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The availability of the double-differential charged-current neutrino cross section, measured by the MiniBooNE Collaboration by using a carbon target, allows for a systematic comparison of nuclear effects in quasielastic electron and neutrino scattering. The results of our study, based on the impulse approximation scheme and a state-of-the-art model of the nuclear spectral functions, suggest that the electron cross section and the flux averaged neutrino cross sections, corresponding to the same target and comparable kinematical conditions, cannot be described within the same theoretical approach using the value of the nucleon axial mass obtained from deuterium measurements. We analyze the assumptions underlying the treatment of electron-scattering data and argue that the description of neutrino data will require a new paradigm, suitable for application to processes in which the lepton kinematics is not fully determined.

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The data set of charged-current quasielastic (CCQE) events recently released by the MiniBooNE Collaboration [\[1\]](#page-3-0) provides an unprecedented opportunity to carry out a systematic study of the double-differential cross section of the process,

$$
\nu_{\mu} + {}^{12}\text{C} \rightarrow \mu^{-} + X,\tag{1}
$$

averaged over the neutrino flux. Comparison between the results of theoretical calculations and data may provide valuable new information on nuclear effects, whose quantitative understanding is critical to the analysis of neutrino oscillation experiments, as well as on the elementary interaction vertex.

The charged-current elastic neutrino-nucleon process is described in terms of three form factors. The proton (p) and neutron (*n*) vector form factors $F_1^{p,n}(Q^2)$ and $F_2^{p,n}(Q^2)$ $(Q^2 = -q^2, q$ being the four-momentum transfer) have been precisely measured up to large values of Q^2 in electron-proton and electron-deuteron scattering experiments, respectively (for a recent review, see, e.g., Ref. [\[2](#page-3-1)]). The Q^2 dependence of the nucleon axial form factor $F_A(Q^2)$, whose value at $Q^2 = 0$ can be extracted from neutron β -decay measurements, is generally assumed to be of dipole form and parametrized in terms of the so-called axial mass M_A :

$$
F_A(Q^2) = g_A(1 + Q^2/M_A^2)^{-2}.
$$
 (2)

The world average of the measured values of the axial mass, mainly obtained from low statistics experiments carried out by using deuterium targets, turns out to be $M_A = 1.03 \pm 0.02$ GeV [3-[5](#page-3-3)], while the analyses performed by the K2K [[6](#page-3-4)] and MiniBooNE [\[7\]](#page-3-5) Collaborations using oxygen and carbon targets, respectively, yield $M_A \sim 1.2{\text -}1.35$ GeV.

It would be tempting to interpret the large value of M_A reported by MiniBooNE and K2K as an effective axial mass, modified by nuclear effects not included in the Fermi gas model employed in data analysis. However, most existing models of nuclear effects (for recent reviews, see Ref. [\[8\]](#page-3-6)) fail to support this explanation, suggested by the authors of Ref. [\[7\]](#page-3-5), a prominent exception being the model of Ref. [\[9](#page-3-7)].

Obviously, a fully quantitative description of the electron-scattering cross section, driven by the known vector form factors, is a prerequisite for the understanding of the axial vector contribution to the CCQE neutrinonucleus cross section.

Over the past two decades, the availability of a large body of experimental data has triggered the development of advanced theoretical descriptions of the nuclear electromagnetic response. The underlying scheme, based on nuclear many-body theory and realistic nuclear Hamiltonians, relies on the premises that (i) the lepton kinematics is fully determined and (ii) the elementary interaction vertex can be extracted from measured proton and deuteron cross sections.

The above paradigm has been successfully applied to explain the electron-nucleus cross section in a variety of kinematical regimes (for a recent review of the quasielastic sector, see Ref. [[10\]](#page-3-8)). However, in view of the uncertainties associated with the energy of the incoming beam, the identification of the reaction mechanisms, and the determination of the interaction vertex, its extension to the case of neutrino scattering may not be straightforward.

In this work we compare theoretical results obtained from the approach described in Refs. [\[11](#page-3-9)[,12\]](#page-3-10) to the measured CCQE cross sections of Ref. [\[1\]](#page-3-0), discuss the differences involved in the analyses of electron and

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neutrino-nucleus scattering, and argue that modeling neutrino interactions may require a paradigm shift.

