few hundred nm. We have found that spin lensing is effective for a contact of lengths up to λ_N. Thus, our assumption that the intervalley mixing is negligible is well justified for contacts with a length smaller than λ_N, but much larger than a.

In conclusion, we have proposed a solid-state electronic spin lens based on a ferromagnetic graphene which has an exchange potential higher than its Fermi energy. The key property is that an interface between such a spin-chiral FM and a N graphene region exhibits an electronic refractive index which has different signs for electrons with different spin directions. We have shown that in a corresponding NFN structure, a pointlike nonmagnetic source in one N region produces an image in the other N region which is a point spin accumulation with associated Friedel-like oscillations of spin LDOS.

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