Electron-nucleus scattering cross sections are usually analyzed at fixed beam energy E_e and electron-scattering angle θ_e as a function of the electron energy loss ω . As an example, Fig. [1](#page-1-0) shows the double-differential cross section of the process

$$
e + {}^{12}\text{C} \to e' + X,\tag{3}
$$

at $E_e = 730 \text{ MeV}$ and $\theta_e = 37^\circ$, measured at Massachusetts Institute of Technology–Bates [[13](#page-3-11)]. The peak corresponding to quasielastic (QE) scattering and the bump at larger ω , associated with excitation of the Δ resonance, are clearly visible and well separated. The three-momentum transfer $|q|$ turns out to be nearly constant, its variation over the range shown in the figure being $\leq 5\%$. As a consequence, the cross section of Fig. [1](#page-1-0) can be readily related to the linear response of the target nucleus to a probe delivering momentum q and energy ω , defined as

$$
S(\mathbf{q}, \omega) = \sum_{n} \left| \langle n | \sum_{\mathbf{k}} a_{\mathbf{k}+\mathbf{q}}^{\dagger} a_{\mathbf{k}} | 0 \rangle \right|^{2} \delta(\omega + E_{0} - E_{n}). \tag{4}
$$

In the above equation, $|0\rangle$ and $|n\rangle$ represent the target initial and final states, with energies E_0 and E_n , respectively, while $a_{\mathbf{k}+\mathbf{q}}^{\dagger}$ and $a_{\mathbf{k}}$ are the nucleon creation and annihilation operators, respectively.

The magnitude of the momentum transfer, $|q| \sim$ 450 MeV, is large enough to make the impulse approximation (IA) scheme, in which Eq. (4) (4) (4) reduces to $[14]$

$$
S_{\text{IA}}(\mathbf{q},\,\omega) = \int d^3k dE P_h(\mathbf{k},E) P_p(\mathbf{k}+\mathbf{q},\,\omega-E), \quad (5)
$$

safely applicable [\[15\]](#page-3-13). In Eq. [\(5](#page-1-2)), $P_h(\mathbf{k}, E)$ and $P_p(\mathbf{k} +$ $\mathbf{q}, \omega - E$ are the spectral functions describing the energy and momentum distributions of the struck nucleon in the initial (hole) and final (particle) states, respectively.

FIG. 1 (color online). Inclusive electron-carbon cross section at beam energy $E_e = 730 \text{ MeV}$ and electron-scattering angle $\theta_e = 37^\circ$, plotted as a function of the energy loss ω . The data points are taken from Ref. [\[13\]](#page-3-11).

The solid line in Fig. [1](#page-1-0) represents the results of a theoretical calculation of the QE contribution, carried out within the IA by using the hole spectral function of Ref. [[16](#page-3-14)] and the recent parametrization of the vector form factors of Ref. [[4](#page-3-15)]. Final state interactions between the struck nucleon and the recoiling spectator system [[11\]](#page-3-9), whose main effect is a \sim 10 MeV shift of the QE peak, have been also taken into account.

It is apparent that height, position, and width of the QE peak, mostly driven by the energy and momentum dependence of the hole spectral function, are well reproduced.

Applying the same scheme employed to obtain the solid line in Fig. [1](#page-1-0) to neutrino scattering, one gets the results shown in Fig. [2.](#page-1-3) The data points represent the doubledifferential CCQE cross section averaged over the MiniBooNE neutrino flux, whose mean energy is $\langle E_{\nu} \rangle =$ 788 MeV, plotted as a function of the kinetic energy of the outgoing muon at different values of the muon scattering angle. The solid lines show the results (integrated over the $\cos\theta_{\mu}$ bins) obtained by using the same spectral functions and vector form factors employed in the calculation of the electron-scattering cross section of Fig. [1,](#page-1-0) and a dipole parametrization of the axial form factor with $M_A =$ 1:03 GeV. Comparison of Figs. [1](#page-1-0) and [2](#page-1-3) indicates that the electron and neutrino cross sections corresponding to the

FIG. 2 (color online). Flux averaged double-differential CCQE cross section measured by the MiniBooNE Collaboration [[1\]](#page-3-0), shown as a function of kinetic energy of the outgoing muon. The upper and lower panels correspond to different values of the muon scattering angle. Theoretical results have been obtained by using the same spectral functions and vector form factors employed in the calculation of the electron-scattering cross section in Fig. [1](#page-1-0), and a dipole parametrizaition of the axial form factor with $M_A = 1.03$ GeV.

same target and comparable kinematical conditions (the position of the QE peak in Fig. [1](#page-1-0) corresponds to kinetic energy of the scattered electron $~610~MeV$) cannot be explained by using the same theoretical approach and the value of the axial mass resulting from deuterium measurements. In this instance, the paradigm of electron scattering appears to conspicuously fail.

Note that the comparative analysis of electron and neutrino data, based on double-differential cross sections depending on measured kinematical variables, is made possible for the first time by the availability of the data set of Ref. [[1\]](#page-3-0).

The results of a global comparison between the MiniBooNE data and the calculated cross sections show that theory sizably underestimates the measured cross section for any values of muon energy and scattering angle.

It has to be emphasized that the above conclusion, while being based on a calculation carried out within the scheme of Refs. [[11](#page-3-9),[12](#page-3-10)], is largely model-independent. Theoretical approaches providing a quantitative description of the electron-nucleus cross section in the QE channel are bound to predict CCQE neutrino-nucleus cross sections significantly below the MiniBooNE data if the value of the axial mass is set to 1.03 GeV.

In spite of the fact that the large value of M_A reported by the MiniBooNE Collaboration was first obtained from a shape analysis of the Q^2 distribution, the effect of the axial mass on the CCQE cross section can be best analyzed by studying the flux averaged muon kinetic energy spectrum and angular distribution, obtained by integrating the double-differential cross section over $\cos\theta_{\mu}$ and T_{μ} , respectively. These quantities depend only on the measured muon kinematical variables, thus being unaffected by the assumptions associated with the reconstruction of the incoming neutrino energy E_{ν} [\[15,](#page-3-13)[17\]](#page-3-16), entering the definition of the reconstructed Q^2 .

The upper panel in Fig. [3](#page-2-0) shows a comparison between the MiniBooNE flux averaged muon kinetic energy spectrum and the results of our calculations, corresponding to three different values of M_A . The behavior of the curve corresponding to $M_A = 1.03$ GeV is consistent with that shown in Fig. [2](#page-1-3), as the data turn out to be largely underestimated. Increasing M_A to 1.35 GeV, the value resulting from the MiniBooNE analysis of Ref. [\[1\]](#page-3-0), while improving the agreement between theory and experiment, still does not lead to reproduce the data at $T_{\mu} \lesssim 1$ GeV. The dotdashed curve has been obtained by using the value M_A = 1.6 GeV, yielding the best χ^2 fit to the MiniBooNE flux averaged Q^2 distribution within our approach.

The M_A dependence of the flux averaged muon angular distribution is shown in the bottom panel in Fig. [3,](#page-2-0) together with the data of Ref. [\[1\]](#page-3-0). The overall picture is clearly the same as in the upper panel.

In Fig. [4](#page-3-17), we compare the results of our calculations to the MiniBooNE flux unfolded total cross section. It is apparent that in this case using $M_A = 1.6$ GeV leads to

FIG. 3 (color online). Upper panel: Flux averaged muon kinetic energy spectrum. The dot-dashed, solid, and dashed lines have been obtained by setting the value of the axial mass to $M_A = 1.03$, 1.35, and 1.6 GeV, respectively. The data are taken from Ref. [\[1\]](#page-3-0). Bottom panel: The same as the upper panel, but for the flux averaged muon angular distribution.

overestimating the data in the region of high energy $(E_{\nu} >$ 800 MeV), where the choice $M_A = 1.35$ GeV provides a better fit. The different pattern emerging from Fig. [4](#page-3-17), compared to Fig. [3](#page-2-0), clearly points to the uncertainty associated with the interpretation of flux averaged and flux unfolded data.

The results of our work indicate that the theoretical approach based on the IA and realistic spectral functions, successfully applied to QE electron scattering, fails to reproduce the CCQE neutrino-nucleus cross section, unless the value of the nucleon axial mass resulting from deuteron measurements is significantly increased. In addition, the possibility of interpreting the large M_A resulting from the K2K and MiniBooNE analyses as an effective axial mass, modified by nuclear effects beyond the Fermi gas model, appears to be ruled out [\[17\]](#page-3-16). This statement should be regarded as largely model-independent, as calculations carried out by using different descriptions of nuclear dynamics yield similar results [\[18\]](#page-3-18).

A different scenario is suggested by the results of Ref. [\[9\]](#page-3-7), whose authors obtain a quantitative account of the MiniBooNE flux unfolded total cross section without *increasing* M_A . Within the model of Ref. [[9\]](#page-3-7), the main mechanism responsible for the enhancement that brings the theoretical cross section into agreement with the data is multinucleon knockout, leading to n particle- n hole $(np-nh)$ nuclear final states $(n = 2, 3, ...)$. Within the

FIG. 4 (color online). Flux unfolded total CCQE cross section, as a function of neutrino energy. The dot-dashed, solid, and dashed lines have been obtained by setting the value of the axial mass to $M_A = 1.03$, 1.35, and 1.6 GeV, respectively. The data are taken from Ref. [\[1](#page-3-0)].

approach employed in our work, the occurrence of $2p-2h$ final states is described by the continuum part of the spectral function, arising from nucleon-nucleon correla-tions [[16](#page-3-14)]. It gives rise to the tail extending to large ω , clearly visible in Fig. [1.](#page-1-0) However, its contribution turns out to be quite small (less than 10% of the integrated spectrum). The analysis of the momentum distribution sum rule indicates that the contributions of $np-nh$ final states with $n \geq 3$ are negligibly small [\[19\]](#page-3-19).

According to the philosophy outlined in this Letter, in order to firmly establish the role of multinucleon knockout in CCQE neutrino interactions, the model of Ref. [\[9\]](#page-3-7) should be thoroughly tested against electron-scattering data.

In our opinion, the available theoretical and experimental information suggests that the main difference involved in the analysis of neutrino-nucleus scattering, as compared to electron-nucleus scattering, lies in the flux average.

Unlike the electron cross section shown in Fig. [1,](#page-1-0) the flux averaged CCQE neutrino cross section at fixed energy and scattering angle of the outgoing lepton picks up contributions from different kinematical regions, where different reaction mechanisms dominate. As a consequence, it cannot be described according to the paradigm successfully applied to electron scattering, based on the tenet that the lepton kinematics is fully determined.

A new paradigm, suitable for studies of neutrino interactions, should be based on a more flexible model of nuclear effects, providing a consistent description of the broad kinematical range corresponding to the relevant neutrino energies.

Besides single- and multinucleon knockout, such a model should include the contributions of processes involving the nuclear two-body currents, which are known to provide a significant enhancement of the electromagnetic nuclear response in the transverse channel [[20](#page-3-20)]. The occurrence of processes leading to pion production and excitation of nucleon resonances should also be taken into account.

A great deal of information could be obtained by applying the new paradigm to the analysis of inclusive neutrino-nucleus cross sections, preferably, although not necessarily, through direct implementation of the resulting nuclear model in the Monte Carlo simulation codes. This kind of analysis may help to reconcile the different values of the axial mass obtained from different experiments and would be largely unaffected by the problems associated with the possible misidentification of CCQE events, recently discussed in Ref. [\[21\]](#page-3-21).

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- [1] A. A. Aguilar-Arevalo et al. (MiniBooNE Collaboration), Phys. Rev. D 81[, 092005 \(2010\).](http://dx.doi.org/10.1103/PhysRevD.81.092005)
- [2] C. F. Perdrisat, V. Punjabi, and M. Vanderhaeghen, [Prog.](http://dx.doi.org/10.1016/j.ppnp.2007.05.001) [Part. Nucl. Phys.](http://dx.doi.org/10.1016/j.ppnp.2007.05.001) 59, 694 (2007).
- [3] V. Bernard et al., J. Phys. G **28**[, R1 \(2002\)](http://dx.doi.org/10.1088/0954-3899/28/1/201).
- [4] A. Bodek, S. Avvakumov, R. Bradford, and H. Budd, [Eur.](http://dx.doi.org/10.1140/epjc/s10052-007-0491-4) Phys. J. C 53[, 349 \(2008\)](http://dx.doi.org/10.1140/epjc/s10052-007-0491-4).
- [5] V. Lyubushkin et al. (NOMAD Collaboration), [Eur. Phys.](http://dx.doi.org/10.1140/epjc/s10052-009-1113-0) J. C 63[, 355 \(2009\).](http://dx.doi.org/10.1140/epjc/s10052-009-1113-0)
- [6] R. Gran et al. (K2K Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.74.052002) 74, [052002 \(2006\).](http://dx.doi.org/10.1103/PhysRevD.74.052002)
- [7] A. A. Aguilar Arevalo et al. (MiniBooNE Collaboration), Phys. Rev. Lett. 100[, 032301 \(2008\).](http://dx.doi.org/10.1103/PhysRevLett.100.032301)
- [8] Proceedings of the Sixth International Workshop on Neutrino-Nucleus Interactions in the Few-GeV Region (NUINT-09), edited by F. Sanchez, M. Sorel, and L. Alvarez-Ruso, AIP Conf. Proc. No. 1189 (AIP, New York, 2010).
- [9] M. Martini, M. Ericson, G. Chanfray, and J. Marteau, Phys. Rev. C 80[, 065501 \(2009\)](http://dx.doi.org/10.1103/PhysRevC.80.065501); 81[, 045502 \(2010\).](http://dx.doi.org/10.1103/PhysRevC.81.045502)
- [10] O. Benhar, D. Day, and I. Sick, [Rev. Mod. Phys.](http://dx.doi.org/10.1103/RevModPhys.80.189) 80, 189 [\(2008\)](http://dx.doi.org/10.1103/RevModPhys.80.189).
- [11] O. Benhar, N. Farina, H. Nakamura, M. Sakuda, and R. Seki, Phys. Rev. D 72[, 053005 \(2005\)](http://dx.doi.org/10.1103/PhysRevD.72.053005).
- [12] O. Benhar and D. Meloni, Nucl. Phys. A789[, 379 \(2007\).](http://dx.doi.org/10.1016/j.nuclphysa.2007.02.015)
- [13] J. S. O'Connell et al., Phys. Rev. C 35[, 1063 \(1987\)](http://dx.doi.org/10.1103/PhysRevC.35.1063).
- [14] O. Benhar, [AIP Conf. Proc.](http://dx.doi.org/10.1063/1.2834460) **967**, 111 (2007).
- [15] A. M. Ankowski, O. Benhar, and N. Farina, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.82.013002) 82[, 013002 \(2010\)](http://dx.doi.org/10.1103/PhysRevD.82.013002).
- [16] O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, [Nucl.](http://dx.doi.org/10.1016/0375-9474(94)90920-2) Phys. A579[, 493 \(1994\)](http://dx.doi.org/10.1016/0375-9474(94)90920-2).
- [17] O. Benhar and D. Meloni, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.80.073003) 80, 073003 [\(2009\)](http://dx.doi.org/10.1103/PhysRevD.80.073003).
- [18] A. V. Butkevich, [arXiv:1006.1595v1](http://arXiv.org/abs/1006.1595v1).
- [19] O. Benhar, A. Fabrocini, and S. Fantoni, [Nucl. Phys.](http://dx.doi.org/10.1016/0375-9474(89)90374-6) A505[, 267 \(1989\).](http://dx.doi.org/10.1016/0375-9474(89)90374-6)
- [20] J. Carlson and R. Schiavilla, [Rev. Mod. Phys.](http://dx.doi.org/10.1103/RevModPhys.70.743) **70**, 743 [\(1998\)](http://dx.doi.org/10.1103/RevModPhys.70.743).
- [21] T. Leitner and U. Mosel, Phys. Rev. C 81[, 064614 \(2010\).](http://dx.doi.org/10.1103/PhysRevC.81.064614)